Low-complexity Coordinated Beamforming Transmission for Multiuser MISO Systems and its Performance Analysis

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Abstract—In this paper, we present a low-complexity coordinated beamforming transmission scheme for multiuser multiple-input-single-output (MISO) systems. With a minimum overhead signaling, the proposed scheme selects the beamforming vector and users while considering the interference from other cells. We develop the exact analytical expressions of the ergodic sum-rate for the resulting system for dual-cell scenario. Based on the analytical results, we compare the performance of the proposed scheme with other MISO transmission schemes in multiple cell environment, and then analyze their respective superiority and shortcomings. We show through selected numerical examples that our proposed coordinated beamforming scheme achieves tremendous performance gain over conventional schemes while only requiring beam index sharing between base stations.

Index Terms—network-MIMO, coordinated beamforming, sum-rate analysis and wireless communications.

I. INTRODUCTION

MIMO (Multiple-Input Multiple-Output) techniques can improve the spectral efficiency of wireless communications and provide significant throughput gains. In future wireless systems, the performance will be limited by inter-cell interference, due to the increasing number of interfering sources [1]. To meet the increasing demand, future wireless systems are evolving towards universal frequency reuse, where two neighboring cells may utilize the same radio spectrum. Such scenario also applies to the emerging femtocell systems. In this circumstance, network-MIMO, also called coordinated multicell transmission has drawn significant research attention recently, due to its potential effectiveness to suppress inter-cell interference through coordination among multiple base stations (BSs) [2].

With conventional network-MIMO, multiple coordinated BSs effectively constitute a ‘super-BS’, which transforms the interference channel into a MIMO broadcast channel, and eliminates the intercell interference completely [3]-[5]. The optimal dirty paper coding (DPC) [6] [7] and sub-optimal linear precoders have been studied for network MIMO scenario [8]- [13]. With simplified network models, analytical results have appeared in [14]- [17]. These schemes require, however, the complete channel state information, and, sometimes, even the user data to be shared among coordinating BSs, which can be challenging in practical systems. Note that, although BSs are usually connected with wired connections with each other through the switching center, these connections are already fully loaded with the increasing amount of multimedia traffics.

We consider the low-complexity coordinated beamforming transmission systems in this paper. To limit the amount overhead signal between base station and minimize the additional burden to the back haul connections, we consider the schemes that exchange a minimum amount control information to achieve coordinate beamforming. Specifically, we look into the coordinated transmission schemes for multiuser MISO systems, where only the selected beam index is shared among BSs, which leads to a minimum amount of overhead signaling.

In this paper, we analyze the sum rate performance of the resulting system through mathematical analysis. In particular, the exact statistics of users’ SINRs are derived and applied to system sum-rate evaluation. Selected numerical examples show that our proposed scheme can offer significant sum-rate capacity gain over other multicell random processing options with low system complexity.

The rest of the paper is organized as follows. In section II, we describe the system and channel models used in this paper. After introducing the practical mode of operations in section III, we develop its exact sum-rate expressions and provide its selected numerical examples in section IV. Some reference schemes are presented and analyzed in section V. The paper concludes with some remarks in section VI.

II. SYSTEM AND CHANNEL MODELS

The system under consideration as shown in Fig. 1 consists two base stations, utilizing the same radio spectrum to serve their selected users. Both base stations are equipped with N antennas, which facilitates beamforming transmission,
whereas each user has only a single receive antenna due to its size or complexity constraint. The user set in cell 1 is denoted by $\mathcal{I} = \{1, 2, \ldots, i, \ldots, K_1\}$, and that in cell 2 by $\mathcal{J} = \{1, 2, \ldots, j, \ldots, K_2\}$. The channel vectors are defined as following:

- $\mathbf{h}_{1i}$ is the $N \times 1$ channel vector from the base station 1 to the $i$th user in cell 1, i.e. $i \in \mathcal{I}$.
- $\mathbf{h}_{2i}$ is the $N \times 1$ channel vector from the base station 2 to the $i$th user in cell 1, i.e. $i \in \mathcal{I}$.
- $\mathbf{h}_{1j}$ is the $N \times 1$ channel vector from the base station 1 to the $j$th user in cell 2, i.e. $j \in \mathcal{J}$.
- $\mathbf{h}_{2j}$ is the $N \times 1$ channel vector from the base station 2 to the $j$th user in cell 2, i.e. $j \in \mathcal{J}$.

We assume that, with proper power control mechanism, the users experience flat homogeneous Rayleigh fading. Thus, each component of $\mathbf{h}_{1i}$ and $\mathbf{h}_{2j}$ is modeled as independent and identically distributed (i.i.d) complex Gaussian random variables with zero mean and unit variance. And each component of $\mathbf{h}_{1j}$ and $\mathbf{h}_{2i}$ is modeled as independent and identically distributed (i.i.d) complex Gaussian random variables with zero mean and variance $\delta$. In this scenario, $\delta$ attributes to the path loss, and thus usually we have $\delta < 1$. \footnote{The i.i.d assumption on the entries of $\mathbf{h}_{1i}$ and $\mathbf{h}_{2j}$ here is just for the sake of presentation clarity. As can be seen later, our analysis is general enough and applicable to the non-i.i.d case.}

We assume that with the wired connection from the base stations to the switching center, they can exchange a limited amount of control information for coordinated beamforming transmission. In particular, each base station employs a codebook-based random beamforming strategy to serve one selected user in its coverage area. The base station will communicate the utilized beamforming vectors to each other and to the users using the index of the code book. The code book is assumed to consist of $B$ unit-norm vectors of length $N$, generated from an isotropic distribution. With the proper design of the beamforming vectors and user selection, the inter-cell interference can be controlled. The specific design and selection scheme proposed in this work will be discussed in the following sections. For the multi-transmit antenna case under consideration, the received signal at the $i$th user in cell 1 and $j$th user in cell 2 can be written as:

$$
y_i = \mathbf{h}_{1i}^T \mathbf{w}_1 s_1 + \mathbf{h}_{2i}^T \mathbf{w}_2 s_2 + n_i,
$$

$$
y_j = \mathbf{h}_{1j}^T \mathbf{w}_1 s_1 + \mathbf{h}_{2j}^T \mathbf{w}_2 s_2 + n_j,
$$

respectively, where $s_i(i = 1, 2)$ are data symbols to selected users and $\mathbf{w}_i(i = 1, 2)$ are the corresponding beamforming vectors.

### III. MODE OF OPERATIONS

The mode of operation of the proposed coordinated beamforming strategy is summarized as follows.

- Without loss of generality, we assume that one of the base stations starts its user selection for beamforming transmission first, which is denoted by BS$_1$. In particular, BS$_1$ randomly selects a vector from its code book as beamforming vector and transmits a pilot symbol with this vector, denoted by $\mathbf{w}_1$. Every user in the coverage area of BS$_1$ will estimate and feed back its received signal to noise ratio (SNR), which will be proportional to the projection power of users channel vector on to the beamforming direction, i.e. $|\mathbf{h}_{1i}^T \mathbf{w}_1|^2$, where $\mathbf{h}_{1i}$ denotes the MISO channel vectors from BS$_1$ to user $i$ in its coverage area. Note that users will not need to estimate its channel vector in this process and each needs only to feed back a real number for user selection.

- BS$_1$ will select the user achieving the largest SNR among all users, i.e. user $i^*$, where $i^* = \arg \max_i |\mathbf{h}_{1i}^T \mathbf{w}_1|^2$. With conventional random beamforming strategy, transmission will then start without any mechanism for controlling the interference from the other base station. With the proposed coordinated transmission strategy, we ask user $i^*$ to estimate its MISO channel from the interfering base station BS$_2$, denoted by $\mathbf{h}_{2i^*}$. With this channel state information, user $i^*$ will determine the beamforming vector that leads to the smallest amount of interference and feedback its index back. Mathematically speaking, the beamforming vector $\mathbf{w}_2$ should satisfy $|\mathbf{h}_{2i^*}^T \mathbf{w}_2|^2 = \min_j |\mathbf{h}_{2j}^T \mathbf{w}_2|^2$.

- BS$_1$ will inform BS$_2$ the desired beamforming vector to use through the wired backhaul connection. Then BS$_2$ will broadcast training symbol using the selected beamforming vector for its own user selection. Every user in the coverage area of BS$_2$ will estimate and feedback its received signal to noise and interference ratio (SINR), with signal power proportional to $|\mathbf{h}_{1j}^T \mathbf{w}_2|^2$ and interference power to $|\mathbf{h}_{2j}^T \mathbf{w}_1|^2$. Here, $\mathbf{h}_{2j}$ and $\mathbf{h}_{1j}$ stand for the channel vectors from BS$_2$ and BS$_1$ to user $j$ in the coverage area of BS$_2$, respectively. Specifically, the SINR of the $j$th user in the BS$_2$’s coverage is given by

$$
\gamma_{2,j} = \frac{|\mathbf{h}_{1j}^T \mathbf{w}_1|^2}{|\mathbf{h}_{1j}^T \mathbf{w}_2|^2 + \rho},
$$

where $\rho$ is the normalized noise power, equal to $N_0/E_a$. BS$_2$ will select the user that achieves the maximum SINR among all users to serve, i.e. user $j^*$ where $j^* = \arg \max_j \gamma_{2,j}$.

Based on the above mode of operation, we can determine the SINRs of the selected users as

$$
\gamma_1 = \frac{|\mathbf{h}_{1j}^T \mathbf{w}_1|^2}{|\mathbf{h}_{1j}^T \mathbf{w}_2|^2 + \rho} = \frac{\max_i |\mathbf{h}_{1i}^T \mathbf{w}_1|^2}{\min_j |\mathbf{h}_{2j}^T \mathbf{w}_2|^2 + \rho},
$$

and

$$
\gamma_2 = \max_j \left( \frac{|\mathbf{h}_{1j}^T \mathbf{w}_2|^2}{|\mathbf{h}_{1j}^T \mathbf{w}_1|^2 + \rho} \right),
$$

### IV. SUM-RATE ANALYSIS

In this section, we analyze the ergodic sum rate performance of the proposed coordinated beamforming scheme. We first derive the exact statistics of the selected users’ SINR.
A. First user’s SINR analysis

The first user’s SINR, denoted as $\gamma_1$, is represented as below,

$$\gamma_1 = \frac{|h_{1i}^T w_1|^2}{|h_{2i}^T w_2|^2 + \rho} = \max_j \frac{|h_{1i}^T w_j|^2}{\min_l |h_{1i}^T w_l|^2 + \rho} = \frac{y}{z + \rho}, \quad (5)$$

For the nominator term $y$, since $w_j$ is a normalized vector, while $h_{1i}^T$ is a vector with i.i.d. $CN(0, 1)$ entries, thus $|h_{1i}^T w_j|^2$ are i.i.d. over $K$ with $\chi^2$ distribution with two degrees of freedom. This implies that the probability density function (PDF) of the nominator term $y$ could be obtained, presented as,

$$f_y(x) = K_1(1 - e^{-x})^{K_1 - 1} e^{-x}. \quad (6)$$

Since the first user $i^*$ in BS$_1$’s coverage area is selected, the denominator term in (5), $\min_l |h_{1i}^T w_l|^2$ can be rewritten as

$$z = \min_l |h_{1i}^T w_l|^2 = u \times v. \quad (7)$$

It can be shown that $|h_{1i}^T w_l|^2$ is beta distributed with parameters $1$ and $N-1$, whose PDF is expressed as

$$f(x) = (N-1)(1-x)^{N-2}, \quad x \in (0, 1) \quad (8)$$

Therefore, the PDF of the $u = \min_{l \neq 1} \frac{|h_{1i}^T w_l|^2}{|h_{1i}^T w_1|^2}$ term in (7) can be obtained as

$$f_u(x) = (B-1)(N-1)(1-x)^{(N-1)^2} \frac{1}{2(B-2)(1-x)} = x \in (0, 1) \quad (9)$$

Noting that $v = |h_{2i}^T w_1|^2$ follows a modified $\chi^2_{(2N)}$ distribution, with PDF given by:

$$f_v(x) = \frac{1}{(N-1)\delta^2} \left( \frac{x}{\delta^2} \right)^{N-1} e^{-x/\delta^2}, \quad (10)$$

the PDF of the $z$ could be obtained as the product of two random variables [18], as:

$$f_z(z) = \int_0^1 \frac{1}{(N-1)\delta^2} \left( \frac{z}{\delta^2} \right)^{N-1} e^{-z/\delta^2} \frac{1}{2(B-2)(1-x)^{N-2}} \quad (11)$$

After carrying out the integration and some simplification, we obtain the following closed-form expression for the PDF of $z$.

$$f_z(z) = \frac{(B-1)z^{N-1}}{\delta^{2N}(N-2)!} \left( \frac{z}{\delta^2} \right)^{N-1} e^{-z/\delta^2} \frac{1}{2} \left( \frac{i + N - A - 2}{2} \right)^{A-i} \frac{1}{i - A + N - 1} \frac{1}{\delta^2}, \quad (12)$$

where $\mathcal{M}(\nu, \mu, \zeta)$ denotes the Whittaker hypergeometric function and $A = (N-1)(B-1) - 1$.

Conditioning on $y$, the PDF of $\gamma_1$, $f_{\gamma_1}(x)$, can be obtained as

$$f_{\gamma_1}(x) = \int_0^\infty f_{\gamma_1 | y}(x|y) f_y(y) dy $$

And then we obtain the PDF of $\gamma_2$, $f_{\delta_2}(x)$, which can be written as

$$f_{\delta_2}(x) = K_2(F_{\delta_2}(x) - 1) \frac{1}{\delta^2} e^{-x/\delta^2}$$

where $F_{\delta_2}(x)$ and $f_{\delta_2}(x)$ are the PDF and CDF of the $\gamma_2$ term, given by

$$f_{\delta_2}(x) = \int_0^\infty f_{\gamma_2}(x|\gamma_1) f_{\gamma_1}(\gamma_1) d\gamma_1 \quad (13)$$

C. Ergodic Capacity

The ergodic capacity of network-MIMO system with codebook based coordinated beamforming can be calculated as:

$$R = \int_0^\infty \log_2(1 + \gamma)(f_{\gamma_1}(\gamma) + f_{\gamma_2}(\gamma)) d\gamma $$

D. Numerical Examples

In this section, we present selected numerical examples in order to illustrate the mathematical formalism on the sum-rate analysis of the proposed coordinated beamforming scheme. Noting that all the analytical results in this paper have been verified through Monte-Carlo simulation.

In Fig. 2, we present the sum-rate performance of the two cells respectively with the proposed coordinated beamforming scheme. Here we have $(B, N, \delta) = (16, 4, 0.7)$ for different volume of active users in both cells and average channel SNR. Both analytical and simulation-based curves have been
provided. From the specific figure, it is obvious that the sum-rate capacity gain of cell 1 outperforms that of cell 2 for any channel conditions, for that the second beam is specifically designed in order to minimize the intercell-interference from cell 2 to 1. For fairness in practice, the two cells would work as cell 1 in turns.

In Fig. 3, we provide the overall sum-rate performance at different average channel SNR and path loss parameter $\delta$. Here we have $(B, N, K_1, K_2) = (16, 4, 40, 40)$. The figure gives a clear view through the performance for different channel conditions and cell distances.

V. REFERENCE SCHEMES

In this section, we compare the sum-rate performance of the proposed coordinated beamforming scheme with several popular multicell processing schemes in the dual-cell scenario.

- **Selfish random beamforming (SRB)**

  BS$_1$ and BS$_2$ follow the operations of conventional random beamforming independently [19]. This option is based on that the system is fully unaware of the intercell interference. Therefore, we can determine the SINRs of the selected users as

  $$\gamma_1 = \max_i \frac{|h_{i1}^T w_1|^2}{|h_{i2}^T w_2|^2 + \rho}, \quad \gamma_2 = \max_j \frac{|h_{j2}^T w_2|^2}{|h_{j1}^T w_1|^2 + \rho}. \quad (23)$$

  Then it is easy to get the PDF of $\gamma_1$, $f_{\gamma_1}$ and that of $\gamma_2$, $f_{\gamma_2}$, which are presented as

  $$f_{\gamma_1} = f_\gamma(K_1), f_{\gamma_2} = f_\gamma(K_2), \quad (24)$$

  where

  $$f_\gamma(K) = \frac{1}{\delta^2} \sum_{i=0}^{K-1} (-1)^{K-1-i} K \times e^{-\rho(K-i)x} \times \left(\frac{\rho}{Kx - ix + \frac{1}{\delta^2}} + \frac{1}{(Kx - ix + \frac{1}{\delta^2})^2}\right). \quad (25)$$

- **Interference aware random beamforming (IA-RB)**

  The operations share a lot in common with the previous option, the only difference is that each user will estimates and feeds back its received SINR, instead of SNR, for its BS to select which achieves the largest. Note that as long as the two BSs do not begin their pilot symbol transmission simultaneously, all users will not need to estimate their channel vectors, but only to feed back a real number for user selection. And the achieved SINRs of the two selected users can be presented as

  $$\gamma_1 = \max_i \frac{|h_{i1}^T w_1|^2}{|h_{i2}^T w_2|^2 + \rho}, \quad \gamma_2 = \max_j \frac{|h_{j2}^T w_2|^2}{|h_{j1}^T w_1|^2 + \rho}. \quad (26)$$

  Obviously, in this approach, the PDF expression of $\gamma_1$, $f_{\gamma_1}$ and that of $\gamma_2$, $f_{\gamma_2}$ can be referred to (18).

- **Coordinated ZF beamforming with user selection (CZF)**

  A lot of work have been focused on the coordinated ZF beamforming option, also called ‘super BS’ [20]. However, most of them do not concern about user selection. Here, the CZF option relies on a simple multiuser scheduling method, i.e. to select the user with the largest channel vector norm square. After the full CSI sharing between two cells, the new ‘super BS’ uses zero-forcing method to transform the interference channel into a MIMO broadcast channel [3]- [5]. Suppose that $h_{i1}$, $h_{i2}$ are the two selected user’s channel vector respectively in cell 1 and 2. Then, without loss of generality, taking cell 1 as an example, to cancel its interference for cell 2, the beamforming vector $w_1$ needs to satisfy the orthogonality condition $h_{i1}^H w_1 = 0$. Meanwhile, we also want to maximize the desired signal power $|h_{i1}^T w_1|^2$. This corresponds to design $w_1$ in the direction of projection of $h_{i1}$.

  In Fig. 4, we have $(B, N, \delta, SNR) = (16, 4, 0.7, 20dB)$ for different number of active users $K_1$ and $K_2$ in both cells. From the figure, we can find that our proposed scheme outperforms SRB and IA-RB for any volume of active users, with only a beam index sharing, while CZF can offer optimal capacity gain, requiring full CSI sharing between two cells.

VI. CONCLUSION

In this paper, we studied the ergodic capacity of dual-cell MISO broadcast channels with the proposed coordinated random beamforming. In particular, we derived the exact analytical expressions of the ergodic sum-rate for the low-complexity scheme with the help of some new statistical results, and compared the performance with other MISO transmission schemes in multi-cell environment. We showed through selected numerical examples that our proposed coordinated beamforming scheme achieves tremendous performance over SRB and IA-RB for any volume of active users, with only a beam index exchange, while CZF can offer optimal capacity gain, requiring full CSI sharing among cells. In the future work, we are proposing the specific coordinated scheme with orthogonal beamforming vector design and extending it into the multi-cell scenario and femtocell system, which still preserves its low-complexity.
REFERENCES


