JOINT ADAPTIVE TRANSMISSION AND COMBINING WITH OPTIMIZED RATE AND POWER ALLOCATION

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ABSTRACT
We consider the problem of finding optimal transmission rates and power allocation under the framework of diversity combining. Capitalizing on recent results for both link adaptation schemes and adaptive combining, we design and analyze two joint link adaptation and diversity combining schemes. Based on the channel fading, the proposed schemes adaptively select both the signal constellation and diversity combiner structure. We show that the novel schemes provide significant throughput gains compared to existing joint adaptive QAM and diversity schemes. Further, contrary to previous results on power control for discrete rate link adaptation, power control does not give significant average spectral efficiency gains in this jointly adaptive setting. However, power control yields significant outage probability gains over the constant power schemes.

1. INTRODUCTION
The need for ever higher spectrum efficiency motivates the search for optimization of the wireless resources. Resource management in wireless communications is a difficult task due to user mobility and highly time-variant propagation environments, thus implying adaptive solutions. Key adaptive techniques are those of adaptive modulation [1,2], power control [3, 4], and adaptive combining [5–8].

In [5, 6] new jointly adaptive modulation and combining schemes based on a multiple threshold idea are introduced. [5, 6] evaluate the proposed schemes by adopting a constant-power variable-rate uncoded $M$-QAM scheme. Given a target bit error rate (BER), the adaptive modulator thresholds are then predetermined. Since the thresholds are predetermined, the scheme will not be able to fully take advantage of the time-varying nature of the fading channel, in terms of spectral and power efficiency. It is then natural to seek optimal thresholds based on the fading statistics. Further, by utilizing the framework of [4] power control can be introduced.

There is a rich literature on diversity combining schemes, e.g., [7–9]. In Generalized Selection Combining (GSC), the receiver will combine a fixed number of the resolvable paths with the highest signal-to-noise ratio (SNR), reducing the complexity relative to the optimal maximum ratio combining (MRC) scheme. However, both the GSC and MRC combining schemes always combine the maximum number of allowed branches, even if the combined SNR of less branches would satisfy the transmission requirements. Proposed in [8], the minimum selection GSC (MS-GSC) attempts to solve this by combining the least number of highest SNR branches such that the combined SNR is above a given threshold.

In this paper we tackle the problem maximizing the average spectral efficiency (ASE) by finding optimal transmission rates and power control schemes for adaptive diversity combining over Rayleigh fading channels. We show that the proposed schemes offer large ASE gains over the previous QAM schemes discussed in [5, 6], by better taking advantage of the time-varying nature of the wireless channel. Introducing power control gives a significant reduction in outage probability, however, as opposed to previous results on power control for discrete rate link adaptation, power control does not give significant ASE gains in this jointly adaptive setting [4].

The remainder of the present paper is organized as follows. In Section 2, we introduce the diversity model and review results on QAM-based adaptation. Optimal transmission schemes are derived and analyzed in Section 3. Numerical examples and plots are presented in Section 4. Finally, conclusions are given in Section 5.

2. DIVERSITY SYSTEM AND QAM ADAPTATION

2.1. GSC Diversity
We assume a generic diversity system with $L$ available diversity branches, as shown in Fig. 1. The received signal on each diversity branch is assumed to experience independent identically distributed (iid) Rayleigh fading. Due to hardware complexity and power considerations, a maximum of $L_c$ branches can be combined at the receiver side ($L_c \leq L$).

By estimating and ranking the $L$ instantaneous branch SNRs in descending order, i.e., $\gamma_1 > \gamma_2 > \cdots > \gamma_L$. 

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then GSC will combine the \( L_c \) best branches only, using MRC weights, and giving a combined SNR of \( \gamma_{\text{GSC}} = \sum_{l=1}^{L_c} \gamma_l \).

Let \( \gamma \) denote the common average SNR on each branch. Then, the probability density function of the received SNR at the output of a GSC combiner is given by [9, Eq. (16)]:

\[
\begin{align*}
&f_{\gamma_{\text{GSC}}}^{(L/L_c)}(\gamma_c) = \left( \frac{L_c}{L} \right) \left[ \frac{\gamma_c L_c - 1}{\gamma_c L_c (L_c - 1)!} \right] \frac{1}{\gamma_c} \\
&\quad \times \sum_{l=1}^{L-L_c} (-1)^{L_c+l-1} \left( \frac{L}{L_c} L_c \right) \left( \frac{L_c}{l} \right) \frac{1}{\gamma_c} L_c - 1
&\quad \times \left( e^{-\gamma_c \gamma / L} - \frac{1}{m!} \left( -\frac{\gamma_c \gamma}{L} \right)^m \right). \\
\end{align*}
\]

(1)

Now, by integrating (1) we find the cumulative density function of the received SNR as [9, Eq. (24)]

\[
\begin{align*}
&F_{\gamma_{\text{GSC}}}^{(L/L_c)}(\gamma_c) = \left( \frac{L_c}{L} \right) \left[ 1 - e^{-\gamma_c / \gamma} \sum_{l=0}^{L_c-1} \left( \frac{\gamma_c l}{l!} \right) \right] \\
&\quad + \sum_{l=1}^{L-L_c} (-1)^{L_c+l-1} \left( \frac{L}{L_c} L_c \right) \left( \frac{L_c}{l} \right) \frac{1}{\gamma_c} L_c - 1
&\quad \times \frac{1}{1 + \frac{\gamma_c}{L_c}} \frac{1}{m!} \left( -\frac{\gamma_c \gamma}{L_c} \right)^m
&\quad \times \left( 1 - e^{-\gamma_c \gamma / L} \sum_{k=0}^{m} \left( \frac{\gamma_c k}{k!} \right) \right)\].
\]

(2)

The average number of combined branches will quantify the processing power consumed by the diversity combining [5]. We denote by \( B_c \) and \( \gamma_{T_{1,1}} \) the number of combined branches and the minimum required combined SNR for transmission, respectively. Then, \( L_c \) branches are combined when \( \gamma_{\text{GSC}} \geq \gamma_{T_{1,1}} \), and 0 branches otherwise, i.e., the system gives up on the channel and buffers the data. The average number of GSC-combined branches is then given as

\[
\mathcal{B}_c = L_c (1 - f_{\gamma_{\text{GSC}}}^{(L/L_c)}(\gamma_{T_{1,1}})).
\]

(3)

In Section 3, to simplify the notation we write \( F_{\gamma_{\text{GSC}}}^{(L/L_c)}(\cdot) \) and \( f_{\gamma_{\text{GSC}}}^{(L/L_c)}(\cdot) \) as \( F_{\gamma_c}(\cdot) \) and \( f_{\gamma_c}(\cdot) \), respectively.

2.2. M-QAM Based Link Adaptation

As the baseline case we consider the constant power \( M \)-ary QAM scheme studied in [5, 6]. This scheme, denoted by \( \eta_N \), consists of \( N \) signal constellations with spectral efficiencies \( M = 2^n \), where \( 1 \leq n \leq N \). Given a target bit error rate \( \text{BER}_0 \), the switching threshold corresponding to constellation \( n \) is given as follows [6, Eq. (4)]

\[
\gamma_{T_{n,1}} = -\frac{2}{3} \ln(5 \text{BER}_0)(2^n - 1).
\]

(4)

2.3. Bandwidth Efficient Scheme with MS-GSC

For MS-GSC, the minimum number of branches is combined with MRC weights, such that the output of the combiner is larger than or equal to a given threshold \( \gamma_T \). Thus, if \( B_c \) denotes the number of branches selected out of \( L \) available branches, then \( B_c \) is the minimum number \( \in \{1, 2, \cdots, L_c\} \), for which \( \sum_{l=1}^{B_c} \gamma_l L \geq \gamma_T \) is satisfied [7].

Our objective is to design bandwidth efficient schemes, so to maximize the spectral efficiency we employ the bandwidth efficient version of the link adaptation and diversity schemes presented in [5]. As a consequence, the receiver tries to combine a minimum number of branches to support the highest transmission rate. When MS-GSC is combined with link adaptation in a bandwidth efficient mode, its spectral efficiency is equal to that of GSC combining [6], but at the same time MS-GSC offers a reduction in the average number of combined branches, and thus processing power savings. Therefore, we derive the optimal switching thresholds and power levels based on the GSC receiver and evaluate the power savings by utilizing these values in a MS-GSC system.

For MS-GSC, \( \mathcal{B}_c \) can be found by generalizing [5, Eq.(14)], in which \( \mathcal{B}_c \) is derived only for the case of \( L_c = L \). To obtain \( \mathcal{B}_c \) for \( L_c \leq L \), we modify \( [5, \text{Eq. (14)}] \), yielding

\[
\mathcal{B}_c = 1 + \sum_{l=1}^{L-L_c-1} F_{\gamma_{\text{GSC}}}^{(L/L_c)}(\gamma_{T_{N,1}}) - L_c F_{\gamma_{\text{GSC}}}^{(L/L_c)}(\gamma_{T_{1,1}}).
\]

(6)

3. OPTIMAL SCHEMES FOR GSC-BASED SYSTEMS

3.1. System Model

We consider a wireless channel with additive white gaussian noise (AWGN) and fading. Under the assumption of slow, frequency-flat fading, we may use a block-fading model to approximate the wireless fading channel by an AWGN channel

\(1\)The upper summation of [5, Eq. (14)] limit is changed from \( L - 1 \) to \( L_c - 1 \), and the combining scheme in the last term is changed from \( L \) branch MRC to \( L_c \) GSC.
within the length of a codeword [10]. Thus, the system may use codes which typically guarantee a certain target spectral efficiency within a range of SNRs on an AWGN channel.

We denote the instantaneous pre-adaptation combined SNR by $\gamma_c[i]$. This is the SNR that would be experienced using signal constellations of average power $S$ without power control [2]. Assuming, as in [1], that the transmitter receives perfect channel predictions, sent over a zero-error feedback channel, we can adapt the transmit power instantaneously at time $i$ according to a power adaptation scheme $S(\gamma_c[i])$, as shown in Fig. 1. The received post-adaptation SNR at time $i$ is then given by $\gamma_c[i] = S(\gamma_c[i])$. By virtue of a stationarity assumption, the distribution of $\gamma_c[i]$ is independent of $i$, and is denoted by $f_{\gamma_c}(\gamma)$. To simplify the notation we omit the time reference $i$ from now on.

The transmission scheme is to be based on a set of $N$ codes, each associated with $K$ power levels. The choice of code and power level is at any time based on the fading channel state. Following [1, 4], we partition the range of the combined SNR $\gamma_c$ into $NK + 1$ pre-adaptation regions, which are defined by the switching thresholds $\{\gamma_{T_n,k}\}$, as illustrated in Fig. 2. Code $n$, with spectral efficiency $R_n$, is selected whenever $\gamma_c$ is in the interval $[\gamma_{T_n,k}, \gamma_{T_{n+1,k}})$. Within this interval the transmission rate is constant, but the system can adapt the transmitted power to the channel conditions in order to maximize the average spectral efficiency. When $\gamma_c < \gamma_{T_{n,1}}$ data is buffered, and the system suffers an outage. For convenience, we let $\gamma_{T_{n,1}} = 0$ and $\gamma_{T_{N+1,1}} = \infty$.

### 3.2. Spectral Efficiency Analysis

Using $N$ distinct codes we analyze the obtainable spectral efficiencies of the $L/L_r$-GSC schemes, under discrete and constant transmit power adaptation. We shall assume that the fading is so slow that Shannon capacity-achieving codes for AWGN channels can be employed $^2$.

Now, recall that the pre-adaptation combined SNR range is divided into regions lower bounded by $\gamma_{T_{n,1}}$, $1 \leq n \leq N$. Thus, we let $R_n = C_n$, where $C_n = \log_2(1 + S(\gamma_{T_{n,1}})/\gamma_{T_{n,1}})$ is the highest spectral efficiency that can be supported within the range $[\gamma_{T_{n,1}}, \gamma_{T_{n+1,1}})$ for $1 \leq n \leq N$, after transmit power adaptation [4]. Each SNR region’s contribution to the ASE of the scheme is the spectral efficiency of the $n$th code,

\[ \psi_{N \times K} = \sum_{n=1}^{N} \log_2(1 + \beta_n) \int_{\gamma_{T_{n,1}}}^{\gamma_{T_{n+1,1}}} f_{\gamma_c}(\gamma) \, d\gamma, \]

such that

\[ \sum_{n=1}^{N} \beta_n \sum_{k=1}^{K} \frac{1}{\gamma_{T_{n,k}}} \int_{\gamma_{T_{n,k}}}^{\gamma_{T_{n,k+1}}} f_{\gamma_c}(\gamma) \, d\gamma \leq 1, \]

where $\gamma_{T_{n,K+1}} \triangleq \gamma_{T_{n+1,1}}$. The solution is found by using constrained numerical optimization, cf. [4, Appendix].

### 3.3. Discrete-Power Transmission Scheme

For practical scenarios the resolution of power control will be limited, e.g., for the Universal Mobile Telecommunications System (UMTS) power control step sizes on the order of 1 dB are proposed [11]. Further, continuous power control is not feasible as it would require an infinite capacity feedback channel. We thus analyze discrete power adaptation, by considering the $\psi_{N \times K}$ scheme where we allow for $K \geq 1$ power regions within each of the $N$ rate regions.

The optimal discrete-level power control was shown in [4] to be discretized piecewise channel inversion. Now, by defining $^3\beta_n \triangleq \frac{S(\gamma_{T_{n,1}})}{S(\gamma_{T_{n+1,1}})} \gamma_{T_{n,1}}$, the ASE maximization problem can be formulated as follows [4]. Find $\{\beta_n, \gamma_{T_{n,k}}\}$ to maximize

\[ \psi_{N \times K} = \sum_{n=1}^{N} \log_2(1 + \beta_n) \int_{\gamma_{T_{n,1}}}^{\gamma_{T_{n+1,1}}} f_{\gamma_c}(\gamma) \, d\gamma, \]

and times the probability $P_n = \int_{\gamma_{T_{n,1}}}^{\gamma_{T_{n+1,1}}} f_{\gamma_c}(\gamma) \, d\gamma$ that it is employed. An upper bound $\psi$ on the ASE —for a given set of codes switching levels—is therefore given as

\[ \psi = \sum_{n=1}^{N} \log_2\left(1 + \frac{S(\gamma_{T_{n,1}})}{S} \gamma_{T_{n,1}}\right) \int_{\gamma_{T_{n,1}}}^{\gamma_{T_{n+1,1}}} f_{\gamma_c}(\gamma) \, d\gamma, \]

subject to the average power constraint,

\[ \sum_{n=1}^{N} \int_{\gamma_{T_{n,1}}}^{\gamma_{T_{n+1,1}}} S(\gamma_c) f_{\gamma_c}(\gamma) \, d\gamma \leq S, \]

where $S$ denotes the average transmit power.

If arbitrarily long codewords can be used, the bound can be approached from below with arbitrary precision for an arbitrarily low BER. Our goal is now to find optimal switching levels and power adaptation schemes in order to maximize the ASE in a GSC diversity environment.

### 3.4. Constant-Power Transmission Scheme

When a single transmission power is used for all codes, the term constant power transmission scheme is used. From [4]...

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$^2$The existence of such codes is guaranteed by Shannon’s channel coding theorem. However, we do not address the important problem of constructing such codes, which is a research problem in itself.

$^3$\beta_n$ corresponds to the minimum post-adaptation combined SNR in the region $[\gamma_{T_{n,1}}, \gamma_{T_{n+1,1}}]$. 

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Fig. 2. The pre-adaptation SNR range is partitioned into regions where $\gamma_{T_{n,k}}$ are the switching thresholds.
with increasing efficiency gains are possible by clever system design. and the optimized schemes indicates that significant spectral efficiency gains over the constant power transmission schemes, contrary to the findings for a SISO link in [4]. Then it is clear from the figure that optimal power adaptation seems to yield insignificant gains over the constant power transmission schemes, contrary to the findings for the case of variable-rate continuous-power transmission scheme [4, Section III-A]. Then it is clear from the figure that optimal power adaptation seems to yield insignificant gains over the constant power transmission schemes, contrary to the findings for a SISO link in [4].

Second, from 0 \leq \bar{\gamma} \text{ dB} \leq 10, the four \psi schemes give an increase in spectral efficiency of approximately 2 bps/Hz over the QAM scheme. As \bar{\gamma} increases the QAM scheme saturates at 4 bps/Hz, corresponding to utilizing 16-QAM with a probability close to 1. The large gap between the QAM and the optimized schemes indicates that significant spectral efficiency gains are possible by clever system design.

4.2. Probability of No Transmission

When \gamma_c is less than the smallest signal constellation threshold \gamma_{T_{1}}, no data is sent. The probability of no transmission \text{P}_{\text{out}} can then be calculated as

\[
P_{\text{out}} = \Pr(\gamma_c < \gamma_{T_{1}}) = \Phi_{\gamma_{T_{1}}}(\gamma_c).
\]

(12)

In Fig. 4 we have plotted \text{P}_{\text{out}} as a function of \bar{\gamma}. As expected from Table 1, for the \psi schemes the probability of no transmission decreases both as power adaptation is introduced, as well as with increasing \bar{\gamma}. For the QAM scheme \text{P}_{\text{out}} decreases much faster than for the \psi schemes, which is due to the fact that \gamma_{T_{1}} for the QAM scheme is fixed and independent of \bar{\gamma}, whereas for the other schemes it is adapted to the underlying fading distribution for every value of \bar{\gamma}.

It is not necessarily a disadvantage that the probability of no transmission is high. For data-centric services, the most important thing from a quality-of-service point of view is probably the total time of data downloading experienced by a user. For large data sets, this time will be minimized independently of the value \text{P}_{\text{out}}, as long as the average spectral efficiency is maximized. However, for multimedia services with delay requirements, the probability of no transmission can be important. Thus it is interesting to see that \text{P}_{\text{out}} is considerably reduced when power adaptation is used.

4.3. Processing Power

For terminals with limited battery, receiver processing power is of high importance. By keeping fewer branches active, the
power consumption in the receiver can be reduced [5]. Fig. 5 shows the average number of combined branches as a function of $\bar{\gamma}$, for GSC and MS-GSC implementations of the $\psi_4$ and $\eta_4$ schemes. It is seen that MS-GSC reduces the average number of combined branches for both schemes over the entire SNR range. Compared to the M-QAM scheme, the optimal schemes show less reduction from an MS-GSC implementation, which is a direct consequence of the fact that the $\psi$ schemes use the increased combined SNR at higher $\gamma$ to facilitate higher spectral efficiencies, while the QAM saturates and will on average need to combine fewer branches.

5. CONCLUSIONS

We have analyzed optimal rate and power allocation for a joint link adaptation and adaptive combining system, under an average spectral efficiency maximization criterion and both average power and diversity combining constraints. Given the channel fading condition information the proposed schemes maximizes the obtainable channel spectral efficiency.

Compared to a joint adaptive QAM scheme, our proposed schemes give significant ASE gains. Contrary to previous work on power control for discrete rate link adaptation, we have shown that when jointly optimized with adaptive combining, power control does not significantly increase the average spectral efficiency. However, it does significantly decrease the probability of no transmission, which is important for some multimedia networks which not only offer high-rate data-based services, but also low-rate services with real-time or low-latency requirements.

6. REFERENCES


