Abstract— This paper uses a linear transformer model to investigate the effect of the lift-off on the results of the thickness measurement of non-ferromagnetic metallic plates. The transformer model previews that the time derivative of the magnetization curves obtained for different gaps between the excitation coil and the plate should intercept in a single point when low magnetic coupling factors are considered. To assess the validity of the model, results are compared with experimental data obtained with a giant magnetoresistive (GMR) sensor probe. For comparison, the sensor output voltage time derivative must be performed as well. The similarity of the theoretical model results and those obtain experimentally with pulsed excitation, confirms the correctness of the transformer approach.

Keywords: Pulsed Eddy Currents; GMR sensors; thickness measurement; Lift-off Point of Intersection.

I. INTRODUCTION

Traditional eddy currents testing (ECT) methods designed for non-destructive evaluation use a sinusoidal excitation current to produce the magnetic field required to create the eddy currents in the metallic sample under study. These currents generate a secondary magnetic field to be measured that contains all the information concerning the sample defects or conductivity discontinuities. Recently, transient eddy currents produced by a square wave received interest in applications such as the detection of defects [1,2], the characterization of sub-surface cracks [3,4], and the measurement of a specimen thickness [5,6] or of its electrical conductivity [7]. Due to the rich frequency content of the signal used deeper penetration of the electromagnetic field on the sample is achieved [8-10] and simultaneous evaluation of the material at different depths can be carried out. These are considered the main advantages of this method over the sinusoidal eddy currents method [11,12] but data interpretation still remains a difficulty.

In order to understand the basic physical phenomena present at the thickness measurements when transient eddy currents techniques are used an analogy with the theory of the linear transformer is proposed within this paper.

The analogy of the eddy currents non-destructive testing method with a transformer has already been published. In [13,14] an analytical expression for the eddy currents sensor impedance is established when a sinusoidal excitation is imposed in a simple coil used both for excitation and sensing purposes.

The approach of the present paper is to use the electrical equivalent circuit of a transformer to investigate the origin and the characteristics associated with the Lift-Off Point of Intersection (LOI). The LOI is a point observed experimentally where the time-domain transient eddy currents signals intersect when only the lift-off (distance between the EC sensor and the sample) changes and all the other testing conditions remain identical in the experiment.

The LOI has been reported with transient and harmonic eddy currents [15-18] and with different inductive probes (single coil or concentric reflection probe with and without ferrite core). Its importance has been verified experimentally in several applications like corrosion [19] or cracks detection [20] but its physical interpretation is not clear yet. In order to understand the basic physical phenomena present numerical or analytical tool are useful. This is the case in [21] where an analytical series development was used to model the magnetic field penetration inside a layered medium in the presence of an excitation coil. This model, when used under pulsed excitation, has shown that the response curves don't intersect exactly, but all of them pass through a small region that could be identified.

The present paper gives some insight to the Lift-off Point of Intersection with an intuitive analogy with the electrical transformer behavior. The results obtained with the transformer model are compared with experimental data obtained with a giant magnetoresistive EC probe.

The rest of the paper is organized as follows. Section 2 describes the transformer model to predict the LOI and presents the results obtained. As stated, the idea is to study a typical problem of magnetic coupling between two circuits: the probe excitation coil and the conductive sample, with a lumped electrical circuit. Section 3 describes the experimental setup, hardware and software, for the measurements taken with the GMR based probe for thickness evaluation of a metallic plate and presents the experimental data. Further work and conclusions are outlined in Section 4.

II. TRANSFORMER MODEL

The magnetic coupling between the excitation coil included in an EC probe and the non-conductive plate under test, appears to be analogous to a two winding transformer as the one represented in Fig. 1(a).
In this approach, Fig. 1(b), the primary of the transformer corresponds to the excitation coil included in the EC probe and the eddy currents produced in the plate are pictured as the short-circuited secondary of the transformer

\[ u_2(t) = 0 \]  

(1)

The time domain equations for the transformer circuit are obtained by using Faraday’s induction law:

\[
\begin{align*}
\psi_1(t) &= \psi_1^0 + \frac{du_1}{dt} \\
\psi_2(t) &= \psi_2^0 + \frac{du_2}{dt}
\end{align*}
\]

(2)

where \((u_1, u_2)\) and \((i_1, i_2)\) are the primary and secondary voltages and currents and \((\psi_1, \psi_2)\) the primary and secondary fluxes. The following two conventions apply to obtain (2):

\((i_1, i_2)\) are oriented so as to create concordant magnetic field in the core of the transformer and \((u_1, u_2)\) to produce a positive power from the primary terminals into the transformer and from the transformer into the load. Assuming that the transformer core has a linear behavior, then:

\[
\begin{align*}
\psi_1(t) &= L_1 i_1(t) + L_M i_2(t) \\
\psi_2(t) &= L_M i_1(t) + L_{22} i_2(t)
\end{align*}
\]

(3)

where \(L_1\) and \(L_{22}\) represent self-inductance of the primary and secondary coils and \(L_M\) the mutual inductance.

Considering that in a single core ideal transformer, the primary and secondary voltage ratio is equal to the corresponding winding turns ratio and the currents to its reverse, equations (2) and (3) can be represented by the equivalent circuit diagram, as presented in Fig. 2, including an ideal transformer with the primary and secondary number of turns ratio given by \(\nu = n_1 / n_2\).

To predict and understand some phenomena within the eddy currents physical problem, the ideal transformer can be

\[ \lambda_1 = L_{11} - \nu L_M \]  

(4)

is the induction coefficient relative to the leakage flux linked to the current that runs in the EC probe (relates this excitation current with the magnetic flux not coupled with the plate) and

\[ \lambda_2 = L_{22} - L_M / \nu \]  

(5)

the induction coefficient relative to the leakage flux linked with the eddy currents produced in the plate. The induction coefficient, \(l_{11}\):

\[ l_{11} = \nu L_M \]  

(6)

is the induction coefficient relative to the transformer main flux, or in the ECT system the main linkage flux linked simultaneously to the excitation current and to the eddy currents created in the plate. The probe and the plate are magnetically coupled by \(L_M\) that depends on the coupling factor \(k\) and varies inversely to the lift-off distance mutual inductance:

\[ k = \frac{L_M}{\sqrt{L_{11} L_{22}}} \]  

(7)

In Fig. 2, the relation between the currents is:

\[
\begin{align*}
i_1 &= -\frac{i_2}{\nu} \\
i_{10} &= i_1 - i_2
\end{align*}
\]

(8)

The current \(i_{10}\) is called the magnetizing current reduced to the primary because the magneto-motive force relative to \(i_{10}\) is equal to the magneto-motive force resulting from the currents \(i_1\) and \(i_2\), actuating simultaneously:

\[ n_1 i_{10} = n_1 i_1 + n_2 i_2 \]  

(9)

Making the analogy with the eddy current testing experiment, the current \(i_{10}\) must be proportional to the magnetic field that experimentally is accessed by the GMR sensor included in the EC probe.

The turns ratio, \(\nu = n_1 / n_2\), of the ideal transformer is an arbitrary parameter. The real transformer and the equivalent circuit are formally equivalent from the accessible voltages \((u_1, u_2)\) and currents \((i_1, i_2)\). Nevertheless, \(\nu\) must be chosen in order the actual components of the circuit are physically realizable, corresponding to inductances with positive values. Considering the analogy between the eddy currents in the plate and the transformer secondary, to assume \(\nu = n_1\), (secondary with a single coil) seems appropriate.
removed from the equivalent circuit (secondary short-circuited, \( n_s=0 \)) and one gets the circuit shown in Fig. 3.

The meaning of:

\[ r_2' = v^2_t f_2 \quad \text{and} \quad \lambda_{12} = v^2 \lambda_{12} \quad (10) \]

are respectively the resistance of the secondary as seen from the primary and \( \lambda_{12} \) as seen from the primary. \( \xi_1 \) is the induction coefficient relative to the main flux reduced to the primary.

A resistance \( R_1 \) in Fig. 3 is now the sum of \( r_1 \) with an external resistance, with the minimum value that guarantees a regime without current oscillations in the excitation coil.

The analysis of the transient phenomena can be studied in the time-domain using the equations for the “T” circuit shown in Fig. 3, when a voltage step is applied to the primary, in accordance to the real experimental situation.

\[
\begin{align*}
&u_G(t) = R_1 i_1 + \xi_1 \frac{di_1}{dt} + \lambda_{12} \frac{di_2'}{dt} + r_2 i_2' \\
&l_1 \frac{di_1}{dt} = l_1 \frac{di_1}{dt} - l_2 \frac{di_2'}{dt} = \lambda_{12} \frac{di_2'}{dt} + r_2 i_2'
\end{align*}
\]

with the assumption that \( u_G(t) \) is a step function:

\[
u_G(t) = \begin{cases} 0, & t < 0 \\ E, & t \geq 0 \end{cases}
\]

The system above can be transformed in a second order equation in \( i_2' \):

\[
\begin{align*}
\left( \xi_1 + \frac{\lambda_{12}^2}{l_1} \right) \frac{d^2 i_2'}{dt^2} + & \left( r_2 + \frac{\lambda_{12}^2}{l_1} + \frac{r_2 \xi_1}{l_1} + r_2' \right) \frac{di_2'}{dt} + \frac{R_2 r_2'}{l_1} i_2' = 0
\end{align*}
\]

with a solution of the type

\[
i_2' = K_1 e^{st} + K_2 e^{-st}, \quad s_{12} = -\frac{B \pm \sqrt{B^2 - 4AC}}{2A} \quad (14)
\]

In (14) the constants \( K_1 \) and \( K_2 \) are specified by the application of the physical current continuity in the inductances. Thus,

\[
i_2' (t = 0^+) = i_2' (t = 0^-) = 0 \Rightarrow K_1 = -K_2 = I_0 \quad (15)
\]

Both currents \((i_1 \text{ and } i_2')\) are zero in the initial moment \( t = 0^+ \). Therefore all the voltage \( E \) appears instantaneously in the inductances:

\[
\xi_1 \frac{di_1}{dt}\bigg|_{t=0^-} + \lambda_{12} \frac{di_2'}{dt}\bigg|_{t=0^-} = E
\]

The application of this constraint allows the determination of the constant \( I_0 \) in (15):

\[
I_0 = \frac{E}{\xi_1 + \lambda_{12}^2 + \frac{\lambda_{12}^2 r_2'}{l_1}} \quad (17)
\]

The current \( i_0(t) \) may be determined from \( i_2(t) \) and taking into account that \( i_{10}(t = 0^-) = 0 \):

\[
i_{10}(t) = I_0 \left( r_2' \frac{di_2'}{dt} \right) + I_0 \left( r_2' \frac{\lambda_{12}^2}{s_{12} l_1} \right) e^{st} + \frac{1}{s_{12} - \frac{r_2'}{l_1}}
\]

According to (9) and (13) the relevant current equations may be condensed as

\[
i_{10}(t) = I_1 e^{st} - I_2 e^{-st}
\]

\[
i_{10}(t) = I_1 e^{st} + I_2 e^{-st} - (I_1 + I_2)
\]

\[
i_{10}(t) = i_{10}(t) + i_2(t)
\]

where \( I_1 \) and \( I_2 \) are the coefficients that affect the exponentials in (13) and \(- (I_1 + I_2)\) the symmetric of the third term in the same equation.

The time derivative of \( i_{10}(t) \) is also a useful quantity:

\[
\frac{di_{10}(t)}{dt} = s_{12} I_1 e^{st} + s_{12} I_2 e^{-st}
\]

\[\text{A. The LOI Effect}\]

In the analogy of the eddy current technique metallic plate testing with the commonly known transformer model, it is assumed that the metallic plate can be simulated by a lumped single coil secondary with a given inductance \( L_{22} \) and a resistance \( r_2 \) and that the distance (lift-off) variation between the excitation coil and the plate can be simulated by varying the magnetic coupling factor \((k)\). It is also assumed that the distance variation between the excitation coil and the plate, in the real experimental setup, is small in order that the parameters \( r_2 \) and \( L_{22} \) connected to the secondary (testing plate) remain unchanged.

To simulate the effect of coil to plate lift-off variation, different magnetic coupling factors were introduced. The currents in the model were calculated, with a special attention to those that are observable in the experimental setup.

The transformer equivalent circuit is of second order and as a consequence the currents are the combination of two exponentials. The numerical results obtained from the model indicate that the two exponentials present very different time constants. Thus, from now on, we will consider these regimes as the slow regime and the fast regime.

In Fig. 4 the magnetization current \( i_{10}(t) \) is represented. This current is such that it contains the combined magneto-motive forces of the currents \( i_1(t) \) and \( i_2(t) \) as stated in (9).
The secondary (plate) current $i_2(t)$ is displayed in Fig. 5. Although, this current is not experimentally measurable in our setup the picture contains useful information. The negative sign of $i_2(t)$ means that its magnetizing effect is contrary to the primary current. On the other hand it is also clear that $i_2(t)$ presents a fast increase in the first instants and afterwards decreases slowly to zero.

![Figure 4: Magnetization current for different coupling factors.](image)

The presence of the two exponential regimes, is also evident when we compute the time derivative of $i_{10}(t)$, which is depicted in Fig. 6. In this derivative the slow regime is negligible when compared to the fast one.

![Figure 5: Secondary current for different coupling factors.](image)

Fig. 6 shows a point of intersection for several magnetic coupling factors representing an equal number of lift-off situations.

III. EXPERIMENTAL SETUP

In contrast to conventional eddy currents sinusoidal methods, in the transient (or pulsed) eddy currents technique a pulse or square wave is used as the voltage source. The resulting current through the exciting coil generates transient eddy currents at the surface of the conductive test sample that propagate down into the material. In the experimental setup the probe includes an excitation coil and a giant magnetoresistive (GMR) as a sensing element to measure the magnetic field in the plate vicinity.

A. Description of the Experimental Setup

Fig. 7 shows a schematic of the implemented experimental setup. A programmable arbitrary waveform generator Agilent 33201A controlled by Matlab over USB, outputs the driving pulses. This voltage signal is boosted into a mosfet module switching on and off a power voltage source chip. An exponential current is then generated by applying this feeding signal to the probe, since the current is running through an excitation winding and a resistor controlling its value. The pulse active state and the time period during which data is acquired take as long as the secondary generated transient response on the sensing element reaches steady state. The entire transient is captured using an oscilloscope.

![Figure 7: Block diagram of the experimental setup.](image)

The used probe is described in Fig 8. Its parameters and arrangement have been optimized based on finite element modeling results. The GMR sensor is placed vertically inside of the excitation coil in order to have its sensing axis parallel to the magnetic field produced. It is an AA002-02 sensor from NVE configured in a Wheatstone bridge so it must be powered. The mutual position of the probe itself and a tested specimen is settled by a precise positioning stage enabling to set the desired lift-off with a 10μm resolution.

![Figure 8: Cross section of a probe with GMR.](image)

The direct probe output must be pre-processed before digitalization to preserve the signal without noise. The output of the GMR bridge is amplified by an instrumentation amplifier and the response averaged and digitalized by an oscilloscope Tektonix TDS 2014. Data acquisition to the computer is performed over the GPIB bus. A Matlab program carries out instrument controlling and synchronization, data acquisition and analytical estimation of responses for post-processing.
B. Results Obtained with the GMR Based Probe

Sensor’s output based on giant magneto-resistance effect is directly proportional to magnetic field strength in its immediate vicinity. Fig. 9 depicts the probe pulsed responses with different lift-offs corresponding to different magnetic coupling factors.

![Figure 9. Output voltage of the GMR sensor for lift-offs between the probe and a 2 mm thick stainless steel 304 plate.](image)

The curves show different rise times due to the effect of the eddy currents induced in the plate. In the same conditions, lower lift-offs imply stronger eddy currents inside the aluminum plate. As the resultant magnetic field opposes the primary magnetic field created by the excitation current that runs in the coil probe, the total magnetic field accessed by the GMR sensor along the coil axis is decreased.

As referred in Part A of Section II, the shown curves represent indirectly the magnetization current \( i_{10}(t) \). Reporting to the linear electrical transformer model, the magneto-motive force relative to \( i_{10} \) is equal to the magneto-motive force resulting from the currents \( i_1 \) and \( i_2 \), actuating simultaneously. Thus, the current \( i_{10} \) must be proportional to the magnetic field that in the eddy currents testing experiment is accessed by the GMR sensor included in the EC probe placed in the axis of the excitation coil.

![Figure 10. Exponential components of pulsed GMR responses for different lift-offs.](image)

As derived in Section 2, the magnetization current \( i_{10}(t) \) is composed of two exponential curves and a constant determining the steady state level. The GMR output curves depicted in Fig. 9 have been fitted by (19) and are presented in the Fig. 10. The same identification numbers apply. The influence of the lift-off effect described with reference to Fig. 9 is more evident especially in the low transient regimes. The increase in the lift-off, decreases the time constant and the initial values change inversely to keep the steady state level constant.

The lift-off point of intersection (LOI) has been reported before [16-18] when inductive detection coils were used in eddy current testing methods. Those coils produce output voltages proportional to the time derivative of the magnetic flux across them. When GMR sensors are being used the output signal is proportional to the magnetic field in its vicinity. Therefore to access the LOI, the time derivative of the GMR output signal must be performed in the experimental data. In Fig. 11 depicts the time derivatives of the instantaneous output GMR voltages represented in Fig. 9. Having the analytical description of the investigated step responses, Fig. 10, the time derivation is obtained analytically as well. As in Fig. 6 a point of intersection for four different lift-off situations is evident.

![Figure 11. LOI illustration for the 2 mm thick stainless steel plate.](image)

Figs. 9-11 were obtained for a 2 mm thick stainless steel 304 plate. Five material thicknesses were obtained by stacking different plates of the same material. For each of these thicknesses the response of the GMR was acquired for four different lift-off values for the same step excitation current. The time derivative of these experimental data was determined. Curves similar to those depicted in Fig.11 were obtained for each thickness except that the LOI is reached at a different instant of time. A set of the LOIs for the tested thicknesses is shown in Fig. 12.

The LOI moves along the fitted exponential curve depending on the thickness of the sample. Each curve of the magnetic field acquired for a different specimen thickness reaches steady state at a different instant of time. That is ruled by the penetration time of the eddy currents through a specimen weakening the measured magnetic field. This time is thus related to the thickness of the specimen.
The same data represented in Fig. 12 can be displayed with the thickness as a function of the time derivative of the output voltage of the GMR taken at the time of interception (instant of time of the LOI). The thickness as a function of the voltage is presented in Fig. 13 for the stainless steel 304. The crosses in the picture represent the measured data. Being experimentally verified the existence of a point of interception, one can access to the thickness of a plate by measuring the output voltage of the GMR sensor at that instant of time. This result will be independent of the lift-off distance.

IV. CONCLUSION

In this article, it was demonstrated that the theory of the linear transformer is adequate and proved to be a valuable tool to understand the physics fundamentals underneath the observed data acquired when pulsed eddy currents are used for thickness measurements. The results obtained theoretically with the transformer approximation model fit well the experimental data. It was shown that some aspects of the technique are not connected to the eddy currents phenomenon, but to the magnetic coupling between the plate and the excitation coil. Therefore the conventional model of the electrical transformer describes well the LOI effect.

Special attention was dedicated to the lift-off effect because the determination of the LOI point can be cross-correlated with the values obtained by derivation of the acquired GMR output voltages. When inspecting a metallic plate using the eddy currents method the signals acquired by a sensor depend on the plate thickness, on the material conductivity and on the lift-off. Using GMRs that respond directly to the magnetic field, it was observed that the time derivatives of the sensor output for different lift-off distances presented the same instantaneous values at singular instants of time (LOI). Therefore, it was concluded that it was possible to decrease the degrees of freedom from three to two by performing the measurements at that instant of time. The method was verified experimentally and proved to be suitable for measuring material thickness. Its simplicity makes it an adequate technique for the implementation in small and portable instruments employing microcontrollers with limited computational power and memory.

Due to the higher sensitivity the GMR based probe is expected to have a better performance as a wider range of values is obtained for the same plate thicknesses span.

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