Local Straightness: A contrast independent statistical edge measure for color and gray level images

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Abstract

Most existing methods for edge detection rely on contrast dependent thresholds. We show that a local measurement defined by the ratio of the smallest to the largest eigenvalue of the second moment matrix of filter kernels, can be used to separate smooth, low curvature curves and straight lines from noise, independent of contrast, in both color and gray level images. This is done without applying a threshold to the gradient magnitude. The edge images are defined as zero crossings in the gradient direction. The covariance matrix can easily be computed for both gray level images and color images. Further we show the potentiality of such a measure by integrating it with the Hough transform to extract long straight lines in noisy color images. The method is shown to successfully extract consistent line features from color images of a scene, captured under drastically different lightening conditions.

1. Introduction

Figure ground segmentation can be explained as separating the image structure into two different classes. One class defines interesting structure and the other non-interesting structure, often called noise.

Edge images are a traditional starting point for this process. The most common definition of an edge point is a point where the gradient magnitude is maximal in the gradient direction [2]. The influence of noise is usually reduced by applying a threshold on the gradient magnitude. This is a reasonable strategy since noise generally has a low gradient magnitude. Two main problems occur: First it is difficult to estimate the correct threshold, which typically changes for different images. Secondly it is not always true that noise has lower gradient magnitude than other structure. If we instead observe a local neighborhood and associate the characteristics of that neighborhood with its central point, we can use this as a description. The descriptors should be chosen so that interesting structure can be separated from noise. In this article we propose to use the ratio of the smallest eigenvalue to the largest eigenvalue of the second moment matrix of the gradient as a descriptor that enhances low curvature curves and straight lines. This is based on the assumption that gradient directions of a local neighborhood are highly correlated for these structures and less correlated for noise. The second moment matrix is computed for all color channels, over gaussian filter kernels of edge pixels.

2. Related Work

The second moment matrix has previously been used as a measure of edge strength by e.g. Harris and Stephens [5]. They define the edge strength as a weighted difference of the determinant and the square of the trace of the second moment matrix. Implicitly this is an evaluation of the eigenvalues since the determinant equals the product of the two eigenvalues and the trace equals the sum of the two eigenvalues. Both the determinant and the trace of the second moment matrix will change if the image contrast changes, which makes this measure dependent on contrast. Guru et al [4] have recently used the smallest eigenvalue to detect straight lines. They claim that the smallest of the two eigenvalues, computed from a set of connected edge points is an indicator of the straightness of that edge point. In general, this is not true. The eigenvalues, both the larger and the smaller, are dependent on the gradient magnitude. If the lightening conditions changes, then the values of both of
the eigenvalues will also change. This should not indicate a change in structure as happens if only one of the eigenvalues is used. (The only exception is when at least one eigenvalue is equal to zero.) In general the structure depends on the joint behavior of the two eigenvalues [1]. All approaches based on the second moment matrix can easily be extended to color images or multi images [3]. By computing the second moment matrix over all color channels, edge evidence along a given direction in one channel will reinforce edge evidence along the same direction in other channels. We will show that by using the ratio of the smallest to the largest eigenvalue, we overcome the problem of contrast dependency and still have a robust measure that separates structure from noise.

3. Theory

We will now go through the basic theory that we need to construct an algorithm for extracting our features. First the basic theory of edge detection is presented and later the explicit expressions needed for calculating the second moment matrix are given. We end this section by defining local straightness as a measure of the alignment of the gradient directions in a local neighborhood.

3.1. Edge Detection

We start up by defining an edge point as a point where the gradient magnitude is maximal in the gradient direction [2]. This can be implemented in terms of directional derivatives by detecting points where the second order distance [2] can be implemented in terms of directional derivatives by detecting points where the second order distance [2] is zero and the second order derivative in the gradient direction is zero and the third order directional derivative in the gradient direction is negative [7]. We define

\[ L(x, y; t_1) = g(x, y; t_1) \ast f(x, y) \]  

as being the convolution of an image \( f \) with a gaussian kernel \( g \), given by

\[ g(x, y; t_1) = \frac{1}{2\pi t_1} \exp\left(-\left(x^2 + y^2\right)/(2t_1)\right) \]  

where \( t_1 \) is the variance of the kernel. We will later refer to this variance as the deriviation scale. We now introduce a right handed coordinate system, \((u, v)\), where the \(v\)-direction is defined as the gradient direction. The second and third order derivatives in the gradient direction are written as \( L_{uv} \) and \( L_{uuv} \). An edge point can now be expressed as follows:

\[
\begin{align*}
L_{uv} &= 0 \\
L_{uuv} &< 0
\end{align*}
\]  

3.2. Second Moment Matrix

The second moment matrix, \( M \), is defined as:

\[
M = W((\nabla L)(\nabla L)^T) = W \begin{pmatrix}
\frac{\partial L}{\partial x} & \frac{\partial L}{\partial y} & \frac{\partial L}{\partial t} \\
\frac{\partial L}{\partial x} & \frac{\partial L}{\partial y} & \frac{\partial L}{\partial t} \\
\frac{\partial L}{\partial x} & \frac{\partial L}{\partial y} & \frac{\partial L}{\partial t}
\end{pmatrix}
\]  

where \( W \) is an averaging window. The second moment matrix can also be computed for color images. The partial derivatives must then be summed over all the different channels [3]. We choose the window to be gaussian with variance \( t_2 \) and by doing so we can now define the color second moment matrix, \( M_{col} \), to be:

\[
M_{col} = \begin{pmatrix}
c_{11} & c_{12} \\
c_{21} & c_{22}
\end{pmatrix}
\]  

where,

\[
c_{11} = g(x, y; t_2) \ast \left(\frac{\partial L_R}{\partial x} \frac{\partial L_R}{\partial x} + \frac{\partial L_G}{\partial x} \frac{\partial L_G}{\partial x} + \frac{\partial L_B}{\partial x} \frac{\partial L_B}{\partial x}\right)
\]  

\[
c_{12} = g(x, y; t_2) \ast \left(\frac{\partial L_R}{\partial y} \frac{\partial L_R}{\partial y} + \frac{\partial L_G}{\partial y} \frac{\partial L_G}{\partial y} + \frac{\partial L_B}{\partial y} \frac{\partial L_B}{\partial y}\right)
\]  

\[
c_{22} = g(x, y; t_2) \ast \left(\frac{\partial L_R}{\partial y} \frac{\partial L_R}{\partial y} + \frac{\partial L_G}{\partial y} \frac{\partial L_G}{\partial y} + \frac{\partial L_B}{\partial y} \frac{\partial L_B}{\partial y}\right)
\]  

\[
c_{21} = g(x, y; t_2) \ast \left(\frac{\partial L_R}{\partial x} \frac{\partial L_R}{\partial y} + \frac{\partial L_G}{\partial x} \frac{\partial L_G}{\partial y} + \frac{\partial L_B}{\partial x} \frac{\partial L_B}{\partial y}\right)
\]  

The sub-index denotes that the different color channels \((R, G, B)\) are filtered separately. By looking at the ratio of the eigenvalues of \( M_{col} \) we get information about the alignment of the gradient directions. Note that the importance of different directions are naturally weighted both by the gradient magnitude and the distance to the point under consideration.

3.3. Local Straightness

As stated earlier the ratio of the two eigenvalues of the color second moment matrix, \( M_{col} \), gives us a weighted measure of the alignment of the gradient directions. We now introduce local straightness, \( s \), \( s \in [0, 1] \). If this value is close to one then the weighted gradient directions can be well approximated by one direction, otherwise this is not the case. Definition:

\[
s = 1 - \frac{\lambda_{min}}{\lambda_{max}}
\]  

where,

\[
\begin{align*}
\lambda_{max} &= \frac{c_{11} + c_{22} + \sqrt{(c_{11} - c_{22})^2 + 4c_{12}^2}}{2} \\
\lambda_{min} &= \frac{c_{11} + c_{22} - \sqrt{(c_{11} - c_{22})^2 + 4c_{12}^2}}{2}
\end{align*}
\]
The local straightness, \( s \), is calculated only for edge points. Only edge points are integrated when evaluating the color second moment matrix, \( M_{col} \). This guarantees that \( \lambda_{\text{max}} \) will never be zero, since an edge point is defined as a maxima of the gradient in the gradient direction. By taking the ratio of the two eigenvalues we guarantee that the measure will be independent of local contrast changes. If the contrast changes, the two eigenvalues will change equally relative to themselves.

4. Experiments

We have carried out experiments on both real and synthetic images to test how our straightness measure separates edge points generated by noise from edge points that belong to curves of low curvature and straight lines.

4.1. Synthetic Images

The first row of Figure 1 contains three synthetic images of a circle, with different contrast to the background. Gaussian noise of different variance has been added separately. The second, third and forth row show scan lines (the middle row of the images), of the gradient magnitude (second row), the smallest eigenvalue (third row) and local straightness (forth row). The two values marked with two stars in each plot corresponds to the edge points that separates the circle from the background. The gradient magnitude (second row) clearly changes with contrast. In this special case one would not be able to choose a threshold that separates the edge points from noise in all images. The smallest eigenvalue (third row) can not be used to extract edge points. It varies with contrast and it fails to separate the two edge points from the noise. If it is to be used for edge extraction, noise has to be removed and contrast kept constant. Our straightness measure (forth row) is independent of contrast and gives the same response for all images. It also tells us how well the local neighborhood can be approximated by one direction. (Note that the marked points are close to one). The derivation scale and the integration scale were both set to 3 for these experiments.

4.2. Real Images

We have integrated our local straightness measure with the Hough transform [6] to show the potentiality for this measure to be used together with already existing methods developed for edge detection. Local straightness can be used to separate curves of low curvature and straight lines from noise, as we have showed previously. It is independent of contrast and tells us how well the directions of a neighborhood can be approximated by one direction. These properties are important if we want to robustly extract structural information from images. Figure 2 shows the result of extracting straight lines from two color images of a church. The images were taken with two different cameras at different times of the day. No care has been taken to set the camera parameters and the camera positions are just approximately the same. All algorithmic parameters were fixed during the experiment (derivation scale, 3; integration scale, 3; local straightness 0.95).

The following steps are involved in the algorithm:

1. Extract all edges, Figure 2, (1b and 2b).
2. Threshold edges on local straightness, Figure 2, (1c and 2c).
3. Apply the Hough transform to the remaining edges, Figure 2, (1d and 2d).

The algorithm successfully extracts the same structure in both images without changing the parameter settings. This could then further be used for matching the two images.
5. Further Work

One problem has not been investigated here: How to choose the scale parameters? For this to work they were chosen manually. It is important to have a strategy for the selection of these parameters and this will be further studied. Choosing the derivation scale depends on the scale at which we wish to detect structure. This depends on the task and the resolution of the image, so for specific applications this scale can be, at least, approximately known. Further, the derivation scale also affects the position of the edges, which is a problem if we want to use information from different scales.

The integration scale is used to collect statistics for the local straightness measure. When this scale is small, it will not produce a good estimate. When on the other hand the scale is increased, structure further and further away from the interest point increases its influence on that point.

6. Conclusions

We have presented a local straightness measure for both color and gray level edge images, based on the ratio of the smallest to the largest eigenvalue of the second moment matrix. We show that this ratio is contrast independent and that we do not need to put any threshold on the gradient magnitude. The potential use of this local straightness measure in combination with other edge detection algorithms is demonstrated by applying it together with the Hough transform. The method is shown to successfully extract consistent line features from color images of a scene captured under drastically different lightening conditions.

References