Self-calibration of radially symmetric distortion by model selection

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Abstract

For self-calibration of general radially symmetric distortion (RSD) of omnidirectional cameras such as fish-eye lenses, calibration parameters are usually estimated so that curved lines, which are supposed to be straight in the real-world, are mapped to straight lines in the calibrated image, which is assumed to be taken by an ideal pin-hole camera. In this paper, a method of calibrating RSD is introduced based on the notion of principal component analysis (PCA). In the proposed method, the distortion function, which maps a distorted image to an ideal pin-hole camera image, is assumed to be a linear combination of a certain class of basis functions, and an algorithm for solving its coefficients by using line patterns is given. Then a method of selecting good basis functions is proposed, which aims to realize appropriate calibration in practice. Experimental results for synthetic data and real images are presented to demonstrate the performance of our calibration method.

1 Introduction

Recently, importance of omnidirectional cameras is growing because of their large FOV (field of view). Omnidirectional cameras such as fish-eye cameras have many potential applications, for example, 3-D reconstruction, surveillance system, and so on. Since images taken with these cameras usually have distortions, they have to be transformed into distortion free images. There are many sources of distortions which come from lenses, and they can be classified into 3 categories: radial distortion, decentering distortion, and thin prism distortion [8, 9]. In the case of perfectly centered fish-eye lenses, their geometrical distortion is caused by the shape of lenses, and thus the radial distortion is dominant. In computer vision, the calibration of radially symmetric distortion (RSD) has been actively studied [4, 5, 6, 7], and most of them are based on the plumpline principle, that is, straight lines in the real scene should be straight in the perspective projection plane [1]. Hence stripes and checker board patterns are used for calibration.

This paper only considers RSD, and throughout this paper, the center of RSD is assumed to coincide with the center of the image. This assumption can be satisfied by using a method proposed by Tardif et al. [6], for example. The FOV of the camera is also assumed to be less than 180 degrees. In this situation, the calibration is regarded as the determination of the continuous 1-D mapping from the distance between the origin and a point in the distorted image, to the distance between the origin and the corresponding point in the undistorted image. In this paper, the representation of this mapping is assumed to be a linear combination of basis functions. In this case, the coefficients of the linear combination are estimated by an algorithm based on PCA [3]. This algorithm estimates the coefficients of the linear combination by hyper-plane fitting in a high-dimensional space, which is easily solved by an eigenvalue problem of matrix. In the hyperplane fitting, the choice of basis functions is very important. If prepared basis functions are not appropriate, the calibration fails. Unfortunately, distortions vary depending on characteristics of cameras, therefore, universally appropriate basis functions do not exist. In this paper, a method of selecting a good set of basis functions is discussed. To demonstrate the performance of proposed method, experiments for synthetic data and real images are presented.

2 Radially symmetric distortion (RSD)

When the center of RSD coincides with the center of the image, the calibration of RSD is the determination of the continuous 1-D mapping from \( r_d \) to \( r_u = f(r_d) \), where \( r_d \) is the distance between the origin and a point in the distorted image, and \( r_u \) is the distance between the origin and the corresponding point in the undistorted image, respectively [2]. Figure 1 depicts the correspondence between \( r_d \) in the distorted observation plane and
Let $x \in \mathbb{R}^2$ be the image coordinate of an observed feature point under distorted camera, and $\phi(x) \in \mathbb{R}^2$ be the image coordinate of the corresponding perspective camera. The mapping from $x$ to $\phi(x)$ can be represented by a function $f(r)$, which only depends on the radius $r = ||x||$, as

$$\phi(x) = f(r) \frac{x}{||x||} = \frac{f(r)}{r} x.$$  \hfill (1)

In this paper, it is considered that the distortion function $f(r)$ can be approximated by a linear combination of basis functions $\{f_n(r)\}_{n=1}^N$ as

$$f(r) = \sum_{n=1}^N c_n f_n(r).$$  \hfill (2)

Consequently, the calibration of RSD is achieved by determining the coefficients vector $c = (c_1, \ldots, c_N)^\top$. Usually, a 3-D real scene contains a number of straight lines, and these lines are distorted by omni-directional cameras and projected to curves, which is called distorted lines. In other words, a number of distorted lines, which are images of straight lines, are observed in a distorted image. The vector $c$ can be estimated from the images of distorted lines so that calibrated distorted lines by mapping $f$ are as straight as possible. This idea is called the plumbline principle [1]. The determination of the coefficients is reformulated as hyper-plane fitting in a high-dimensional space, and it is easily solved in the framework of linear algebra [3].

3 Calibration method

This section explains how to calibrate an RSD image from distorted lines. The calibration method explained here is a non-iterative method for estimating the coefficient vector $c$ of the distortion function (2). To derive a constraint with respect to each distorted line, points on the same distorted line $\{x_{[d]}\}_{d=1}^D$ are considered at first. From Eqs.(1) and (2), the mapping from the distorted image to the calibrated image is rewritten as

$$\phi(x_{[d]}) = \sum_{n=1}^N c_n \phi_n(x_{[d]}) = P(x_{[d]})c,$$

where $\phi_n(x) = f_n(||x||) \frac{x}{||x||}$ and $P(x_{[d]}) = (\phi_1(x_{[d]}), \ldots, \phi_N(x_{[d]}))$. Since the points $\{\phi_{[d]} := \phi(x_{[d]})\}_{d=1}^D$ should be on a straight line, there exists a vector $a$ and a scalar $b$, those of which satisfy $D$ constraints:

$$(a)^\top \phi_{[d]} + b = \Phi_{[d]}^\top C = 0 \quad (d = 1, \ldots, D),$$

where $\Phi_{[d]} = (\text{vec} P(x_{[d]}))^\top, 1 \in \mathbb{R}^{2N+1}$ and $C = ((c \otimes a)^\top, b)^\top \in \mathbb{R}^{2N+1}$, in which homogeneous coordinates are used. In this expression, \text{vec} $X$ denotes a column vector expansion of a matrix $X$, and $\otimes$ denotes the Kronecker product.

Defining $\Phi = (\Phi_{[1]}, \ldots, \Phi_{[D]}) \in \mathbb{R}^{(2N+1) \times D}$, the plumbline principle is represented as $\Phi^\top C = 0$. Then the least squares (LS) estimate of $C$ is the eigenvector of the matrix $\Phi \Phi^\top$ corresponding to the minimum eigenvalue.

When $S$ distorted lines are observed in a distorted image, $S$ estimates $\{C^S\}_{s=1}^S$ are obtained. To estimate $c$, the first $2N$ rows of $C^s$, that is, $\{c \otimes a^S\}_{s=1}^S$, is used in the method. Note that the unknown coefficient $c$ is common for all $S$ estimates.

Using $A = (a^1, \ldots, a^S) \in \mathbb{R}^{2 \times S}$, the set of vectors $\{c \otimes a^S\}_{s=1}^S$ consists of the column vectors of $c \otimes A$.

Letting $\Psi = (I_N \otimes 1_2)((c \otimes A) = c (1_2 A)$ where $I_N$ is the $N$-D identity matrix and $1_2$ is the 2D column vector with one in all entries, the LS estimate of $c$ is given by the eigenvector of the matrix $\Psi \Psi^\top$ corresponding to the minimum eigenvalue.

4 Basis selection of distortion function

Since the appropriate basis functions $\{f_n(r)\}_{n=1}^N$ are generally unknown and there are no universally good basis functions applicable for any kinds of distorted images, selecting appropriate basis functions is required for good calibration. To find good basis functions for each target image, the concept of model selection is important.

Let $B$ be a set of basis functions for calibration. The aim of model selection is to find a good subset $F$ of $B$, which approximates the target distortion function properly. To find the best $F$ from $B$, non-negativity of coefficients for each subset is evaluated at first. After this evaluation, subsets which satisfy the non-negativity condition are selected as candidates of the best subset.
Then the best subset is determined by the straightness of the curves on the calibrated image among the candidates.

The distortion function $f(r)$ should be monotonously increasing for $r \geq 0$. Then, monotonicity of the distortion function should be checked in the selection of the basis. However, it is difficult to investigate monotonicity in general, thus we propose to use non-negativity of coefficients instead. Namely, when all the basis functions are non-negative and monotonously increasing, monotonicity of the linear combination of the basis functions is ensured by non-negativity of their coefficients: $c_n \geq 0, \ (n = 1, \ldots, N)$.

The calibration is done by the plumbline principle, then the performance of calibration should be measured by straightness of $S$ rectified curves on the calibrated image. From this point of view, straightness of the subset $F$ is measured by the contribution rate of the first principal component of the data $\{\phi(x_{[d]})\}_{d=1}^D$, where $\phi$ is a linear combination of $F$. When the contribution rate is small, straightness of $\phi(x_{[d]})$’s is weak. After checking straightness, the best subset which maximizes the straightness criterion is selected.

The best subset is determined as the maximizer of the straightness criterion among the candidates that satisfy the non-negativity condition. The procedures are summarized as follows:

(1) prepare a set of basis functions for calibration, and generate possible subsets.

(2) select the subsets which satisfy non-negativity,

(3) the best subset is the maximizer of the straightness criterion among the candidates selected in (2),

(4) when the straightness of the best subset is under the threshold (an experimentally determined threshold is 0.999), the selected basis function are not sufficient to approximate the distortion function, then go to (1) and retry the calibration by another set of basis functions.

5 Experimental results

First, 10 lines are generated in $[-3, 3] \times [-3, 3]$ region as the perspective image, and $D(\in [21, 40])$ points are randomly and uniformly chosen from each line as the ground truth (Fig. 2 left). Then, points on the lines are mapped to the observation (distorted) image, which is normalized as a subset of the unit disk, by the inverse of the distortion function $f(r) = 1.6 \sin \frac{\pi r}{2} + 0.8 \tan \frac{\pi r}{2}$ (Fig. 2 right). On the distorted image, Gaussian noise with $\frac{1}{300}$-standard deviation are added to observed data for both horizontal and vertical coordinates. The prepared set of basis functions is $B = \{r, r^2, r^3, r^4, r^5, \sqrt{r}, \sqrt[3]{r}, \log(r + 1), \sin \frac{\pi r}{2}, \tan \frac{\pi r}{2}\}$. Note that each element of $B$ is monotonously increasing in the interval $(0, 1)$, therefore, the monotonicity of the distortion function is satisfied by non-negativity in the unit disk.

To avoid overfitting, the number of basis functions to represent the distortion function $f(r)$ is restricted to 2 or 3. Then the total number of combinations of the basis functions is only 165.

By applying the proposed method, 30 subsets among 165 combinations satisfy non-negativity. Among 30 non-negative subsets, 14 subsets exceed 0.999-threshold. Figure 3 shows the result of calibration by $\{r, r^4\}$, which has the best straightness (0.9997) among 14 subsets. The left plot of Fig. 3 is the distortion function of the ground truth (red line) and the estimation (black dashed line), and the right plot is the calibrated images (green circles and black solid lines) and the ground truth (red circles). It can be seen that
this subset represents the distortion function very well. The estimated distortion function is \( f(r) = 3.6126r + 1.9458r^4 \). The result of this experiment indicates that the non-negativity condition and the straightness criterion proposed in this paper are useful to find a good subset of basis functions. Note that the basis of the estimated distortion function is different from the ground truth \( \{ \sin \frac{\pi r}{2}, \tan \frac{\pi r}{2} \} \). This implies, in the estimation of the distortion function from noisy data, selecting an appropriate basis set is more important than detecting the true basis set.

To confirm the validity of the proposed method, an experiment with a real image is carried out. 20 images of a distorted line such as upper left of Fig. 4, are observed and a total of 398 sample points are selected for calibration (Fig. 4 upper right). The coefficients \( c \) of the distortion function (2) are estimated using the proposed calibration method with model selection. The prepared basis \( B \) is the same with the previous simulated experiment. Among the basis, 2 or 3 elements are chosen to represent the distortion function \( f(r) \). By the proposed method, the estimated distortion function is \( f(r) = 0.4679r + 0.8838 \tan \frac{\pi r}{2} \). The input natural image and its calibrated image by the estimated distortion function is shown in Fig. 4, and it can be seen that our model selection method works very well in practical situation also.

6 Conclusion

In this paper, a calibration method based on PCA is introduced for RSD images, and the method is improved by a model selection procedure. The model selection criteria are monotonicity of the distortion function and straightness of rectified lines in the calibrated image. This method is simple and completely algebraic and contains no iteration, therefore the solution is free from local optima. The proposed method is shown to work properly by experiments. To realize more precise calibration, it is important to investigate what kinds of basis functions should be prepared.

References