On the robust linear control of the “buck-buck” converter: An active disturbance rejection approach

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Abstract—In this article, the cascade connection of two “buck” converters is examined from the perspective of differential flatness and Generalized Proportional Integral (GPI) observer–based control. The multi-variable nonlinear “buck-buck” converter is subject to unknown, time-varying, load current demands, at each one of the outputs of the two constituent stages. The linear output feedback controllers have to actively counteract the uncertain exogenous load demands as well as the surprisingly highly nonlinear endogenous interactions, arising between the converters, due to their cascade connection. Illustrative simulation results are presented.

I. INTRODUCTION

Controllers based on disturbance estimation, followed by cancelation from the control actions, have been developed in the past along different, but closely related, directions. The pioneering work of C.D. Johnson, known as Disturbance Accommodation Control (DAC), dates from the nineteen seventies (see [9]). The developments initially considered linear state space formulation and dealt with the on-line estimation of exogenous perturbation inputs alone. DAC has been extended to nonlinear systems while including estimation of endogenous (i.e., state-dependent) perturbations. The historic evolution of the DAC approach is found in [10]. Related developments, known as Active Disturbance Rejection (ADR), are addressed in the work of J. Han [8]. This includes nonlinear observers, with developments related to: efficient time derivatives computation, practical relative degree computation and nonlinear PID control extensions. Han’s contributions in applications of ADR include many industrial and laboratory applications. Z. Gao, and his colleagues (see [6], [7], [19], [17], [18]) have made important contributions in ADR and have proposed a new control paradigm, closely related to Han’s vision. M. Fliess and C. Join (see [2] and [5]), proposed Intelligent PID Control (IPIDC). This method is based on Differential Algebra and it implies to resort to low order, phenomenological plant models. The effects of nonlinear uncertain additive terms, locally modeled as time-polynomials that need to be updated, are suitably annihilated. Time-varying state dependent control gains are on line identified using algebraic methods (see [3]). Recently, an entirely linear active disturbance rejection approach has been proposed for the control of uncertain non-linear differentially flat systems (see [13]). Linear GPI observers are used to approximately estimate, in a self-updating manner, the joint effects of exogenous and endogenous additive disturbances, while the control input gain is assumed to be known and, at most, output dependent ([14]). The approach has significant implications in nonlinear chaotic systems estimation as demonstrated in [11]. The GPI observer based approach has also been extended to the control of nonlinear multi-variable systems exhibiting known time delays in the control inputs (see [16]).

In this article, we examine, from the perspective of GPI observer-based active disturbance rejection, the problem of linearly controlling a perturbed “buck-buck” DC-to-DC, multi-variable, power converter in an output trajectory tracking task specified for each one of the cascaded components of the system. We exploit the input-output approach by resorting to differential flatness, a key concept in nonlinear systems control (see Fliess et al. [4] and, also, [12]). Regulating a “buck” converter, including an unknown time-varying load current demand acting as an exogenous perturbation input, represents a challenging control problem in spite of the low dimensionality of the system. This is evidenced from the results of [13] where the reader is referred to for some background results. Here, we deal with the vastly more complex problem of regulating two cascaded boost converters subject to unknown, time-varying, load current demands on each stage.

The remainder of the article is organized as follows: Section 2 presents some generalities about the unperturbed “buck-buck” converter. In particular, the differential flatness property is established along with the need for a dynamic extension of the second converter’s control input. Section 3 deals with the perturbed version of the “buck-buck” converter and formulates the trajectory tracking problem under unknown, time-varying, exogenous current load demands for each converter module output. It is shown, thanks to flatness, how the exogenous perturbation vector components affect the additive disturbance as well as the control input gain matrix components. A certainty equivalence controller is then proposed which is based on an extra linear GPI estimation effort carried out for the approximate, but close, determination of one of the exogenous perturbation current demand. Active disturbance rejection and estimated input gain matrix cancelation are carried out using certainty equivalence linear observers and linear controllers. Simulation results are presented which illustrate the viability, and efficiency, of the proposed multi-variable linear controller design scheme, in spite of rather wild output current demands and, unknown, resistive load variations. Section V contains the conclusions and suggestions for further work.
II. THE IDEAL "BUCK-BUCK" CONVERTER MODEL

The ideal switched "buck-buck" converter, depicted in Figure 1, admits the following average state model

\[
\begin{align*}
L_1 \frac{d}{dt} i_1 &= -v_1 + u_{1,av} E \\
C_1 \frac{d}{dt} v_1 &= i_1 - \frac{v_1}{R_1} - u_{2,av} i_2 \\
L_2 \frac{d}{dt} i_2 &= -v_2 + u_{2,av} v_1 \\
C_2 \frac{d}{dt} v_2 &= i_2 - \frac{v_2}{R_2} y_1 = v_2, y_2 = v_4
\end{align*}
\]

where \( i_1 \) and \( i_2 \) are, respectively, the average inductor (input) currents on the first and second stage. \( v_1 \) and \( v_2 \) are, respectively, the average capacitor (output) voltages of the first stage and of the second stage, \( u_{i,av} \in [0, 1] \) for \( i = 1, 2 \), represent the average control inputs. \( R_i, i = 1, 2 \) denote the load resistances placed at the output of each converter stage.

\[ u_{2,av} = \frac{1}{L} (\alpha \beta \ddot{F} + \frac{\alpha F}{Q_2} + F) \]

\[ u_{1,av} = \dot{\alpha} + \frac{\dot{F}}{Q_1} + \dot{\alpha} \]

Contrary to the average normalized "buck" converter model, the average "buck-buck" model is clearly nonlinear, thanks to the multiplication of currents, or voltages, by the average control inputs.

B. Flatness

The average normalized multi-variable model of the "buck-buck" converter is differentially flat. Indeed, all variables are differentially parameterizable in terms of the flat outputs: \( \mathcal{F} = x_2, \mathcal{L} = x_4 \):

\[ x_2 = \mathcal{L}, \quad x_4 = \mathcal{F}, \quad x_3 = \beta \dot{\mathcal{F}} + \frac{\mathcal{F}}{Q_2} \]

\[ u_{2,av} = \frac{1}{L} (\alpha \beta \ddot{F} + \frac{\alpha F}{Q_2} + F) \]

Flatness readily allows for the parameterized equilibrium expressions in terms of the, desired, normalized average flat outputs equilibrium voltages, \( x_2 = \mathcal{F} = x_4 \). The system exhibits, from setting time derivatives to zero in (4), the following corresponding equilibrium point:

\[ \begin{align*}
\mathcal{F}_{1,av} &= \mathcal{F}_{2,av} \\
\mathcal{F}_1 &= \frac{\mathcal{F}}{Q_1} + \frac{\mathcal{F}}{Q_2} \\
\mathcal{F}_3 &= \frac{\mathcal{F}}{Q_2} \\
\mathcal{F}_4 &= \frac{\mathcal{F}}{Q_2}
\end{align*} \]

Fig. 1. The "buck-buck" converter

A. The normalized "buck-buck" converter model

Define the following set of normalized variables:

\[ x_1 = \frac{i_2}{E} \sqrt{\frac{L_1}{C_1}}, \quad x_2 = \frac{v_1}{E}, \quad x_3 = \frac{i_2}{E} \sqrt{\frac{L_1}{C_1}}, \quad x_4 = \frac{v_2}{E} \]

and, further, let:

\[ Q_i = R_i \sqrt{\frac{C_1}{L_1}} i = 1, 2 \quad \alpha = \frac{L_2}{L_1} \quad \beta = \frac{C_2}{C_1} \quad \tau = t/\sqrt{LC}, \quad \frac{d}{d\tau} = \omega \]

The ideal, normalized, "buck-buck" converter, admits the following average normalized model (see [15]),

\[ \begin{align*}
\dot{x}_1 &= -x_2 + u_{1,av} \\
\dot{x}_2 &= x_1 - \frac{x_2}{Q_1} - u_{2,av} x_3 \\
\alpha \dot{x}_3 &= -x_4 + u_{2,av} x_2 \\
\beta \dot{x}_4 &= x_3 - \frac{x_4}{Q_2} \\
y_1 &= x_2, \quad y_2 = x_4
\end{align*} \]

where \( x_1 \) and \( x_3 \) are, respectively, the normalized average inductor (input) currents on each stage. The variables \( x_2 \) and \( x_4 \) are, respectively, the normalized average capacitor (output) voltages of the first stage and the second stage. \( Q_i, i = 1, 2 \) are the normalized load resistances placed at the output of each converter stage.
\[
\dot{u}_{2,av} = \frac{1}{L} \left( \alpha \beta F^{(3)} + \frac{\alpha \dot{F}}{Q_2} + \ddot{F} \right) - \frac{\dot{L}}{L^2} \left( \alpha \beta \ddot{F} + \frac{\alpha \ddot{F}}{Q_2} + \dot{F} \right)
\]
\[
u_{1,av} = \dot{\dot{L}} + \frac{\dot{L}}{Q_1} + L + \ldots \text{to obtain an invertible extended input to flat outputs}
\]

highest derivatives relation. The differential parametrization

We can readily extract the following relation, exhibiting the fundamental gain-integration structure of the nonlinear multi-variable system:

\[
\begin{bmatrix}
\dot{I} \\
F^{(3)}
\end{bmatrix} = \begin{bmatrix}
1 - \left( \beta \dot{\dot{F}} + \frac{\dot{F}}{\tau} \right) & 0 \\
\frac{\dot{L}}{\tau} & 0
\end{bmatrix} \begin{bmatrix}
u_{1,av} \\
u_{2,av}
\end{bmatrix} + 
\begin{bmatrix}
\psi_1(\dot{L}, \dot{\dot{F}}, \dot{\dot{F}}, \dot{\dot{F}}) \\
\psi_2(\dot{L}, \dot{F}, \dot{F}, \dot{F})
\end{bmatrix}
\]

where \(\psi_1(\dot{L}, \dot{\dot{F}}, \dot{\dot{F}}, \dot{\dot{F}})\), and, \(\psi_2(\dot{L}, \dot{F}, \dot{F}, \dot{F})\), are readily determined from (7).

\[\text{III. A Flatness-based Active Disturbance Rejection Approach to the Control of the Perturbed \text{"buck-buck" Power Converter}}\]

Consider the perturbed “buck-buck” converter, in Figure 2, consisting of the cascade connection of two “buck” converter topologies. Contrary to the traditional case, the output of each converter will be subject to an unknown load current drain. \[\text{Fig. 2. A perturbed “buck-buck” converter including unknown output load current drains.}\]

The normalized, average, model of the perturbed converter is expressed as,

\[
\begin{align*}
\dot{x}_1 &= -x_2 + u_{1,av} \\
\dot{x}_2 &= x_1 - x_2 - u_{2,av} - x_3 - I_1(t) \\
\alpha \dot{x}_3 &= -x_2 + u_{2,av} \\
\beta \dot{x}_4 &= x_3 - x_4 - I_2(t) \\
y_1 &= x_2 \\
y_2 &= x_3
\end{align*}
\]

\[1\text{Note that, provided } \mathcal{L}, \text{ and } F \text{ are measurable, these two quantities: } \\
\psi_1(\cdot), \psi_2(\cdot), \text{ are observable in the, Differential Algebra oriented, definition of Diop and Fliess [1]}\]

where, \(I_1(\tau)\), and, \(I_2(\tau)\), represent the normalized (exogenous) drain current loads, \(I_1(t)\), \(I_2(t)\), located, respectively, at each one of the two output modules voltage nodes. These time-varying perturbation inputs are assumed to be of positive magnitude but, otherwise, completely unknown.

\[\text{A. Problem formulation}\]

The trajectory tracking problem for the perturbed “buck-buck” converter is stated as follows:

\[\text{Given a set of desired compatible average output reference trajectories: } \mathcal{L}^*(\tau), F^*(\tau), \text{ it is desired to, respectively, asymptotically drive, via a linear observer-based linear multivariable controller, the average normalized output voltages}\]

\[\text{tracking error trajectories: } e_1(\tau) = y_1 - \mathcal{L}^*(\tau), e_2(\tau) = y_2 - F^*(\tau), \text{ towards small as desired open neighborhoods of the origins of the multivariable tracking error phase spaces: } (e_1, e_1), (e_2, e_2, e_2), \text{ where they will remain uniformly bounded, regardless of the magnitudes of exogenous perturbation inputs, } I_1(\tau), I_2(\tau).\]

\[\text{B. Perturbed differential parameterizations}\]

The average normalized multi-variable model of the unperturbed “buck-buck” converter is differentially flat with flat outputs given by \(F = x_2, L = x_4\). The perturbed differential parametrization of the system variables, in terms of the flat outputs \(F = x_2, L = x_4\), and the exogenous perturbation inputs, \(I_1(\tau), I_2(\tau)\), is given by:

\[
x_2 = \mathcal{L}, \quad x_4 = \mathcal{F}, \quad x_3 = \beta \ddot{F} + \frac{\mathcal{F}}{Q_2} + I_2(\tau)
\]
\[
u_{2,av} = \frac{1}{L} \left( \alpha \ddot{F} + \frac{\alpha \ddot{F}}{Q_2} + \ddot{F} + \alpha \ddot{I}_2(\tau) \right)
\]
\[
x_1 = \dot{\dot{L}} + \frac{\dot{L}}{Q_1} + \frac{1}{L} \left( \beta \ddot{F} + \frac{\mathcal{F}}{Q_2} + I_2(\tau) \right) \times 
\left( \alpha \ddot{F} + \frac{\alpha \ddot{F}}{Q_2} + \ddot{F} + \alpha \ddot{I}_2(\tau) \right) + I_1(\tau)
\]
\[
u_{1,av} = \ddot{\dot{\dot{L}}} + \frac{\ddot{\dot{\dot{L}}}}{Q_1} + \ddot{\dot{\dot{I}}}_1(\tau) + 
\frac{1}{L} \left( \beta \ddot{F} + \frac{\mathcal{F}}{Q_2} + \ddot{\dot{I}}_2(\tau) \right) \times 
\left( \alpha \ddot{F} + \frac{\alpha \ddot{F}}{Q_2} + \ddot{F} + \alpha \ddot{I}_2(\tau) \right) + 
\left( \beta \ddot{F} + \frac{\mathcal{F}}{Q_2} + I_2(\tau) \right) \times 
\left( \alpha \ddot{F} + \frac{\alpha \ddot{F}}{Q_2} + \ddot{F} + \alpha \ddot{I}_2(\tau) \right)
\]

The relation linking the highest order time derivatives of the flat outputs and the average control inputs is non-invertible. This reveals an ill-defined vector relative degree of the flat outputs. As in the unperturbed case, a dynamic extension of the average control input, \(\nu_{2,av}\), proves to be sufficient to obtain an invertible extended input to flat outputs highest derivatives relation. The differential parameterization
of the perturbed control input extension for $u_{2,av}$ is thus given by:

$$
\dot{u}_{2,av} = \frac{1}{L} \begin{pmatrix}
\alpha \beta F^{(3)} + \alpha \frac{\dot{F}}{Q_2} + \dot{F} + \alpha \ddot{I}_2(	au)
\end{pmatrix}
- \frac{\dot{L}}{L^2} \begin{pmatrix}
\alpha \beta \dot{F} + \alpha \frac{\ddot{F}}{Q_2} + \ddot{F} + \alpha \dddot{I}_2(	au)
\end{pmatrix}
$$

(10)

The extended input to flat output multi-variable relation is given by

$$
\begin{bmatrix}
\dot{L} \\
\dot{F}^{(3)}
\end{bmatrix} = \begin{bmatrix}
1 & -\beta \dot{F} + \frac{F}{Q_2} + \ddot{I}_2(	au)
0 & \frac{\ddot{L}}{L}
\end{bmatrix} \begin{bmatrix}
u_{1,av} \\
u_{2,av}
\end{bmatrix}
+ \begin{bmatrix}
\nu_1(L, \dot{L}, F, \dot{F}, \ddot{F}, \dot{I}_1, \ddot{I}_2, \dot{I}_2)
\nu_2(L, \dot{L}, F, \dot{F}, \ddot{F}, \dot{I}_2, \ddot{I}_2, \dot{I}_2)
\end{bmatrix}
$$

(11)

C. Estimation of the Load current $I_2(\tau)$.

While, $\nu_1(\cdot)$, and, $\nu_2(\cdot)$, can be on-line estimated via a linear GPI observer, the control input gain matrix explicitly depends on the unknown current source drain magnitude $I_2(\tau)$. This makes it difficult for the canceling of the nonlinear input gain matrix. An on-line estimation of this quantity may be obtained whenever a full state variables measurement assumption is enforced. Under these circumstances, we set, for a given integer, $q$, $\hat{x}_2 = x_2 - \ddot{x}_2$, is seen to satisfy:

$$
\begin{align*}
\dot{x}_2 &= x_2 - \ddot{x}_2 - u_{2,av} x_3 + z_1 + \lambda_q(x_2 - \ddot{x}_2) \\
\dot{z}_1 &= z_2 + \lambda_{q-1}(x_2 - \ddot{x}_2) \\
\vdots \quad & \\
\dot{z}_q &= \lambda_0(x_2 - \ddot{x}_2)
\end{align*}
$$

(12)

The state reconstruction error, $e_2 = x_2 - \ddot{x}_2$, is seen to satisfy:

$$
e_2^{(q+1)} + \left( \lambda_0 + \frac{1}{Q_1} \right) e_2^{(q)} + \cdots + \lambda_0 e_2 = -(I_2(\tau))^{(q)}
$$

If one lets, respectively, $N(x_2(\tau), \rho)$, $N(-I_2(\tau), \rho)$, denote balls of radii, $\rho$, uniformly centered around, $x_2(\tau)$, and, $I_2(\tau)$ at each $\tau$, it follows that $\ddot{x}_2 \to N(x_2(\tau), \rho)$ and $z_1 \to N(-I_2(\tau), \rho)$ where, $\rho$, can be made as small as desired by appropriate choice of the set of parameters $\{\lambda_0, \cdots, \lambda_3\}$.

Approximate knowledge of the load current perturbation, $I_2(\tau)$, in an asymptotic fashion allows us to use the following certainty equivalence GPI observer for the estimation of the flat outputs phase variables and the, exogenously influenced, phase variable-dependent perturbation inputs, $\nu_1(\cdot)$, and, $\nu_2(\cdot)$.

D. GPI observers for the perturbed “buck-buck” converter.

We make the following additional assumptions for the perturbed case,

1) For any given smooth, closed loop, trajectories, $F(\tau)$ and $L(\tau)$, and for uncertain, sufficiently smooth, load current demands: $I_2(\tau)$, $I_2(\tau)$, the corresponding control input, $u_{2,av}(\tau)$, satisfying (10), is uniformly absolutely bounded.

2) The time derivatives of $u_{2,av}(\tau)$, up to order $m$, are all assumed to be uniformly absolutely bounded.

3) The variables $\nu_j(\tau)$, $j = 1, 2$, are of the form:

$$
\nu_1(\tau) = \nu_1(L(\tau), \dot{L}(\tau), F(\tau), \dot{F}(\tau), \ddot{F}(\tau), \dot{I}_2(\tau), \ddot{I}_2(\tau))
$$

$$
\nu_2(\tau) = \nu_2(L(\tau), \dot{L}(\tau), F(\tau), \dot{F}(\tau), \ddot{F}(\tau), \dot{I}_2(\tau), \ddot{I}_2(\tau))
$$

These two signals are assumed to be uniformly absolutely bounded with, respectively, time derivatives up to order $m$ and $p$ being equally uniformly absolutely bounded.

Theorem.: The following certainty equivalent GPI observer:

$$
\begin{align*}
\dot{L}_1 &= L_2 + \gamma^L_{m+1}(L - L_1) \\
\dot{L}_2 &= u_{1,av} - \nu_1(L, \dot{L}, F, \dot{F}, \ddot{F}, \dot{I}_2, \ddot{I}_2) \\
\dot{L}_3 &= \nu_2(L, \dot{L}, F, \dot{F}, \ddot{F}, \dot{I}_2, \ddot{I}_2)
\end{align*}
$$

(13)

with $\ddot{I}_2(\tau)$ obtained from the GPI observer (12) as $-z_1$, and the GPI observer,

$$
\begin{align*}
\dot{F}_1 &= F_2 + \gamma^F_{p+2}(F - F_1) \\
\dot{F}_2 &= F_3 + \gamma^F_{p+1}(F - F_1) \\
\dot{F}_3 &= \left( \frac{L}{\alpha F^3} \right) u_{2,av} + z_1 F + \gamma^F_p(F - F_1)
\end{align*}
$$

(14)

with a suitable choice of the gain parameters: $\gamma^j_k$, $j = L, F, k = 1, 2, \cdots$, such that the roots of the corresponding associated characteristic polynomials in the complex variable $s$,

$$
p_F(s) = s^{p+3} + \gamma^F_{p+2}s^{p+2} + \cdots + \gamma^F_1 s + \gamma^F_0
$$

$$
p_L(s) = s^{m+2} + \gamma^L_{m+1}s^{m+1} + \cdots + \gamma^L_1 s + \gamma^L_0
$$

(15)

are located sufficiently far into the left hand side of the complex plane, produce estimation error trajectories, $e_F(\tau) = F - F_1$, and, $e_L(\tau) = L - L_1$, that asymptotically converge, in an exponentially dominated manner, towards small as desired neighborhoods of the origins of the estimation error phases spaces, $(e_F, \dot{e}_F, \cdots e^{(2p+2)}_F)$, $(e_L, \dot{e}_F, \cdots e^{(m+1)}_L)$. Moreover, the trajectories of the estimation errors, $z_{1L}(\tau) - \nu_1(\tau)$, and, $z_{1F} - \nu_2(\tau)$, converge towards small as desired neighborhoods of zero where they remain uniformly bounded.

Proof
The proof is based on the fact that the injected dynamics, associated with the estimation errors $e_F(\tau)$ and $e_L(\tau)$, are given by the following perturbed linear systems:

$$
e^{(m+2)}_L + \gamma_{m+1}^{L}e^{(m+1)}_L + \cdots + \gamma_{1}^{L}e^{F} + \gamma_{0}^{L}e_L = \psi^{(m)}_1(\tau)$$

$$+ \frac{d^{m}}{dt^{m}} \left[ (\hat{I}_2(\tau) - \hat{I}_2(\tau)) \right] u_{2,av}(\tau)$$

$$e^{(p+3)}_F + \gamma_{p+2}^{F}e^{(p+2)}_F + \cdots + \gamma_{1}^{F}e^{F} + \gamma_{0}^{F}e_F = \psi^{(p)}_2(\tau)$$

The right hand sides of these linearly dominated dynamics are assumed to be uniformly absolutely bounded by a small neighborhood of zero. The estimation errors: $e_L$ and $e_F$, converge towards small as desired neighborhoods of zero where they will remain uniformly bounded.

**E. A certainty equivalence controller for the perturbed “buck-buck” converter**

The perturbed “buck-buck” converter is controlled by the following certainty equivalence controller based on knowledge of the drain current estimate $\hat{I}_2(\tau)$ obtained from (12).

Theorem. Suppose $k_{L1}$, $k_{L0}$ and $k_{F2}$, $k_{F1}$, $k_{F0}$, are constant gains, chosen in such a way that the following polynomials in the complex variable $s$

$$p^{F}(s) = s^3 + k_{F2}s^2 + k_{F1}s + k_{F0}$$

$$p^{L}(s) = s^2 + k_{L1}s + k_{L0}$$

are Hurwitz polynomials, with roots located sufficiently far to the left of the imaginary axis in the complex plane. Then the following, certainty equivalence, GPI observer-based average multivariable controller with approximate disturbance cancelation:

$$u_{1,av} = v_1 + \left( \frac{\alpha \beta}{L} \right) (\beta \hat{F} + \frac{F}{Q_2} + \hat{I}_2(\tau)) v_2 - \hat{\psi}_1(\tau)$$

$$\hat{u}_{2,av} = \left( \frac{\alpha \beta}{L} \right) v_2 - \hat{\psi}_2(\tau)$$

(17)

where $\hat{I}_2(\tau)$ is given by $-z_1$, as obtained from (12), and,

$$v_1 = \hat{L}^{\ast}(\tau) - k_{L1}(L - \hat{L}^{\ast}(\tau)) - k_{L0}(L - \hat{L}^{\ast}(\tau))$$

$$v_2 = \frac{d^3 \hat{F}^{\ast} (\tau)}{dt^3} - k_{F2}(\hat{F} - \hat{F}^{\ast}(\tau))$$

$$- k_{F1}(\hat{F} - \hat{F}^{\ast}(\tau)) - k_{F0}(\hat{F} - \hat{F}^{\ast}(\tau))$$

(18)

asymptotically drives the tracking errors: $e^{F} = \hat{F} - F^{\ast}(\tau)$, and, $e^{L} = L - L^{\ast}(\tau)$, to small as desired neighborhoods of the tracking error phase spaces, ($e^{F}, e^{\hat{F}}, \hat{e}^{F}$) and ($e^{L}, \hat{e}^{L}$), where they remain ultimately uniformly bounded for all times $\tau$.

Proof

The closed loop tracking error systems are seen to satisfy the following linear perturbed dynamics

$$[e^{F}]^3 + k_{F2}e^{\hat{F}} + k_{F1}e^{F} + k_{F0}e^{F} = \psi_1(\tau) - \hat{\psi}_1(\tau)$$

$$+ \left( \frac{\alpha \beta}{L} \right) (\hat{I}_2(\tau) - \hat{I}_2(\tau))v_2(\tau)$$

$$\hat{e}^{L} + k_{L1}e^{L} + k_{L0}e^{L} = \psi_2(\tau) - \hat{\psi}_2(\tau)$$

(19)

The auxiliary control input $v_2(\tau)$ is clearly uniformly absolutely bounded. By the results of previous theorems, the rest of the terms in the right hand side of the above expressions represent time functions which are ultimately uniformly bounded to small vicinities of zero. Under suitable choice of the feedback gains, the closed loop tracking errors are seen to asymptotically converge towards small neighborhoods of the origin of the tracking error phase spaces. By Lyapunov stability theory arguments, already used in the previous theorem, these neighborhoods become smaller as the roots of the dominant closed loop characteristic polynomials are chosen further to the left of the imaginary axis in the complex plane. The result follows.

**IV. SIMULATION RESULTS AND ROBUSTNESS TESTS**

For testing the robustness features of the proposed GPI observer-based controller, we set the following situation:

1) The resistive loads, $Q_1$, and, $Q_2$, are assumed to be nominally known, but they may be subject to piece-wise linear variations, leading towards new unknown constant values.

2) The controller and the observer will be “unaware” of these parameter drifts.

3) The unknown load currents are linearly estimated, on line, under the assumption of full state measurement. They will be used in conforming a certainty equivalence observer and a certainty equivalence controller where the influence of the actual drift currents is replaced by the linear GPI estimated currents.

Figure 5 depicts the performance of the proposed multivariable output feedback GPI observer-based controller on
the system variables for a simultaneous stabilization and trajectory tracking problems on the two flat outputs when the “buck-buck” converter is subject to unknown, time-varying, load current demands on the output of each of the two stages. The resistive loads are subject to piecewise unknown variations causing a significant deviation between the actual and the estimated load currents. The average control inputs are seen to be well within the constraint interval. The switched controller is implemented via a sigma-delta modulator as advocated in [15].

Figure 6 illustrates the performance of the unknown load demand GPI estimator under variations of the resistive loads. While the resistive load adopts its nominal value, the on-line estimate is quite accurate. The lack of knowledge on the part of the observer about the resistive load variations causes the drift of the estimate. The second module feedback controller, though, corrects for this information mismatch, thanks to the relative high gain controller design.

V. CONCLUSIONS

The feedback control of the nonlinear, multivariable, “buck-buck” converter, obtained via the tandem connection of two “buck” converters, has been examined combining differential flatness and active disturbance rejection from the perspective of linear Generalized Proportional Integral (GPI) observer–based control. The decoupled output reference trajectory tracking problem is substantially complex due to 1) an ill defined relative degree of the multivariable converter in need of a control input dynamic extension and 2) the unexpected complexity of the interacting nonlinearity from the second converter over the first converter. The proposed, high gain, linear observer-linear controller approach is shown to be suitable for the case in which the “buck-buck” converter is subject to unknown, time-varying, load current demands, at each one of the outputs of the two constitutive stages, as well as to piecewise linear unknown variations of the resistive output loads. Illustrative, and encouraging, simulation results are presented for, both, stabilization and output reference trajectory tracking cases. The actual laboratory implementation of the GPI observer-based technique for multivariable cascade converters is a topic to be addressed in the future.

REFERENCES