A Maintenance Service Contract for A Warranted Product

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Abstract - In this paper, we investigate a maintenance service contract for a warranted product carried out by the Original Equipment Manufacturer (OEM). The model was developed under the assumption that there are one consumer and one service provider who is the OEM. This is typically applied to the situation where the OEM is the only service provider such as in the mining industry. From the OEM point of view, two contract options were considered, they are: the OEM carried out all repairs and preventive maintenance activities; the OEM carries out failure and the customer undertakes house preventive maintenance actions. The model uses a non-cooperative game formulation by maximizing expected profits. We use a linear function of failure intensity to consider a product with increasing failure intensity. We obtain the pricing structure in the contract. And for the customer, they can choose the optimal option under the term of contract.

Keywords - Maintenance, service contract, game theory

I. INTRODUCTION

In a mining industry, heavy vehicles e.g. dump truck, excavator constitute a core part of its profit center, the availability of the vehicles are critical in achieving the revenue of the company. Moreover to be successful in the mining business requires a high availability of the heavy vehicles. As a result, an economic loses due to the low availability of this vehicle could be high. An appropriate maintenance action for the vehicle is an effective effort to minimize an unplanned maintenance (a preventive maintenance) and to re-set the condition of the vehicle to its best condition (a corrective maintenance). For preventive maintenance (PM) actions, it can be categories into perfect PM actions (restores the vehicle to new condition) and imperfect PM actions (takes to condition between new and current condition), as it eventually reduces the number of the vehicle failures. Meanwhile minimal repair, imperfect repair and perfect repair usually are known as corrective maintenance (CM) actions which give impact in the vehicle failure rate. In another words maintenance can increase the availability of the vehicle.

Many companies have limited skilled labors and maintenance facilities, due to expensive investment and this affects their capability in performing a preventive maintenance. As a result, performing an in-house maintenance service seems to be in-economical solution, and an out-sourcing maintenance service, either preventive or corrective, could be the better solution. The benefits of an out-sourcing maintenance service are two folds: to assure the maximum availability and to reduce the maintenance cost as describe in [1], [2], and [3].

There are some literatures regarding maintenance service contract. Reference [1], [2], and [4] developed a Stackelberg game theory model to obtain an optimal cost strategy with the agent as a leader and consumer as the follower. However, this strategy did not consider any preventive maintenance action. Further, [5], [6], and [9], developed a similar model with the inclusion of a preventive and corrective maintenance policy.

In this paper, we develop a model of maintenance service contract for a warranted product where preventive maintenance (PM) is offered in one package of warranty. The product under consideration is a heavy vehicle product (dump truck) sold directly by the manufacturer to consumer. During the warranty period all failures and preventive actions will be rectified by the manufacturer without any charge to the consumer. After the warranty ceases, all corrective maintenance (CM) and PM actions will be borne to the consumer.

There are two options available to the consumer to carry out PM and CM after the warranty ends i.e:
1. The consumer pays the OEM to repair the product once it fails and to undertake in-house maintenance service for the PM.
2. The consumer buys a service contract (PM and CM) from the manufacturer (OEM).

In the first option, the consumer has to pay an amount of repair cost for every single failure after the warranty expiry. On the other hand, in the second option, the consumer pays the cost of service contract provided by the OEM. The OEM will then rectify all the failures and undertakes preventive maintenances without burdening the consumer with the cost of the rectification and maintenance. Furthermore, if the rectification takes a longer time than it supposed to be, then the OEM would pay a penalty cost to the consumer. The cost structure paid by the consumer includes cost of service contract and costs of repair, the amount of the costs depend on negotiation between the two parties: the OEM and the consumer. This is possible if the OEM is the only possible service provider. The aim of the paper is to obtain an optimal option for the consumer and the structure of the cost for the OEM by considering PM and CM. This cost structure will be solved by a non-cooperative Nash equilibrium.

The paper is organized as follows. Section II gives the methodology which includes model formulation and
model analysis. Section III deals with the result of the solution and numerical examples for the case where the product has a Weibull failure distribution. Section IV presents the discussion. Finally, a brief conclusion and a discussion for future work are presented in Section V.

II. METHODOLOGY

We consider an industrial product sold with warranty period \([0, W]\) and the warranty covers not only CM but also PM. After the warranty expires both CM and PM are borne by the consumer. The consumer can do PM in house if the maintenance facility and the labors are available. If not, the consumer has to buy a maintenance service from the OEM. But for CM services, the consumer needs to buy from the OEM. Let \(L\) denotes as a life time product.

We define a service contract as the following. An OEM gives a service covering preventive and corrective maintenance to the consumer with a fixed cost \(P_G\) within the contract period. If a fail product could not reach an operational condition by \(\tau\) unit time after the time it failed, the OEM should pay a penalty cost. The amount of the penalty cost is proportional to the waiting time after the time \(\tau\) with the cost of \(C_T\). The penalty cost accounted as a compensation given by the OEM.

First we will model the product failure and repair, then the objective function will be defined for both the OEM and the consumer in the form of expected return. To maintain the product, the OEM offers two options of service contract.

Option \(O_0\): After the expiry of warranty in the interval \([0, L]\), OEM does not provide any preventive maintenance. The consumer does an in-house PM to undertake a preventive maintenance. If the product fails, OEM will charge the consumer for the cost of repair \(C_m\) whenever the repair time exceeds \(\tau\) unit time.

Option \(O_1\): For a fixed cost of service contract \(P_G\), the OEM agrees to repair all the failures with PM and CM along the interval \([W, L]\), without any cost. If a fail product could not reach an operational condition by \(\tau\) unit time after the time it failed, the OEM should pay a penalty cost (see Table I below).

<table>
<thead>
<tr>
<th>TABLE I</th>
</tr>
</thead>
<tbody>
<tr>
<td>OEM OPTIONS</td>
</tr>
<tr>
<td>([0, W)) Option (O_0) in ([W, L)) Option (O_1) in ([W, L))</td>
</tr>
<tr>
<td>PM Warranty In House PM Service Contract</td>
</tr>
<tr>
<td>CM Period Service Contract Service Contract</td>
</tr>
</tbody>
</table>

After the expiry of warranty, the consumer must choose the option \(O^*\) taken from the set \(\{O_0, O_1\}\). And the OEM has to determine the optimal cost structure (service contract cost \(P_G\) for option \(O_1\) and repair cost \(C_m\) for option \(O_0\)). The values of service contract cost \(P_G\) and repair cost \(C_m\), will be formulated through a non-cooperative game theory Nash equilibrium.

A. Model Formulation

A.1. Product Failures

We use a black-box approach to model product failure. And it will be modelled by its distribution function. We consider a dump truck as a repairable product and every failure is rectified by a minimal repair. With a minimal repair, the failure rate after repair is the same as that before it fails. It is assumed that the rectification time is relatively small compared to its mean time between failures, so that it can be neglected. As a result, the failure occurs as a Non-Homogenous Poisson process (NHPP) with the intensity function \(r(x)[7]\).

A.2. PM Policy

The PM policy follows [10] by assuming a PM occurs continuously along the time horizon. We consider two failure rate functions given in (1) and (2). Their failure rates are defined as follow:

\[
\begin{align*}
\nu_0(x) &= \begin{cases} 
\eta r(x) & 0 \leq x < W \\
\eta r(W) + \eta r(x) & W \leq x < L 
\end{cases} \quad (1) \\
\nu_1(x) &= \eta r(x) & 0 \leq x < L 
\end{align*}
\]

We assume that \(\eta, \eta'\) denote the gradient of the failure function with \(\eta' > \eta\). This condition \(\eta' > \eta\) indicates that PM done by OEM is better than that of the consumer. Further, \(r(x)\) is an increasing function in \(x\), means that even though there is a maximum effort of the consumer in undertaking the maintenance process, failure rate still increases in time as the product aging.
A.3. Maintenance Cost

We assume that \( C_{pm} \) is a preventive maintenance cost per unit of time and \( C_m \) is the average cost of repair cost. We denote \( C_r \) as the expected penalty cost of the OEM if the repair time exceeds \( \tau \), and \( C_e \) is the average of repair and corrective cost in the interval \([W,L]\) when the consumer does not buy a service contract, and \( C_b \) denotes the pricing of product.

The cost of CM after the expiry of warranty for option \( O_1 \) can be expressed as \( C_m \), where \( C_m \) is the average cost of each rectification. And for option \( O_1 \) the cost of PM is \( C_{pm}(L-W) \). The penalty cost is a function of the repair cost \( Y \) \((r.v \text{ with the distribution function } G(Y))\) and \( \tau \). Let \( Y_i \) denotes the time needed to rectify the \( i\)-th failure and for option \( O_1 \), \( R_i(W,L) \) denotes the number of failure in the interval \([W,L]\) with \( 1 \leq i \leq R(W,L), \) where

\[
R_i(W,L) = \int_{W}^{L} \eta_i(x) \, dx.
\]

The penalty cost in the interval \([W,L]\) is given by \( C_r \sum R_i(W,L) \max(0, Y_i - \tau) \).

A.4. Customer’s Decision Problem

We assume that \( Y_i \) denotes the time needed to return to the operational condition after the \( i\)-th failure and the number of failures in the interval \([W,L]\) whenever choose the option \( O_1 \) is \( N_{O_1} \).

Consumer profit upon choosing the option \( O_1 \) is \( \omega(O_1; P_G, C_r) \), and the value is given by

\[
\omega(O_1; P_G, C_r) = R \left[ L - W - \sum_{i=1}^{N_{O_1}} Y_i \right] - C_b - P_G + C_r \left( \sum_{i=1}^{N_{O_1}} \max(0, Y_i - \tau) \right)
\]  

(3)

Suppose that the number of failures in the interval \([W,L]\) upon choosing the option \( O_0 \) is \( N_{O_0} \). The consumer profit upon choosing the option \( O_0 \), \( \omega(O_0; P_G, C_r) \) is given by

\[
\omega(O_0; P_G, C_r) = R \left[ L - W - \sum_{i=1}^{N_{O_0}} Y_i \right] - C_b - P_G - CN_{O_0}
\]  

(4)

A.5. OEM’s Decision Problem

OEM revenue for option \( O_0 \), after the expiry of the warranty:

\[
\pi(O_0) = C_b + (C_e - C_m) R_0(W,L), C_e > C_m
\]  

(5)

OEM revenue for option \( O_1 \), after the expiry of warranty:

\[
\pi(O_1) = P_G \cdot [\text{Penalty cost}] - [\text{CM cost}] - [\text{PM cost}]
\]  

(6)

In our model, by using Nash bargaining solution for both options \( O_0 \) and \( O_1 \), OEM and consumer will negotiate the pricing of service contract \( P_G \) and the cost of \( C_m \), respectively.

B. Model Analysis

We assume that OEM and consumer have the same attitudes to risk, in order to make the solution reach equilibria.

B.1. OEM’s Decision Problem

The revenue function OEM for option \( O_0 \) is given in (5). Since the failure follows the NHPP, then as in [8]

\[
P(N_0(w,l) = n) = \sum_{n=0}^{\infty} n! e^{-R_0(w,l)} \frac{R_0(w,l)^n}{n!}
\]  

(7)

Hence the expected revenue of the OEM whenever the consumer chose the option \( O_0 \) is \( E[\pi(O_0)] \), and its value is given by

\[
E[\pi(O_0)] = C_b + (C_e - C_m) R_0(W,L)
\]  

(8)

Where \( R_0(W,L) \) is given by \( R_0(W,L) = \int_{W}^{L} \eta_0(x) \, dx \).

From (6) then the expected revenue of the OEM, \( E[\pi(O_1)] \), is given by

\[
E[\pi(O_1)] = P_G - E[\text{Penalty cost}] - E[\text{CM cost}] - E[\text{PM cost}]
\]  

(9)

The expected number of failures after the expiry of the warranty, \( E[N(w,l)] \) is given by

\[
E[N(w,l)] = R_1(W,L) = \int_{W}^{L} \eta_1(x) \, dx
\]  

(10)
Expected cost of rectification is expressed as

\[ EC_m = C_m R_1 (W, L) \]  

(11)

Expected cost of penalty

\[ EC_r = C_r R_1 (W, L) \left( \int_0^\infty (y - \tau) g(y) \right) dy \]  

(12)

Using integral by part, we have

\[
\int_0^\infty (y - \tau) g(y) dy = (y - \tau) G(y) \bigg|_\tau^\infty - \int_\tau^\infty G(y) dy
\]

(13)

Thus (12) becomes

\[ EC_r = C_r R_1 (W, L) \int_\tau^\infty [1 - G(y)] dy \]  

(14)

Expected PM cost is

\[ EC_{pm} = C_{pm} (L - W) \]  

(15)

Then, total expected revenue of the OEM in (9) becomes

\[
E[\pi(O_1)] = P_G - C_r R_1 (W, L) \int_\tau^\infty [1 - G(y)] dy - C_m R_1 (W, L) - C_{pm} (L - W)
\]

(16)

B.2. Customer’s Expected Profit

From (3) then the expected profit of the consumer upon choosing the \( O_1 \) option, \( E[\omega(O_1)] \), is given by

\[
E[\omega(O_1)] = \int L - W
\]

\[
R \left[ -R_1 (W, L) \left( \int_0^\infty [1 - G(y)] dy \right) \right] - C_b - P_G
\]

\[
+ C_r R_1 (W, L) \left( \int_0^\infty [1 - G(y)] dy \right)
\]

(17)

And from (4), the expected profit of the consumer upon choosing the option \( O_0 \), \( E[\omega(O_0)] \), is given by

\[
E[\omega(O_0)] = \int L - W
\]

\[
R \left[ -R_0 (W, L) \left( \int_0^\infty [1 - G(y)] dy \right) \right] - C_b - P_0
\]

\[
+ C_r R_1 (W, L) e^{R_0 (W, L) [C_i - 1]}
\]

(18)

III. RESULTS

We assume that the consumer uses the product up to \( L \), so that there would be a negotiation between the consumer and the manufacturer to determine the value of the service contract cost, \( P_G \), and the repair cost, \( C_m \). The optimal solution is obtained using the method of Nash equilibrium. In principle this method can be used whenever there is a negotiation between the two parties.
TABLE III
SERVICE CONTRACT AND REPAIR COST

<table>
<thead>
<tr>
<th>$\eta$</th>
<th>$P_G^*$</th>
<th>$E[\omega(O_0; P_G^*, C_m)]$</th>
<th>$C_m^*$</th>
<th>$E[\omega(O_0; P_0, C_m^*)]$</th>
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</thead>
<tbody>
<tr>
<td>0.10</td>
<td>7.86</td>
<td>7.12</td>
<td>0.55</td>
<td>6.50</td>
</tr>
<tr>
<td>0.11</td>
<td>7.39</td>
<td>6.59</td>
<td>0.87</td>
<td>5.45</td>
</tr>
<tr>
<td>0.12</td>
<td>6.92</td>
<td>6.06</td>
<td>1.16</td>
<td>4.40</td>
</tr>
<tr>
<td>0.13</td>
<td>6.45</td>
<td>5.53</td>
<td>1.42</td>
<td>3.35</td>
</tr>
<tr>
<td>0.14</td>
<td>5.97</td>
<td>5.00</td>
<td>1.65</td>
<td>2.30</td>
</tr>
<tr>
<td>0.15</td>
<td>5.50</td>
<td>4.47</td>
<td>1.86</td>
<td>1.25</td>
</tr>
<tr>
<td>0.16</td>
<td>5.03</td>
<td>3.94</td>
<td>2.05</td>
<td>0.20</td>
</tr>
<tr>
<td>0.17</td>
<td>4.56</td>
<td>3.41</td>
<td>2.22</td>
<td>-0.85</td>
</tr>
<tr>
<td>0.18</td>
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<td>2.38</td>
<td>-1.90</td>
</tr>
<tr>
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<td>2.53</td>
<td>-2.95</td>
</tr>
<tr>
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<td>2.67</td>
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<td>2.79</td>
<td>-5.05</td>
</tr>
<tr>
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<td>0.76</td>
<td>2.90</td>
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</tr>
<tr>
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<td>0.23</td>
<td>3.01</td>
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</tr>
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</tr>
<tr>
<td>0.26</td>
<td>0.32</td>
<td>-1.36</td>
<td>3.29</td>
<td>-10.30</td>
</tr>
</tbody>
</table>

IV. DISCUSSION

Table III presents the decreasing value of expected revenue [profit] of OEM [consumer] as the level maintenance $\eta$ increases. It also shows in Fig. 2 that option $O_i$ is the optimal option for the consumer.

In Fig. 2, we show that option $O_i$ gives a higher expected value for the OEM (consumer) compared to option $O_j$. It is clear that the larger the gradient of the failure rate the greater the discrepancy between the expected revenues of the different option. It also depicted that although for a high reliability, the effect of service contract in terms of economic revenue is less significant, for a lower reliability (high failure rates), the increased profit gained by the consumer from choosing the service become more apparent.

V. CONCLUSION

In this paper, we have studied a maintenance service contract for a warranted product with increasing failure intensity. Some insights are discussed and one can extend to service contract for product sold with two-dimensional warranty. This topic is currently under investigation.

ACKNOWLEDGMENT

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