Recovery of Surfaces with Discontinuities by Fusing Shading and Range Data Within a Variational Framework

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Abstract—This work provides a variational framework for fusing range and intensity data for recovering regularized surfaces. It is shown that this framework provides natural boundary conditions for the shape-from-shading problem, results in a new shape-from-shading formulation in the absence of range data, and provides a new fusion paradigm when range data is incorporated. The approach is demonstrated on simulated range and intensity images; error analysis with respect to the ground truth surface is presented. It is shown that the formulation performs well even in very noisy images.

I. INTRODUCTION

ONE approach to object recognition is to devise algorithms that yield a three-dimensional (3-D) description of a given scene. Although sensors capable of directly measuring the 3-D structure of a scene—such as laser rangers—are currently available, the cost of such sensors increases rapidly with accuracy. An alternative approach to using high-precision sensors is to permit the fusion of range and intensity data for improving the accuracy of the surface estimation process. This paper describes a variational formulation, as well as implementation details and experimental results, of the fusion of simulated range and intensity data.

Despite the significant amount of past effort in the simultaneous usage of range and intensity data, little has been done in the way of fusing range and intensity images in a single unified formulation. Although there have been some studies that combine range obtained from stereo data with shading data to improve surface estimation, the stereo-based estimation process and the intensity-based estimation process in these studies have been decoupled. For instance, in one study, the surface map initially produced by a stereo sensor is iteratively refined to satisfy the observed intensity image [1]. In a separate study, a lowpass filtered version of the surface obtained by stereo is combined with a highpass filtered version of the surface derived from shading data in order to obtain an improved reconstruction [2]. In both of these cases, there is no coupling between the stereo-based estimation process and the intensity-based estimation process; the stereo-based estimation is used only during the initial surface reconstruction. In contrast, a variational approach to fusing range and intensity data in a single energy functional was recently suggested in [3] and [4]. In these initial works, the smoothness constraint is applied everywhere in the scene and, hence, the resulting surface has low fidelity along the image discontinuities. Moreover, homogeneous Dirichlet conditions were assumed in order to simplify the implementation. The objective of this paper is to expand upon this work by incorporating an edge process term in the variational functional and impose “natural” boundary conditions.

More specifically, this paper describes two innovations. First, a framework for combining range and intensity data into a single functional, while simultaneously incorporating an edge process for facilitating sharper image segmentation, is described. Second, the mathematics of directly applying a plate-based equation (without resorting to intermediate partial derivative representations) so that integrability is automatically enforced, as well as the enforcement of the natural boundary conditions, are described. The efficacy of the approach is demonstrated through the use of simulated data where conventional noise reduction techniques tend to produce oversmooth results. Although the approach described herein has been successfully applied to real data (see [5] for an example with synthetic aperture radar imagery), it is believed that clarity can best be achieved through the use of simple data where ground truth is available and, as such, we have relied on simulated data to illustrate our approach.

This paper is organized as follows. Due to the important role shape-from-shading plays in our formulation, Section II contains a discussion of the shape-from-shading problem. Section III develops our proposed fusion functional. Of particular importance is the treatment of the edge process, which is represented by a continuous variable and may be interpreted as a probabilistic representation of surface creases. Implementational considerations are discussed in Section IV. As the intent of this paper is to advance the theoretical framework for fusion, the experimental results provided in Section V are limited to simulated images. Concluding remarks are provided in Section VI.
II. THE SHAPE-FROM-SHADING PROBLEM

In the shape-from-shading problem, the intensity image of a surface, $S$, is given as an input. In its simplest version, the surface is assumed to be Lambertian; that is, the surface is assumed to have uniform albedo and the reflectance of the surface is proportional to the cosine of the angle between the surface normal and the direction of the light source. Furthermore, it is assumed that there is a single distant light source illuminating the surface such that there are no shadows and no self-reflections. The direction of the light source is assumed to be known. The problem consists of determining the shape of $S$ by calculating the distance of the points on the surface relative to some reference plane normal to the viewing direction. Let $I$ denote the observed image intensity defined over a domain $\Omega$ contained in the reference plane. Let $S$ be defined by the function $f : \Omega \to \mathbb{R}$. Note that the definition of $f$ depends on the choice of the reference plane. Let $N$ denote the outward surface (unit) normal and let $l$ denote the unit vector in the direction of the light source. Then the reflectance map is of the form

$$ R = \gamma N \cdot l $$

where $\gamma$ denotes the surface albedo. The image irradiance equation obtained by setting $I = R$ is a first-order Hamilton-Jacobi differential equation for $f$. Horn’s original solution integrated the differential equation numerically by employing the method of characteristics strips [6]. This approach proved to be numerically unstable, however, and a variational approach was subsequently formulated, in which the image irradiance equation was regularized and a numerical solution was obtained through Euler-Lagrange [7]. Since then, many alternate ways of regularizing the problem have been proposed [8]-[10].

Recently, a new class of fast methods based on the direct integration of the differential equation has been proposed with promising results [11]-[14]. Based on these results, it has been argued that the shape-from-shading problem is not an ill-posed problem after all, and regularization is unnecessary. We argue, however, that the shape-from-shading problem is well-posed only if issues pertaining to boundary conditions and convexity ambiguity are properly accounted for. Specifically, the boundary conditions for the shape-from-shading problem are typically unknown in practice and are frequently ignored. Additionally, local information is needed to resolve the inherent convexity ambiguity of the surface at points where the intensity is maximum. We argue that these and other issues may be most naturally addressed within a variational framework. To make these issues clear, the direct integration method is reviewed next.

For the sake of clarity, consider the simple case in which the viewing direction coincides with the illumination direction and the image intensity $I$ has been normalized so that $\gamma = 1$. Then

$$ R = N \cdot l = \frac{1}{\sqrt{1 + |\nabla f|^2}} $$

and the image irradiance equation takes the form

$$ |\nabla f| = g $$

where

$$ g = \frac{1}{\sqrt{I^2 - 1}}. $$

It has been shown that if $g$ is nowhere zero on $\Omega$ and is Lipschitz continuous, then (3) has at most one (viscosity) solution verifying the boundary condition $f = \varphi$ along the boundary $\partial \Omega$ [15], [16]. If there are “singular” points where $g = 0$, then the solution is no longer unique. At singular points, $I$ assumes its maximal value of one. At such points, $|\nabla f| = 0$ so that the surface may have a maximum, a minimum, or a saddle point. Since (3) is locally compatible with any of these interpretations, it gives rise to nonuniqueness. The solutions of (3) found in [15] and [16] may be interpreted as follows. For simplicity, consider the case where $f = 0$ along the boundary of $\Omega$ and that there is only one singular point, $x_0$. Function $g$ determines a metric on $\Omega$, and a solution to (3) is given by setting

$$ f(x) = \min \{ \text{distance} \ (x, \partial \Omega), \ \theta + \text{distance} \ (x, \ x_0) \} $$

where $\theta = f(x_0)$, and the distance is measured in the metric defined by $g$. Moreover, if $f$ is assumed to be $C^1$-smooth, then $|\theta| = \text{distance} \ (x_0, \partial \Omega)$. This result generalizes to the case where $g$ vanishes on a closed subset of $\Omega$ and $f$ satisfies the general Dirichlet boundary condition $f = \varphi$. These results make clear the essential role played by the boundary conditions. By changing the boundary conditions, it is possible to produce infinitely many different solutions to (3). Moreover, even if the boundary conditions are fixed, it is necessary to impose $C^1$-smoothness on $f$ to ensure that there are only finitely many solutions.

Rouy and Tourin [14] described fast algorithms for calculating these solutions based on dynamic programming. It follows from (4) that there is a neighborhood of each singular point in which the values of $f$ depend only on its value at the singular point and not on the boundary conditions. Dupuis and Oliensis [12] and Pentland [11] exploit this fact to derive local algorithms for calculating $f$ in a neighborhood of a singular point $x_0$ by defining $f(x)$ as the distance of $x$ from $x_0$ in the metric defined by $g$. Furthermore, Oliensis and Dupuis [13] have devised a method for patching these local solutions together to obtain a global solution. They forego the freedom of imposing boundary conditions on $f$; instead, they determine the values of $f$ at the boundary $\partial \Omega$ according to the distance of the boundary points from the singular points. In particular, their algorithm requires the presence of at least one singular point.

In contrast to the direct integration approach, the variational approach provides a way to derive “natural” boundary conditions for the shape-from-shading problem. (Of course, if the values of $f$ are known at the boundary, they can be enforced within the variational formulation.) Furthermore, it is straightforward to include a smoothing term for filtering noise in the data. Lastly, our formulation requires $f$ to be only piecewise $C^1$-smooth; this is necessary when $I$ is...
discontinuous. Since we are assuming uniform albedo, such discontinuities occur when the surface $S$ has creases—i.e., discontinuities in orientation. In such cases, solutions that are no longer $C^1$-smooth must be found. To solve this problem, it is necessary to segment the image $I$. As is well known, segmentation is difficult when the image is noisy. By incorporating both smoothing and segmentation terms, our variational formulation fuses the segmentation problem with the shape-from-shading problem. The formulation contains the pure shape-from-shading problem and the pure segmentation problem as special cases. By omitting the range data from the formulation, we obtain a new formulation for the pure shape-from-shading problem. Omission of the terms involving $I$ and $R$ leads to the problem of segmenting a surface along its creases.

There are two important ways in which we can use the range data for improving shape-from-shading solutions. First, even very noisy range data from low-quality sensors can be used advantageously for providing an initial estimate of the range data for improving shape-from-shading solutions. First, if the range data is of good quality, then we may formulate the problem as special cases. By omitting the range data from the formulation, we obtain a new formulation for the pure shape-from-shading problem. Omission of the terms involving $I$ and $R$ leads to the problem of segmenting a surface along its creases.

A. Fusion Without Regularization

Let $d: \Omega \rightarrow \mathbb{R}$ be the range data. Then the simplest way to combine the image irradiance equation with the range data is to minimize the functional

$$E_0 = \int_\Omega \{(f - d)^2 + \alpha^2(R - I)^2\}. \quad (5)$$

Let $l = (l_1, l_2, l_3)$. Then for the Lambertian reflection case

$$R = \frac{(-f_x, -f_y, 1)}{\sqrt{1 + |\nabla f|^2}} \cdot (l_1, l_2, l_3) \quad (6)$$

where $f_x, f_y$ denote the partial derivatives of $f$ with respect to $x$ and $y$. Let $\nabla f_x, f_y R$ denote the gradient of $R$ with respect to the variables $f_x, f_y$; that is

$$\nabla f_x, f_y R = -\frac{(l_1, l_2)}{\sqrt{1 + |\nabla f|^2}} - \frac{N \cdot l}{1 + |\nabla f|^2} \nabla f. \quad (7)$$

The Euler–Lagrange equation of $E_0$ for gradient descent may be written as

$$\frac{\partial f}{\partial \tau} = C_f [\nabla \cdot V + (d - f)] \quad (8)$$

with boundary condition $n \cdot V|_{\partial \Omega} = 0$, and where

$$V = \alpha^2 (R - I) \nabla f_x, f_y R \quad (9)$$

and $n$ denotes the outward normal to the boundary $\partial \Omega$, $C_f$ is a constant dictating the rate of gradient descent, and $\tau$ is the time parameter (i.e., it indicates the iteration number).

To understand the effect of the $(d - f)$ term in (8), consider the equation without this term; that is, consider the pure shape-from-shading problem. Then $f$ corresponding to a planar surface with constant $\nabla f = (-l_1/l_3, -l_2/l_3)$ is a solution of the equation of the form $I$ arbitrary. The normal $N$ coincides with the light direction $l$ so that $R = 1$, but $R \neq I$. Such a degenerate solution may be prevented in the absence of range data by setting $d = 0$ in (8) and retaining $f$. Even then, if the direction of light coincides with the viewing direction, $f = 0$ is a degenerate solution for arbitrary $I$. Such a degeneracy points to the importance of providing range data, even if it is used only as an initial estimate for $f$.

B. Regularization While Preserving Discontinuities

In many applications, there is a need for calculating some average property of the surface $S$ such as its curvature, requiring smoothing of the surface. If the smoothing constraint is applied everywhere in the image domain $\Omega$, then the meaningful discontinuities in the image are smoothed as well. Note that the discontinuities in the image $I$ correspond to creases in the surface $S$ since the albedo of the surface is assumed to be constant. In this section, a formulation for simultaneously smoothing the surface while preserving its discontinuities, in the spirit of [17], is presented. We define the plate-based functional $E_{pl}$ by

$$E_{pl} = \int_\Omega \{(f - d)^2 + \alpha^2(R - I)^2\} \ dx \ dy + \lambda^4 \int_{\Omega \setminus \Gamma} \{[
abla f_x]^2 + [
abla f_y]^2\} \ dx \ dy + \nu|\Gamma| \quad (10)$$

where $\Gamma$ represents the locus of the surface creases and $|\Gamma|$ is the length of $\Gamma$. Note that $\lambda$ may be interpreted as the nominal smoothing radius.

As is known from the segmentation problem [18], it is difficult to implement gradient descent with respect to $I$ due to the nonconvex nature of the functional. Therefore, we replace $\Gamma$ by a continuous variable $s$ as described in [19]. That is, $|\Gamma|$ in (10) is replaced by

$$\frac{1}{2} \int_\Omega \left\{ \rho |\nabla s|^2 + \frac{s^2}{\rho} \right\}. \quad (11)$$

The continuous variable $s$ varies between zero and one, and may be interpreted as the probability of the presence of $\Gamma$. Functional (11) may also be viewed as blurring of $\Gamma$ with a nominal blurring radius of $\rho$. The second integral in functional (10) must also be modified, since we no longer have $\Gamma$. This is achieved by multiplying the integrand in the second integral by $(1 - s)^2$ and extending the domain of integration to all of
The result is the functional

\[ E_{\text{pla}} = \int_{\Omega} (f - d)^2 + \alpha^2 (R - I)^2 \]
\[ + \lambda^2 (|\nabla f_x|^2 + |\nabla f_y|^2)(1 - s)^2 \]
\[ + \frac{\nu}{2} \left( \frac{\rho |\nabla s|^2 + s^2}{\rho} \right) dx dy. \]  

(12)

where

\[ V = \frac{\alpha^2}{\lambda^4} (R - I) \nabla f_x \cdot f_x R \]
\[ - \{ \nabla \cdot (1 - s)^2 \nabla f_x, \nabla \cdot (1 - s)^2 \nabla f_y \} \]  

(19)

and \( \partial / \partial t \) denotes the derivative operator in the direction tangent to \( \partial \Omega \), and \( C_s \) and \( C_f \) are descent constants chosen to ensure stability. This is the final form of our formulation. Note that without the range data (i.e., \( d = 0 \)), (13) and (14) provide a new computation scheme for the shape-from-shading problem. Furthermore, by dealing with the plate equations directly instead of using intermediate variables, the integrability constraints used by some shape-from-shading formulations (e.g., [7] and [8]) are no longer necessary. Lastly, by setting \( \alpha = 0 \) in the formulation, we obtain a “weak plate” model for surface reconstruction from range data.

IV. IMPLEMENTATION

We have implemented the diffusion system (15), (16) by a finite difference scheme. In our experiments, we found that it was important to calculate the various derivatives by means of minimal templates to avoid numerical instabilities. This was especially true of the derivatives of the highest order. Thus it was necessary to expand the term \( \nabla \cdot V \) of (14) into its individual components before discretization instead of calculating it by means of a series of first order numerical
Fig. 3. Variational fusion results for the wedding cake example. Shown are the surface, the corresponding range and intensity images, and the edge response function.

derivatives. Specifically, let \( w \) denote \( (1 - s)^2 \) and let

\[
Z = \nabla \cdot (\nabla \cdot w \nabla f_x, \nabla \cdot w \nabla f_y).
\] (20)

Then

\[
Z = (wf_{xx})_{xx} + 2(wf_{xy})_{xy} + (wf_{yy})_{yy}.
\] (21)

In the interior of \( \Omega \), we calculate \( Z \) in two stages. We first compute \( f_{xx}, f_{yy}, f_{xy} \) via the following templates: \( f_{xx} \) by \((1, -2, 1)^T\), \( f_{yy} \) by \((1, -2, 1)^T\), and \( f_{xy} \) by

\[
\begin{pmatrix}
-\frac{1}{4} & 0 & \frac{1}{4} \\
0 & 0 & 0 \\
\frac{1}{4} & 0 & -\frac{1}{4}
\end{pmatrix}
\] (22)

and then calculate \( (wf_{xx})_{xx}, (wf_{xy})_{xy}, (wf_{yy})_{yy} \) using the same templates.

Calculations along the boundary of \( \Omega \) are more complicated. In terms of the local coordinates \((n, t)\) at a smooth boundary point (not a corner), the boundary conditions are

\[
(f_{nn})_{\Omega} = 0
\]

\[
(wf_{nn})_{n} + 2(wf_{nt})_{t} = \beta
\] (23)

where

\[
\beta = \frac{\alpha^2}{\lambda^2} (R - I) \frac{\partial R}{\partial f_n}.
\] (24)
Note that the boundary conditions are used only for calculating \( \nabla \cdot V \) at a boundary point. We can write \( Z \) as

\[
Z = A_n + (w_{fn})_{tt}
\]

where

\[
A = (w_{fn})_n + 2(w_{ft})_t + (w_{ft})_n + 2w_{ft}.
\]

The last term in \( Z \) may be calculated as before. Since \( A \) is specified at the boundary, we may calculate \( A_n \) by numerical differentiation as follows. Let \( P \) be the boundary point and let \( Q \) be the grid point next to it in the \( n \)-direction. Let \( M \) be the midpoint of the edge \( PQ \). Then, \( A_n = 2[A(P) - A(M)] = 2[\beta - A(M)] \). The normal derivative of \( (w_{fn})_n + 2(w_{ft})_t \) at \( M \) may be calculated by central difference, \( w_t \) at \( M \) is the average of \( w_t \) at \( P \) and \( Q \) calculated by central differences, and \( f_{nt} \) is calculated by the template

\[
\begin{pmatrix}
-\frac{1}{2} & \frac{1}{2} \\
0 & 0 \\
\frac{1}{2} & -\frac{1}{2}
\end{pmatrix}
\]

The calculation of \( Z \) is much simpler at the corners of \( \Omega \) because many of the terms are set to zero by the boundary conditions \( f_{xx} = f_{yy} = f_{xy} = w_x = w_y = 0 \). In fact, \( Z = (w_{fxx})_{xx} + (w_{fyy})_{yy} \) and the remaining boundary conditions are

\[
(w_{fxx})_x = \frac{\alpha^2}{\lambda^2} (R - I) \frac{\partial R}{\partial f_x}
\]

\[
(w_{fyy})_y = \frac{\alpha^2}{\lambda^2} (R - I) \frac{\partial R}{\partial f_y}
\]

Therefore, we first calculate \( (w_{fxx})_x \) and \( (w_{fyy})_y \) at the midpoints of the grid edges at the corner by central differences; we then calculate \( (w_{fxx})_{xx} \) and \( (w_{fyy})_{yy} \) by forward or backward differences at the corner.

The implementation of the boundary conditions described above illustrates the inherent difficulty in using higher order smoothing such as the “thin plate” smoothing. The boundary conditions for plates become even more complicated if the boundaries are curved. An alternative to these complications is to avoid using gradient descent in such cases and resort to stochastic relaxation. The difficulty with this approach is that convergence is very slow. We are presently exploring another alternative for simplifying the boundary conditions; in this approach, we include the term \( \sigma^2|\nabla f|^2 \) in (12) in addition to the second-order derivatives; that is, we combine “membrane” smoothing with “plate” smoothing. We then change the relative weights \( \sigma \) and \( \lambda \) as we approach the boundary so that at the boundary, the smoothing essentially becomes the much simpler membrane smoothing.

V. EXPERIMENTAL RESULTS

To evaluate the mathematical formulations described in Sections III and IV, experiments were performed using synthetic data where the ground truth was known. In particular, two synthetic data sets were used: The first set attempts to mimic artificial environments where edges and flat surfaces abound; the second set, on the other hand, attempts to mimic more natural scenes where smooth curved surfaces dominate the image. It is shown that the piecewise-smooth reconstruction aspect of our fusion formulation has its greatest efficacy in data of the former type; its utility is not as substantial on data of the latter type.
Fig. 1 shows the noiseless versions of a synthetic "wedding cake" example—shown are the simulated surface, the corresponding range image, and the corresponding Lambertian intensity image with the illumination source located at the lower right. Fig. 2 shows the same images but with noise added; the range image had a signal-to-noise ratio (SNR) of 4.2, while the intensity image had an SNR of 9. Both range and intensity noise were generated from a uniform distribution; range and intensity noise were independent of one another. The images are 128×128 pixels in size.

The result of our processing is shown in Fig. 3. Specifically, shown are the resulting surface, the corresponding range image, the corresponding intensity image, and a plot of the edge response term [i.e., s in (12)]—where brightness indicates the probability of the presence of an edge. Note in particular that relatively sharp delineations along creases in the surface were obtained; furthermore, the resulting solution validates the enforcement of the natural boundary conditions along the image boundaries.

Although the quality of the reconstruction is readily apparent in Fig. 3, a more intuitive feel for the degree of improvement can be obtained by looking at the cross-section through the middle of the image. Fig. 4 shows a cross-sectional plot of the ideal (noiseless) data as well as the noisy input data; Fig. 5 shows a comparison between the noisy input data and the reconstructed solution. Note, once again, that the reconstructed surface demonstrates the piecewise-smooth and sharp crease characteristics of the formulation despite the poor SNR’s.

Table I quantifies the degree of improvement achievable via the use of the variational fusion formulation. In particular, errors with respect to the ideal image are quantified in two ways: as a function of the mean-squared error and as a function of the maximum error. Four cases are shown in Table I: the noisy input surface, the result of a 3×3 moving average filter, the range-data only reconstruction (i.e., the intensity term was not used), and the complete fusion formulation. The results indicate that the fusion approach provided the best answer—one that is significantly superior to more simplistic methods of smoothing such as moving averages. Furthermore, the results also clearly demonstrated the utility of fusing range and intensity data, since the fusion solution is ≈33% more accurate than the range-only reconstruction.

Fig. 6 shows the ideal (top) and noisy (bottom) versions of a synthetic hemisphere image representing a smooth, curved scene. The result of fusion is shown in Fig. 7. Table II quantifies the improvement achieved via the variational formulation. In particular, note that although the fusion formulation clearly yielded superior results, its performance with respect to that of the much simpler 3×3 moving average is not nearly as dramatic as was in the wedding cake example. Through these experiments and other empirical observations, we conclude
that the variational fusion formulation is most useful in the presence of high levels of noise, as well as in the neighborhood of discontinuities.

The selection of weighting terms for variational functionals is, in general, a difficult problem. The fusion solution described above was obtained by using weighting terms that made intuitive sense. Specifically, the fusion solution is obtained in 100 iterations, with the gradient descent coefficients $C_f$ and $C_s$ set conservatively at 0.05 in order to ensure numerical stability. To begin the iteration, the initial value for $f$ was set to a $3 \times 3$ averaged version of the noisy range image $d$, and the initial value of the edge function $s$ was set to zero. Since the ramps in the wedding cake example were eight pixels apart, the values of the smoothing radius $\lambda$ was chosen to be ten, and the blurring radius $\rho$ was set to five. Because the range image had a dynamic range that is approximately 40 times larger than that of the intensity image, the data weighting term $\alpha$ was set to 40 so as to achieve equal emphasis between range and intensity data. The crease locus is governed by the parameter $\rho\lambda^2/\nu$, and its value mainly controls the amplitude of the edge strength.

### TABLE II

<table>
<thead>
<tr>
<th>case</th>
<th>mean-sq. error</th>
<th>max. error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Noisy input surface</td>
<td>1.074</td>
<td>3.44</td>
</tr>
<tr>
<td>$3 \times 3$ moving avg</td>
<td>0.253</td>
<td>1.97</td>
</tr>
<tr>
<td>Range-data only</td>
<td>0.155</td>
<td>1.37</td>
</tr>
<tr>
<td>Fusion</td>
<td>0.148</td>
<td>1.21</td>
</tr>
</tbody>
</table>

Fig. 7. Result of variational fusion on the hemisphere example.
function $s$; the placement of the edge is fairly insensitive to the actual value chosen. In the wedding cake experiment, this parameter was set equal to 150 so that the maximum value of $s$ would be nearly one.

VI. CONCLUDING REMARKS

In summary, a variational formulation for surface reconstruction by fusing range and intensity images while smoothing and segmenting the surface is presented. In the course of developing this formulation, the role of regularization in the shape-from-shading problem as well as the advantages of fusion are discussed. In particular, the importance of boundary conditions as well as the removal of convexity ambiguity by introducing range data are discussed. The edge function serves to identify the locations of surface creases, and it prevents the smoothing process from smoothing surface discontinuities. The efficacy of the formulation is demonstrated via experimental results of the fusion of simulated range and intensity data.

The approach described herein can be improved in several ways. First, we are currently examining statistical approaches (e.g., generalized cross validation) for selecting weighting terms. Specifically, although such statistical techniques have been successfully applied to the regularization parameter selection problem (see [20] and [21]), using such techniques to select relative weights among different sensing modalities introducing range data are developing an adaptive approach in which we also estimate a quantity that is generally unavailable. To this end, we are currently examining statistical approaches or by replacing the gradient descent formulation by Gauss-Seidel and/or multigrid methods. Third, the variational fusion approach requires knowledge of the reflectance function $s$; the placement of the edge is fairly insensitive to the actual value chosen. In the wedding cake experiment, this parameter was set equal to 150 so that the maximum value of $s$ would be nearly one.

REFERENCES


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