Simple-to-Evaluate Error Probabilities for Impulse Radio UWB Multiple Access Systems with Pulse-Based Polarity Randomization

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Abstract—Simple-to-evaluate and accurate bit error probabilities are presented for impulse radio ultra-wideband multiple access systems that use time hopping sequences with pulse-based polarity randomization and binary phase shift keying modulation. Simplified improved Gaussian approximation is used. It is shown that despite having the same processing gain, which is the multiplication of the number of frames per bit and chips per frame, different combinations of the parameters result in different performances. The theoretical results are validated by Monte Carlo simulations.

Index Terms—Ultra-wideband, multiaccess communication, error analysis, simplified improved Gaussian approximation.

I. INTRODUCTION

RECENTLY, time hopping impulse radio (TH-IR) ultra-wideband (UWB) multiple access communications have received considerable attention [1]–[3]. Though initially unipolar communication combined with pulse position modulation (PPM) was focused in [1]–[3], a system with pulse-based polarity randomization combined with binary phase shift keying (BPSK) was recently proposed in [4]. Approximate closed form error probabilities were given for the system in [4] that were based on the improved Gaussian approximation (IGA) previously introduced in [5]. However, the computational complexity of the method is quite intensive which increases with the total number of users. In this letter, we present a simple error analysis for the system that facilitates to obtain simple-to-evaluate and accurate error probabilities based on Holtzman’s simplified improved Gaussian approximation (SIGA) [6].

Previously, the SIGA was extensively used in literature to obtain expressions for error probabilities in DS-CDMA (see [7], [8] and the references therein). However, whether the method can be applied for TH-IR UWB multiple access system has not yet been investigated. In this letter, we extend the method for a TH-IR UWB multiple access system that uses pulse-based polarity randomization. We show that the method is also acceptably accurate for the TH-IR UWB multiple access system.

II. SYSTEM MODEL

The TH-IR UWB system under consideration with pulse-based polarity randomization and BPSK modulation is similar to the one in [4]. There are $K$ simultaneous users in the system. A typical representation of the signal from user $k$ ($k = 1, 2, \ldots, K$) has the form

$$s^{(k)}(t) = \sqrt{\frac{E_k}{N_f}} \sum_{j=-\infty}^{+\infty} d_j^{(k)}(\lfloor j/N_f \rfloor) w(t - jT_f - c_j^{(k)}T_c)$$

where $t$ is time and $w(t)$ is the communication pulse with unit energy. The rest of the signal structure is described as follows:

- $E_k$ is the bit energy of user $k$.
- $N_f$ is the number of frames or pulses representing one information bit. Each frame has one pulse.
- $T_f$ is the average pulse repetition time, also known as frame time. So, the bit duration is $T_b = N_fT_f$.
- $T_c$ is the chip time. Each frame is divided in $N_c$ ($N_c \geq 2$) chips giving $T_f = N_fT_c$. The duration of the pulse $w(t)$ is considered to be equal to $T_c$.
- $\{b_i^{(k)}\}$ is the $i$-th bit of user $k$ which is a random variable (RV) uniform on $\{-1, +1\}$. Here, $i = \lfloor j/N_f \rfloor$ and $\lfloor \cdot \rfloor$ represents floor function.
- $\{d_j^{(k)}\}$ is the random polarity code which is also a RV uniform on $\{-1, +1\}$ and is periodic with period $N_f$.
- $\{c_j^{(k)}\}$ is the TH code which is a RV uniform on $\{0, 1, \ldots, N_c - 1\}$ and is periodic with period $N_f$.
- The processing gain of the system is $PG = N_fN_c$.

III. MULTIPLE ACCESS INTERFERENCE MODELING

Considering an additive white Gaussian noise (AWGN) channel, the received signal can be given by

$$r(t) = \sum_{k=1}^{K} s^{(k)}(t - \tau_k) + n(t)$$

where $n(t)$ is AWGN noise with two sided power spectral density of $N_0/2$ and $\tau_k$ is the random delay of the signal received from user $k$ relative to the desired user 1 ($i.e.$ $\tau_1 = 0$) which is uniformly distributed over $[0, T_b]$. Let $\tau_k = \Delta_k + \gamma_kT_f$ where, $\gamma_k$ is a RV uniform on $\{0, 1, \ldots, N_f - 1\}$ and $\Delta_k$ is the delay from the previous user.
\[ \Delta_k = \alpha_k + \beta_k T_c \] where \( \beta_k \) is a RV uniform on \( \{0, 1, \ldots, N_c - 1\} \) and \( 0 \leq \alpha_k < T_c \).

Considering a template of the form
\[ s_{\text{temp}}^{(i)}(t) = \sum_{j=iN_f}^{(i+1)N_f-1} d_j^{(i)} w(t-jT_f - \epsilon^{(i)}_j T_c) \] (3)
the decision statistics for a coherent correlation receiver while we are detecting the \( i \)-th bit of user 1 can be given by [4], [5]
\[ y^{(i)}(t) = \sqrt{E_1 N_f} b_t^{(i)} + \sum_{j=iN_f}^{(i+1)N_f-1} \sum_{k=2}^K \sqrt{E_k / N_f} a_j^{(k)} + n \] (4)
where the right side of (4) has three parts, of which the first part is the desired signal component, the second part is the multiple access interference (MAI) component and the third part \( n \) is the AWGN component having variance \( \sigma_n^2 = N_c N_f / 2 \). Here, \( a_j^{(k)} \) is the MAI component on the \( j \)-th frame of user 1 contributed by user \( k \), given by
\[ a_j^{(k)} = P_j^{(k)} R_w(\alpha_k) \delta_j(\Delta_k) + Q_j^{(k)} R_w(\alpha_k) \delta_j(\Delta_k) \] (5)
where \( R_w(\alpha_k) \) and \( R_w(\alpha_k) \) are continuous-time partial auto-correlation functions of the pulses \( w(t) \) defined as \( R_w(\alpha_k) = \int_{-\infty}^{\infty} w(t) w(t-\alpha_k) dt \) and \( R_w(\alpha_k) = R_w(T_c - \alpha_k) \). The independent RVs \( P_j^{(k)} \) and \( Q_j^{(k)} \) are uniform on \( \{+1, -1\} \), have zero mean and variances \( E[(P_j^{(k)})^2] = E[(Q_j^{(k)})^2] = 1 \) where \( E[.] \) represents mean value. By defining \( l_1 = \epsilon^{(i)}_j T_c \), \( l_2 = \Delta_k + c_j T_c \), \( l_3 = \Delta_k - (T_f - c_j T_c) \), the functions \( \delta_j(\Delta_k) = 1 \) if \( 0 \leq l_2 - l_3 < T_c \) or, \( 0 \leq l_3 - l_2 < T_c \); \( \delta_j(\Delta_k) = 0 \), otherwise and \( \delta_j(\Delta_k) = 1 \) if \( 0 \leq l_1 - l_2 < T_c \) or, \( 0 \leq l_1 - l_3 < T_c \); \( \delta_j(\Delta_k) = 0 \), otherwise.

The MAI variance \( \Psi = \sum_{k=2}^K \sigma_k^2 \) is a RV which is a function of the delays \( \Delta = [\Delta_1, \Delta_2, \ldots, \Delta_K] \) with \( Z(k) \) given by
\[ Z(k) = \sum_{j=iN_f}^{(i+1)N_f-1} Z_j^{(k)} \] (6)
with
\[ Z_j^{(k)} = \frac{E_k}{N_f} E \left[ (a_j^{(k)})^2 \right] \Delta_k \] (7)
where the expectation in (7) is taken over the RVs \( P_j^{(k)} \) and \( Q_j^{(k)} \) given \( \Delta_k \) (and hence \( \alpha_k \)).

Synchronous system. The system is frame synchronous if \( \Delta_k = 0 \) and bit synchronous if \( \Delta_k = \gamma_k = 0 \). In both cases, \( \alpha_k = 0 \) and \( R_w(0) = 0 \). Here note that the \( (j-\gamma_k) \)-th frame of user \( k \) collides with the \( j \)-th frame of user 1. However, interference may occur only if the pulses in the respective frames coincide (or, collide). Hence, we get
\[ Z_j^{(k)} = \begin{cases} \frac{E_k}{N_f} \mu, & \text{coincidence} \\ 0, & \text{no coincidence} \end{cases} \] (8)

Note that a coincidence in any frame occurs with a probability \( p = 1/N_c \), the probability of no coincidence being \( 1 - p \). Because the position of pulse in any frame of any user is selected randomly and independently, we can write
\[ Z(k) = \frac{E_k}{N_f} J_s \] (9)
where \( J_s \) is a RV binomially distributed over \( \{0, 1, \ldots, N_f\} \). The density function of \( J_s \) is given by [9]
\[ f_j(j) = \sum_{j=0}^{N_f} \left( \frac{N_f}{j} \right) p^j (1-p)^{N_f-j} \] (10)
with \( p = 1/N_c \). Here, \( \binom{N_f}{j} \) is binomial coefficient.

Asynchronous system. In asynchronous system, \( \alpha_k \) is a RV uniformly distributed over \( [0, T_c] \). Hence, there will usually be partial coincidences. However, we note that there is only one pulse in average to interfere with an arbitrary pulse of the desired user. As a result, \( Z_j^{(k)} \) can be approximated given by
\[ Z_j^{(k)} = \begin{cases} \frac{E_k}{N_f} \left( \frac{R_2^2 \mu_c + R_2^2 \mu}{2} \right), & \text{partial coincidence} \\ 0, & \text{no coincidence} \end{cases} \] (11)
However, under the assumption stated above, note that \( R_w(\alpha_k) \) and \( R_w(\alpha_k) \) over the same frame duration are mutually exclusive. Here a partial coincidence in any frame may occur with a probability \( p = 2/N_c \) and the probability of no coincidence is \( 1 - p \) as before. Using the knowledge of the synchronous system, \( Z(k) \) can be approximately given by
\[ Z(k) = \left( \frac{E_k}{N_f} \right) \left( \frac{R_w^2(\alpha_k) + R_w^2(\alpha_k)}{2} \right) J_{as} \] (12)
where \( J_{as} \) is a RV binomially distributed over \( \{0, 1, \ldots, N_f\} \). The density function of \( J_{as} \) is given by (10) with \( p = 2/N_c \).

IV. ERROR PROBABILITIES

If the mean \( \mu \) and the standard deviation \( \sigma \) of the MAI variance \( \Psi \) are known, the simple-to-evaluate bit error probability (BEP) according to the SICA can be given by [6]
\[ P_{e,SICA} = \frac{2}{3} Q \left( \frac{E_1 N_f}{\mu + \sigma_n^2} + \frac{1}{6} Q \left( \frac{E_1 N_f}{\mu + \sqrt{3}\sigma + \sigma_n^2} \right) \right) \] (13)
where \( Q(x) = (2\pi)^{-1/2} \int_x^{\infty} e^{-u^2/2} du \).

Synchronous system. By noting that \( E[J_s] = N_f/N_c \) and \( E[J_s^2] = N_f/N_c (N_f/N_c - 1/N_c + 1) \), the mean of \( \Psi \) can be given by \( \mu = \sum_{k=2}^K E[Z(k)] = \sum_{k=2}^K E_k / N_c \) and the standard deviation, \( \sigma = \sum_{k=2}^K (E[(Z(k))^2] - E[Z(k)^2])^{1/2} \) can be given by
\[ \sigma = \frac{\sqrt{N_c}}{N_c} \left( \frac{N_c - 1}{N_c} \sum_{k=2}^K E_k^2 \right)^{1/2} \] (14)
Asynchronous system. For asynchronous system, \( E[J_{as}] = 2N_f/N_c \) and \( E[J_{as}^2] = 2N_f/N_c (2N_f/N_c - 2/N_c + 1) \) and the mean of \( \Psi \) is given by \( \mu = \sum_{k=2}^K E[Z(k)] = 2m_{wR} \sum_{k=2}^K E_k / N_c \) where \( m_{wR} = \frac{1}{T_c} \int_0^{T_c} R_w^2(\alpha) d\alpha \). The standard deviation takes the form
\[ \sigma = \frac{1}{N_c} \left( \left( \frac{2N_f + N_c - 2}{N_f} \right) w_m - 4m_{wR}^2 \right)^{1/2} \] (15)
where \( w_w = \frac{1}{T_c} \int_0^{T_c} \hat{R}_u(\alpha) \, d\alpha = \frac{1}{T_c} \int_0^{T_c} R^2_u(\alpha) \, d\alpha. \) Note that here we have used the fact \( \frac{1}{T_c} \int_0^{T_c} \hat{R}_u(\alpha) R^2_u(\alpha) \, d\alpha = 0. \)

The BEP based on the well-known standard Gaussian approximation (SGA) is given by the 1st Q-function of (13) without the multiplication factor \( 2/3. \)

V. NUMERICAL EXAMPLES AND CONCLUSION

In this section, the results presented in the previous section are evaluated by illustrative examples. Only asynchronous system with perfect power control (i.e. \( E_k = E_b, k = 1, 2, \ldots, K \)) is considered. Theoretical results from SGA are verified by Monte Carlo simulations and are compared with those from SGA. Though the method can be used for any pulse shape \( w(t) = v(t)/\sqrt{E_v}, \) we use two types of pulses namely 1) the received monocycle pulse where \( v(t) = e^{-(t-T_c/2)^2} \) for TH-IR UWB in [1], [4] and 2) rectangular pulse where \( v(t) = 1, 0 \leq t < T_c; \) \( v(t) = 0, \) otherwise. The energy of \( v(t) \) is \( E_v = \int_{-\infty}^{+\infty} v^2(t) \, dt. \) The partial autocorrelations of \( w(t) \) are given by 1) \( \hat{R}_w(\alpha) = \frac{1}{(1/E_v)}[1 - 4\pi(\alpha/t_m)^2]\exp[-2\pi(\alpha/t_m)^2] \) with \( t_m = 0.39T_c \) which was previously used for TH-IR UWB in [1], [4] and 2) \( \hat{R}_w(\alpha) = (T_c - \alpha)/E_v \) respectively and \( R_w(\alpha) = \hat{R}_w(T_c - \alpha) \) in both cases. Fig. 1 shows the BEP versus total users \( K \) for \( N_f = 4 \) and 7 with \( N_c = 5 \) and signal-to-noise ratio (SNR), \( E_b/N_0 = 14 \) dB. The results from SGA closely match with those from simulation for both pulses and can be considered to be reliable approximations. The SGA, on the other hand, is optimistic for small to medium number of users. Note that the MAI variance converges to a Gaussian RV for large number of users [5] and hence the SGA is accurate in that region.

Fig. 2 presents the BEP versus processing gain, \( PG = N_f N_c \) for \( K = 10 \) and \( E_b/N_0 = 16 \) dB considering only monocycle pulse. As expected, performance improvement is seen with increase in the \( PG \). The effects of \( N_f \) and \( N_c \) on the multiple access performance looks similar from the viewpoint of SGA. However, except in small \( PG \), the actual system performance becomes different for different combinations of \( N_f \) and \( N_c \) despite having the same \( PG \). The \( PG \) is increased by 1) increasing \( N_c \) keeping \( N_f \) fixed at 6 and 2) increasing \( N_f \) keeping \( N_c \) fixed at 2. As seen, better performance is obtained in the second case. It is evident that the SGA accurately tracks this performance difference, whereas the SGA becomes increasingly optimistic. It can be concluded here that the use of long repetition code brings more performance improvement than decreasing signal duty-cycle (= 1/\( N_c \)) to increase the \( PG \).

The simple-to-evaluate SGA previously used mainly for DS-CDMA is found to be reasonably accurate for TH-IR UWB multiple access system as well. This rediscovers the inherent similarity between UWB and CDMA.

REFERENCES


