Impact of emission constraints on DS-UWB communications with arbitrary chip-duty

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Abstract: Peak and average power emissions of the newly emerging ultra wideband (UWB) radio are supposed to be limited by regulatory constraints. Impact of emission constraints on direct sequence (DS-) UWB communications that use arbitrary chip-duty is discussed in this letter. Depending on the communication pulse shape, a minimum chip-duty below which average emission cannot be increased with bandwidth, or in other words, permissible average emission limit cannot be reached due to the peak constraint is pointed. Effect of the emission constraints on the system capacity is also investigated and compared with the same with no constraint. It is shown that a peak constraint slows down the capacity increment with bandwidth.

Keywords: ultra wideband, emission constraints, chip-duty, capacity

Classification: Microwave and millimeter wave devices, circuits, and systems

References

1 Introduction

The design objectives of conventional spread spectrum (SS) and newly emerging ultra wideband (UWB) systems are different in the sense that SS systems focus on making signals more covert by lowering power spectral density (psd) through increasing spreading bandwidth for certain fixed data rate [1]; whereas, UWB systems focus on increasing data rate (possibly at short distance) by increasing spreading bandwidth for certain constant psd [2-4]. Federal Communications Commission (FCC) of the USA has placed both average and peak emissions constraints on UWB signals [3]. Development of regulations in other countries are under way. This imposes some design constraints on direct sequence (DS)-UWB systems that use chip-duty less than unity. However, there has not been any paper except [5] so far to address this important topic. In this letter, we discuss some of the important implications of imposing emission constraints on UWB signals.

To understand the impact of regulatory emission constraints in relation to chip-duty, we consider a DS-UWB system that decreases chip-duty starting from unity, while keeping chip rate constant. In other words, our system increases spreading bandwidth by decreasing chip-duty. Note that lowering chip-duty offers a new option for spreading bandwidth in UWB [6]. Considering an average psd constraint, it can be understood that the average transmission power can be increased with bandwidth, which can be utilized to increase the data rate of the system. However, while a peak psd constraint is placed in addition, the maximum possible average transmission becomes dependent on the peak-to-average ratios of the signal and communication pulse shape. In such a situation, we show that depending on the pulse shape, there exists a minimum chip-duty below which average transmission can no more be increased with bandwidth. This in turn not only renders the capacity of the system to become pulse shape dependent, but also causes the capacity increment rate to slow down with bandwidth.

2 System and Signals

2.1 The DS-UWB System

The DS-UWB system [5, 6] transmits a pulse of duration \( T_p \) per chip of duration \( T_c \) where \( T_p \leq T_c \). The chip-duty is defined as, \( \mu = T_p/T_c \) (0 < \( \mu \leq 1 \)). We note that the spreading bandwidth of the DS-UWB system is \( B \approx 1/T_p = 1/(\mu T_c) \) [7] which, in a system with constant chip rate, is inversely proportional to \( \mu \). Pulse repetition frequency (PRF) i.e. the chip rate of the system is equal to \( 1/T_c \), which in this letter is kept constant.
2.2 Communication Pulses

Three types of pulse shapes are considered in this letter. These are rectangular pulse shape defined by

\[ w(t) = \begin{cases} 
    p_T p(t), & 0 \leq t < T_p, \\
    0, & \text{otherwise} 
\end{cases} \tag{1} \]

equivalent to

\[ w(t) = \begin{cases} 
    1, & 0 \leq t < T_p, \\
    0, & \text{otherwise} 
\end{cases} \]

half-sine pulse shape defined by

\[ w(t) = \sin(\pi t/T_p) p_T p(t) \tag{2} \]

and the second derivative of Gaussian pulse defined by

\[ w(t) = \left[ 1 - 4\pi \left( \frac{t - T_p/2}{\tau_m} \right)^2 \right] \exp \left[ -2\pi \left( \frac{t - T_p/2}{\tau_m} \right)^2 \right] p_T p(t) \tag{3} \]

with \( \tau_m = 0.39 T_p \).

3 Emission Constraints vs. Chip-duty

3.1 Analysis

Let us consider that the average emission of UWB signals is limited by power spectral density (psd) of \( \Psi_a \) and the peak emission is limited by psd of \( \Psi_p \).

We define the difference, \( \Psi_\delta = \Psi_p - \Psi_a \). At chip-duty \( \mu \), let us consider the bandwidth of the system to be \( B(\mu) = 1/(\mu T_c) \). We define

\[ P_{aL}(\mu): \text{ the average power transmission limit at } \mu \]

\[ P_{pL}(\mu): \text{ the peak power transmission limit at } \mu \]

By neglecting the energy in the side-lobes we can write, \( P_{aL}(\mu) = \Psi_a B(\mu) \) and \( P_{pL}(\mu) = \Psi_p B(\mu) \). The average and peak limits, in general, can then be written as

\[ P_{aL}(\mu) = P_{aL}(1)/\mu \tag{4} \]

and

\[ P_{pL}(\mu) = P_{pL}(1)/\mu \tag{5} \]

respectively where \( 0 < \mu \leq 1 \). To maximize data rate for certain signal-to-noise ratio, the system will try to reach the average transmission limit provided the peak limit is not crossed. We represent the maximum possible average power transmission at \( \mu = 1 \) by \( P_{am}(1) \) and the peak power required to maintain \( P_{am}(1) \) by \( P_{pm}(1) \). We note that \( P_{am}(1) \leq P_{aL}(1) \) and \( P_{pm}(1) \leq P_{pL}(1) \). Similarly, the general expression for maximum possible average transmission can be written as

\[ P_{am}(\mu) = P_{am}(1)/\mu, \quad P_{am}(\mu) \leq P_{aL}(\mu) \tag{6} \]

But the system increases spreading bandwidth by decreasing \( \mu \), keeping chip rate constant. As a result, the general expression for the required peak power transmission to maintain the maximum possible average transmission \( P_{am}(\mu) \) is

\[ P_{p_{eq}}(\mu) = P_{pm}(1)/\mu^2 \tag{7} \]
Now we can see, due to the peak constraint, the maximum possible peak transmission is given by

$$P_{pm}(\mu) = \begin{cases} P_{p,req}(\mu) & \text{if } P_{p,req}(\mu) \leq P_{pL}(\mu) \\ P_{pL}(\mu) & \text{otherwise} \end{cases}$$ (8)

So, we define a minimum value of $\mu$, namely $\mu_o$ where $P_{p,req}(\mu_o) = P_{pL}(\mu_o)$. We note $P_{p,req}(\mu) < P_{pL}(\mu)$ for $0 < \mu < \mu_o$ and $P_{p,req}(\mu) > P_{pL}(\mu)$ for $0 < \mu < \mu_o$. As a result, increasing spreading bandwidth by decreasing $\mu$ less than $\mu_o$ will not help to increase average power any more. This is because the peak power cannot be increased by the required amount because of permissible limitation. But still, peak power can be increased according to (5) that will help to keep the average power constant; which means, for $0 < \mu \leq \mu_o$, $P_{am}(\mu) = P_{am}(\mu_o) = P_{am}(1)/\mu_o$.

Now, we focus our effort to find out $\mu_o$, which is the solution of

$$P_{p,req}(\mu_o) = P_{pL}(\mu_o)$$ (9)

Considering that $\Psi_{\delta}$ is represented in decibels/Hz based on proper reference, we can show the solution of (9) to be

$$\mu_o = \frac{P_{pm}(1)}{P_{aL}(1)} 10^{-\Psi_{\delta}/10}$$ (10)

Here we note that $0 < \mu_o \leq 1$.

### 3.2 Examples

We present numerical examples considering three types of pulse shapes namely rectangular, half-sine and 2\textsuperscript{nd} derivative of Gaussian pulse introduced in one of the previous sections. The peak-to-average energy ratios of the pulse shapes are 0 dB, 3 dB and 8.34 dB respectively. We consider $B(1) = 500$ MHz. So, $B(\mu) = 500/\mu$ MHz for $0 < \mu \leq 1$. We also consider $\Psi_a = -41.3$ dBm/MHz. We show results considering two peak limits $\Psi_{p1} = -33.7$ dBm/MHz and $\Psi_{p2} = -23.7$ dBm/MHz.

Considering the first peak limit $\Psi_{p1}$, $\Psi_{\delta1} = \Psi_{p1} - \Psi_a = 7.6$ dBm/MHz. At $\mu = 1$, it can be visualized that while trying to reach the average transmission limit, the maximum possible peak transmissions become $P_{pm}(1) = P_{aL}(1)$ for the rectangular pulse, $P_{pm}(1) = P_{aL}(1) + 3$ dB for the half-sine pulse and $P_{pm}(1) = P_{aL}(1) + 7.6$ dB for the 2\textsuperscript{nd} derivative of Gaussian pulse. For the first two cases $P_{am}(1) = P_{aL}(1)$. However, using the 2\textsuperscript{nd} derivative of the Gaussian pulse, one cannot reach the maximum permissible average transmission limit $P_{aL}$ in this case (i.e. $P_{am}(1) = P_{aL}(1) - 0.74$ dB). This is because, in trying to do so, one exceeds the permissible peak emission limit suggested by $\Psi_{p1}$. Finally, using (10) one gets $\mu_o = 0.1738$ for rectangular pulse, $\mu_o = 0.3476$ for half-sine pulse and $\mu_o = 1$ for the 2\textsuperscript{nd} derivative of Gaussian pulse.

In Fig. 1 we show the average and peak power emission limits ($P_{aL}$ and $P_{pL}$ respectively) in dBm for the three pulse shapes as a function of chip-duty.

We also show the required peak transmissions ($P_{p,req}$) to maintain average
Fig. 1. Effects of chip-duty ($\mu = T_p/T_c$) variations on various transmission powers. The average limit $P_{aL}$ and peak limits $P_{pL1}$, $P_{pL2}$ correspond to psds $\Psi_a = -41.3$ dBm/MHz, $\Psi_{p1} = -33.7$ dBm/MHz and $\Psi_{p2} = -23.7$ dBm/MHz respectively. $B(\mu) = 0.5/\mu$ GHz.

Fig. 2. Maximum possible average power transmission $P_{am}$ vs. bandwidth in a case with $B(\mu) = 0.5/\mu$ GHz. The psd constraints considered are $\Psi_a = -41.3$ dBm/MHz and $\Psi_{p1} = -33.7$ dBm/MHz.

transmission limit for the pulse shapes introduced. Values of $\mu_o$ calculated from (10) agree well with those can be obtained from Fig. 1. Similarly, for the second peak limit $\Psi_{p2}$, we get $\mu_o = 0.0174$ for rectangular pulse, $\mu_o = 0.0348$ for half-sine pulse and $\mu_o = 0.1186$ for the 2nd derivative of Gaussian pulse. We don’t have the same problem with the 2nd derivative of Gaussian pulse this time. However, note that the 2nd derivative of Gaussian pulse has the largest $\mu_o$ in both cases.

Figure 2 shows the maximum possible average power transmission $P_{am}$ versus bandwidth considering $\Psi_a = -41.3$ dBm/MHz and $\Psi_{p1} = -33.7$ dBm/MHz as before. As we can see, depending on the regulatory constraint and
communication pulse shape, we may not be able to increase the average power with bandwidth beyond \( B(1)/\mu_0 \).

### 4 Emission Constraints vs. System Capacity

From the classic Shannon’s theorem, the capacity of the UWB system under consideration can be given by [8]

\[
C(\mu) = B(\mu) \log_2 \left( 1 + \frac{P_{am}(\mu)}{N_0 B(\mu)} \right)
\]

where \( C \) represents the capacity in bits/s and \( N_0 \) represents additive white Gaussian noise psd \( (N_0 = -114 \text{ dBm/MHz}) \). Figure 3 shows the capacity versus bandwidth with and without peak constraint. Average transmission limit of \( \Psi_a = -41.3 \text{ dBm/MHz} \) is considered as before in both cases. If a peak limit is not imposed, the capacity can be increased proportionally with bandwidth. However, when a peak constraint of \( \Psi_{p1} = -33.7 \text{ dBm/MHz} \) is considered, the capacity becomes pulse shape dependent and its increment rate falls while bandwidth is increased beyond \( 0.5/\mu_0 \) GHz. Eventually, a gradual saturation effect is seen in the capacity as bandwidth is increased further.

![Capacity vs. Bandwidth](image)

**Fig. 3.** Capacity vs. bandwidth in the presence and absence of peak constraint of \( \Psi_{p1} = -33.7 \text{ dBm/MHz} \). Average constraint considered in both cases is \( \Psi_a = -41.3 \text{ dBm/MHz}. B(\mu) = 0.5/\mu \text{ GHz} \).

### 5 Conclusion

A study describing some important implications of regulatory constraints on UWB emissions has been presented. Considering a DS-UWB system, the chip-duty, in other words the ratio of pulse width to chip interval has been introduced to be an important design parameter. In addition, selection of pulse shape has been shown to be vital from peak-to-average energy ratio point of view. In a situation with both average and peak constraints, the
system capacity has been shown to become pulse shape and signal chip-duty dependent. Additionally, the achievable capacity has been shown to experience saturating effect with bandwidth increment.

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