A Unified Approach for Performance Analysis in Mobile WiMAX Networks with Adaptive Modulation and Coding Schemes Over Rayleigh Fading Channels

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Abstract—WiMAX technology is considered one of the most important solutions capable to provide a Broadband Wireless Access in metropolitan areas. Mobility issues have become challenging for the rollout for WiMAX networks in order to compete against the other competitive technologies such as 3.5G and LTE. In this paper, a unified approach to derive the overall BER and data rate for mobile WiMAX (in uplink direction) will be introduced. This approach will take into consideration different operational scenarios such as time varying channels with different values of channel estimation precision values, different number of subcarriers, and different symbol durations. An analytical expression for the effect of ICI power on the bit error probability and effective bit rate will be obtained for BPSK, QPSK, and 16QAM in presence of AMC. This paper also investigates the effect of the use of AMC techniques on the obtained system performance.

Keywords— Mobile WiMAX, OFDM, Adaptive Modulation and Coding, ICI, Channel Estimation

I. INTRODUCTION

In MUTIMIDIA wireless communication networks, the demand for high data rates and quality of service is growing rapidly. The bottleneck in such networks is the wireless link because of the overall system performance degradation due to multipath fading, Doppler, and time dispersive effects introduced by the wireless wave propagation. In order to enhance the spectral efficiency while adhering to a target error performance over wireless channels, Adaptive Modulation and Coding (AMC) has been widely used to match the transmission parameters to time varying channel conditions. In this paper, the adaptation is performed based on feedback Channel Quality Indicator (CQI), which allows fully exploiting the characteristics of the multicarrier transmission by dynamically changing the allocation of available resources, such as transmitted power, bandwidth or modulation and coding rate.

In this paper, an adaptive modulation technique for WiMAX systems has been studied, by considering the influence of the wireless channel in terms of Inter-carrier Interference (ICI). ICI can occur for several reasons such as delay and Doppler spread in mobile radio channel and frequency instabilities of oscillators. The ICI degrades the performance of an OFDM system, with the degree of degradation depending on both of users’ mobility and channel estimation quality. So, the paper will develop a technique that improves the system performance in terms of some QoS metric: in particular the attention has been focused on Bit Error Rate (BER) and Effective Bit Rate (EBR). Previously published studies an AMC techniques in WiMAX networks as [1], [2] has been done without the consideration of the effect of ICI on the BER performance. On the other hand, the presented work will investigate the effects of ICI and AMC on the system performance. The presented analysis is based on keeping the error rate below a limit threshold, which is established based on the nature of the application to be realized. The presented model will be analyzed for the uplink direction which is deploying, Binary Phase Shift Keying (BPSK), Quadrature Phase Shift Keying (QPSK), and Quadrature Amplitude Modulation (QAM): 16QAM Mobile WiMAX systems. Also the effect of having different number of subcarriers will be investigated. The rest of the paper is organized as follows. Section II provides the system model. Section III presents the performance with AMC techniques; Section IV contains the results and analysis of BER and Effective Data Rate of AMC in mobile WiMAX using the proposed approach. Finally, Section V concludes the paper.

II. SYSTEM MODEL

Figure 1 shows the OFDM system used in the WiMAX system. A sequence of N data symbols are serial to parallel converted after the M-QAM modulation. For the purpose of channel estimation Np symbols are multiplexed with the data symbols. The output from the IFFT block will be:

\[ x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)e^{j2\pi knT/N} 
\]

\[ x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)e^{j2\pi nk/N}, \quad 0 \leq n \leq N-1 \] (1)
The sequence $x(n)$ corresponds to the samples of the sum $x(t)$ of the subcarrier signals, which is expressed as $[3]$:

$$x(t) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)e^{j2\pi f_{k}t}, \quad 0 \leq t < T$$

(2)

where: $X(k) =$ Information symbol on the subchannel $k$, $T =$ symbol duration, $N =$ FFT size, and $f_k =$ frequency of subcarrier $k$

The OFDM symbols are transmitted through a channel which assumed to have multipath fading composed of $L$ uncorrelated paths with time varying frequency response for the $k^{th}$ subcarrier at time $n$ expressed by:

$$h(n,k) = \sum_{i=0}^{L-1} h(n,k)i e^{-j2\pi f_i n}$$

(3)

The received baseband OFDM symbol is given by:

$$y(n) = \sum_{k=0}^{N-1} h(n,k)x(n-k) + w(n)$$

(4)

where: $w(n) =$ complex Additive White Gaussian Noise (AWGN) with one sided power spectral density $N_0$.

After demodulation by the $N$ point FFT, the demodulated OFDM symbol at the $i^{th}$ subcarrier is given by:

$$Y(i) = \frac{1}{N} \sum_{n=0}^{N-1} Y(n,i) e^{-j2\pi \frac{n}{N}} + \frac{1}{N} w(n) e^{-j2\pi \frac{n}{N}}$$

(5)

From (2), $Y(i)$ may be written as follows:

$$Y(i) = \frac{1}{N} \sum_{k=0}^{N-1} h(n,k)i X(k) e^{-j2\pi \frac{n}{N}} + \frac{1}{N} \sum_{n=0}^{N-1} w(n) e^{-j2\pi \frac{n}{N}}$$

(6)

To study the effect of ICI, through this paper, the received OFDM symbol at the $k^{th}$ subcarrier after FFT can be represented by (9) $[1]$:

$$Y(k) = \frac{1}{N} \sum_{j=0}^{N-1} H(j,k) X(j) + \frac{1}{N} \sum_{j=0}^{N-1} H(i,j) X(i) e^{-j2\pi \frac{n}{N}} + W(k)$$

(7)

Let:

$$Y(k) = \alpha(k) X(k) + \beta(k) + W(k)$$

(8)

where: the first term in (8) is the desired term, consisting of the transmitted symbol $X(k)$ multiplied by the multiplicative distortion $\alpha(k)$, $\beta(k)$ is the ICI term and $W(k)$ is the FFT of the AWGN and having the same statistic as $w(n)$ with variance $\sigma_w^2$, $\alpha(k)$ is a complex Gaussian random variable with zero mean and variance $2\sigma_\alpha^2$ and the ICI term $\beta(k)$ is modeled as a Gaussian random variable with zero mean and variance $\sigma_\beta^2$ $[4]$:

$$\sigma_\beta^2 = \frac{E_S}{N} \sum_{n=1}^{N-1} \sum_{n=0}^{N-1} J_n(2\pi \frac{f_n}{N}) \cos(2\pi \frac{n(k-i)}{N})$$

(9)

where: $f_d =$ Doppler frequency, $E_S =$ Symbol Energy, and $J_0$ is the zero order Bessel function.

To estimate the effect of the channel, a channel estimation using Pilot Symbol Assisted Modulation (PSAM) for each subcarrier in the receiver is used $[5, 6]$.

The quality of the channel estimation will be measured by the complex correlation coefficient ($\rho$) between the actual channel $\alpha(k)$ and its estimate $g(k)$ $[5]$, $g(k)$ is obtained by using Minimum Mean Square Error (MMSE). Hence, ($\rho$) may be expressed as follows $[3, 7, 8]$: 

$$\rho = \frac{\sigma_\alpha}{\sigma_\beta}$$

(10)

where: $\sigma_\alpha^2$ and $\sigma_\beta^2$ are the variances of actual and estimate channel respectively. By using the same methodology which had been presented in $[3, 9]$, the BER for BPSK, QPSK, and QAM may be derived assuming the use of zero forcing equalizer at the receiver.

A. 16-QAM

For 16 QAM, each symbol contains four bits ($b_1, b_2, b_3, b_4$). The average of BER of M-QAM will be equal to the BER of either I or Q components. For 16 QAM: first and third bits compose the inphase bit stream while second and fourth bits compose the quadrature bit stream. For 16 QAM, the decision boundary will be for $M=16$ $[10]$:

$$d = \frac{3E_S}{2(16-1)}$$

(11)

where: $2d =$ the minimum distance between two QAM symbols in the constellation diagram.

So, by using the same methodology which had been presented in $[3]$ the BER of the first bit will be:

$$P_{r_1(b_1)} = \frac{1}{2} \left[ 1 - \frac{\rho^2 \sigma_a^2 \sigma_b^2}{\sigma_a^4 + \sigma_b^4 (1-|\rho|^2) + \sigma_a^2 + \rho^2 \sigma_b^2} \right]$$

(12)

So, the BER of the third bit will be:

$$P_{r_3(b_3)} = \frac{1}{2} \left[ 1 - \frac{\rho^2 \sigma_a^2 \sigma_b^2}{\sigma_a^4 + \sigma_b^4 (1-|\rho|^2) + \sigma_a^2 + \rho^2 \sigma_b^2} \right]$$

(13)
Where:
\[
q^2 = 2 \cdot \frac{\sigma^2}{\sigma_s}, \quad q^2 = 2 + \frac{\sigma^2}{\sigma_s}.
\] (14-a)

\[
q^2 = 2 + 3 \cdot \frac{\sigma^2}{\sigma_s}, \quad q^2 = 2 + 3 \cdot \frac{\sigma^2}{\sigma_s}.
\] (14-b)

After some mathematical manipulations the BER of 16QAM will be:
\[
P_{\text{BER}}(16\text{QAM}) = \frac{1}{2} \left[ 1 - \frac{\psi}{\sqrt{v_1 + \psi}} \right] - \frac{1}{4} \left( \frac{\psi}{\sqrt{v_1 + 10\psi}} \right)
\]
where:
\[
u_1 = \left( \frac{\sigma^2}{\sigma_s} \right) \gamma + 1 + \left( 2 \gamma \cdot \left( 1 - |\rho|^2 \right) \right)
\] (16)
where: $\gamma = \text{Signal to Noise Ratio} = \text{SNR}$

B. QPSK

QPSK are QAM modulation with $M=4$. According to [11], [12] and by using the same methodology which had been presented in [3], after many mathematical manipulations, the BER of QPSK will be:
\[
P_{\text{BER}}(\text{QPSK}) = \frac{1}{2} \left[ 1 - \frac{\psi}{\sqrt{v_2 + \psi}} \right]
\]
where: $v_2 = \left( \frac{\sigma^2}{\sigma_s} \right) \gamma + 1 + \left( 2 \gamma \cdot \left( 1 - |\rho|^2 \right) \right)$

C. BPSK

BPSK are QAM modulation with $M=2$. According to [11] and by using the same methodology which had been presented in [3], after many mathematical manipulations, the BER of BPSK will be:
\[
P_{\text{BER}}(\text{BPSK}) = \frac{1}{2} \left[ 1 - \frac{\psi}{\sqrt{v_1 + \psi}} \right]
\]
where: $v_1 = \left( \frac{\sigma^2}{\sigma_s} \right) \gamma + 1 + \left( 2 \gamma \cdot \left( 1 - |\rho|^2 \right) \right)$

III. AMC ENHANCEMENT

The AMC system is modeled as a Moore’s state machine, where each state is represented by a couple formed by a modulation order and a coding rate: the considered modulations in the uplink are BPSK, QPSK, and 16-QAM with the coding rate $1/2$. The aim of the presented model is to formulate a unified expression that can be adapted easily for different AMC techniques following the user requests and/or the system characteristics. Adaptation algorithm is basically characterized by some thresholds, representing the changing events between different transmission schemes: when a threshold is reached, the modulation order change and the state machine keeps a different state until another threshold is reached. In [13], they consider a Nakagami-m channel model for each sub-channel, in which the channel quality is determined by the instantaneous received SNR ($\gamma$). With adaptive modulation, SNR is divided into $R+1$ non-overlapping regions by thresholds $\Gamma_0, \Gamma_1, ..., \Gamma_R$, where $\Gamma_0 < \Gamma_1 < ... < \Gamma_R < \infty$. The sub-channel is said to be in state $\varphi$ (i.e., rate $ID = \varphi$ will be used), if $\Gamma_\varphi \leq \gamma < \Gamma_{\varphi+1}$. To avoid possible transmission excessive errors, no Protocol Data Unit (PDU) is transmitted when $\gamma < \Gamma_0$. Note that, these thresholds correspond to the required SNR specified in the IEEE 802.16e standard [14], [15], that is, $\Gamma_0 = 6.4$ dB for BPSK, $\Gamma_1 = 9.4$ dB for QPSK, $\Gamma_2 = 16.4$ dB for 16 QAM, $\Gamma_R = \infty$. In [14] it was shown that, the probability of using rate $ID = \varphi$ (i.e., $P_\varphi(\gamma)$) can be obtained using Nakagami-m distribution:
\[
P_\varphi(\gamma) = \frac{\Gamma(\varphi, \gamma)}{\Gamma(\varphi)}
\]
(21)

For Rayleigh fading channel, $m=1$, $\Gamma(m)$ the gamma function, $\Gamma(m, \gamma)$ is the complementary incomplete gamma function and $\gamma$ is the average SNR.
\[
P_\varphi(\gamma) = \frac{\Gamma(\varphi-1, \gamma)}{\Gamma(\varphi)}
\]
(22)

\[
\Gamma(\varphi, \gamma) = \int_0^\infty e^{\gamma t} t^{\varphi-1} dt
\]
(23)

\[
\Gamma(\varphi) = \int_0^\infty e^{-t} t^{\varphi-1} dt
\]
(24)

So, by substituting in equation (21)
\[
P_\varphi(\gamma) = e^{-\frac{\gamma}{\gamma}} - e^{-\frac{1}{\gamma}}
\]
(25)

So, the values of $P_\varphi(\gamma)$ in case of BPSK, QPSK, and 16-QAM according to equation (25) are mentioned in table 1. Then the overall BER for the uplink mobile WiMAX with AMC deployment may be as follows:
\[
P_\text{BER}(\text{AMC}) = P_\varphi(\gamma) \cdot P_{\text{BER}}(\text{BPSK}) + P_\varphi(\gamma) \cdot P_{\text{BER}}(\text{QPSK})
\]
\[
+ P_{\text{BER}}(16\text{QAM})
\]
(26)

\[
P_\text{BER}(\text{AMC}) = P_\varphi(\gamma) \cdot \left[ \frac{1}{2} \left[ 1 - \frac{\psi}{\sqrt{v_1 + \psi}} \right] \right]
\]
\[
+ P_\varphi(\gamma) \cdot \left[ \frac{1}{2} \left( \frac{\psi}{\sqrt{v_2 + \psi}} \right) \right]
\]
\[
+ P_\varphi(\gamma) \cdot \left[ \frac{1}{2} \left( \frac{\psi}{\sqrt{v_3 + \psi}} \right) \right]
\]
(27)

where $\psi$, $v_1$, $v_2$, and $v_3$ as represented in (16), (18),(20).
Applying AMC on the WiMAX system allows the variation of the transmitted bit rate on the uplink. According to the value of the bit rate varies between $R_b=0.855$Mbps for BPSK (under conditions that 10Mbps bandwidth, $N=256$, Frame duration$=5$ms)\cite{15,16}, to $2R_b$ for QPSK to $4R_b$ for 16QAM, the effective bit rate $R_{eff}(AMC)$ will be calculated in the case of AMC mobile WiMAX as follows:

$$R_{eff}(AMC) = P(\phi)R(BPSK) + P(\phi)R(QPSK) + P(\phi)R(16QAM)$$ \hspace{1cm} (28)

These results show the advantage of using AMC in mobile WiMAX, the EBR is 290% of that of using BPSK alone, or 145% of that of using QPSK alone. Note also that when $\gamma$ is less than 6.4dB the system suffers outage with probability 0.02 which may be obtained by integrating equation (25) from 0 dB to 6.4dB. However, EBR is less than that of 16QAM. Never less the 16QAM can’t be used unless $\gamma$ is above 16.4dB, which corresponds to outage probability 0.065. Hence the subscriber will be served with variable rate according to $\gamma$, with a guaranteed BER and higher EBR, as shown in figure 2.

IV. RESULTS AND ANALYSIS

Numerical analyses were implemented for the presented model, which will be used to evaluate the BER for adaptive modulation technique for different speeds, different SNR, and different channel estimation qualities for different cases of the OFDM Symbol Duration with the system parameters mentioned in table 2, which are in accordance with IEEE802.16e standards. The presented analysis will focus only on the uplink direction. The obtained results will focus on the case of having FFT size 256, up to 1024 in different RF channels bandwidths, the symbol duration will be calculated for different FFT sizes as shown in table 2. The results are shown in figures 3 and 4 for the case of $\rho=1$. Repeating for the case of $\rho = 0.9999$, the results are plotted in figures 5, 6 and 7. From figure 3, it is shown that, as the symbol duration increases, the overall BER are going down that is because of by increasing the symbol duration.

The channel estimation may be not as good for the whole symbol duration due to channel time variation. So, severe ICI will occur which has a big effect on BER performance. Also, it can be noticed that, The BER is proportional to the number of subcarriers as they become closer to one another. As the value of $\rho$ decreases, the BER of the system becomes worse whatever, it may be the conditions (QPSK, QAM, and AMC). But AMC still have the best noticed performance due to the allocation of the suitable mapping scheme in the suitable operating SNR values. These results validate that the presented derived analytical model is fitted to the actual system. Figures 5, 6 and 7 have shown that, by increasing of the speed of the user or the symbol duration, the BER increases. That is due to the increase of Doppler frequency shift, so the ICI power increases which has a bad effect on BER.

Table 1. Probability of being in certain modulation scheme

<table>
<thead>
<tr>
<th>Type of modulation</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>BPSK</td>
<td>0.18</td>
</tr>
<tr>
<td>QPSK</td>
<td>0.2</td>
</tr>
<tr>
<td>16QAM</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Figure 2: EBR (Mbps) for AMC for different SNR (dB)

Figure 3: BER for AMC for different SNR (dB), for 60km/hr speed and $\rho=1$ for all symbol durations

Table 2. Mobile WiMAX standard system parameters

<table>
<thead>
<tr>
<th>Number of subcarriers (N)</th>
<th>256</th>
<th>1024</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channel Bandwidth (MHz)</td>
<td>1.75, 3.5 and 10</td>
<td>1.75, 3.5 and 10</td>
</tr>
<tr>
<td>OFDM Symbol Duration (μsec)</td>
<td>25,72, and 146</td>
<td>103,288, and 576</td>
</tr>
<tr>
<td>Speed (km/hr)</td>
<td>3,30,60,90, and 120 km/hr</td>
<td></td>
</tr>
<tr>
<td>Doppler Frequency (Hz)</td>
<td>9.72, 97.2, 194.4, 291.66, and 388.88</td>
<td></td>
</tr>
<tr>
<td>Carrier Frequency</td>
<td>3.5 GHz</td>
<td></td>
</tr>
</tbody>
</table>

Figure 4: BER for different SNR (dB), $N=256$, $\rho=1$, 25μsec symbol duration and 60km/hr speed
V. CONCLUSION AND FUTURE DIRECTIONS

In this paper, BER performance of mobile WiMAX was analyzed. The current work is focused on the case of having time varying channels and its estimations. Numerical solutions were deployed in order to study the behavior of the obtained BER against different values of channel estimation qualities, mobile speed, and number of carriers. The obtained results were for adaptive modulation technique applied for uplink WiMAX system, the adaptive modulation gives better performance than using any of BPSK, QPSK or 16 QAM alone. This work may be extended to evaluate the downlink mobile WiMAX system. It was also shown that applying AMC in mobile WiMAX systems given higher bit rate with guaranteed BER.

REFERENCES


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