Obstacle surpassing and posture control of a stair-climbing robotic mechanism

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A R T I C L E   I N F O

Article history:
Received 20 December 2011
Accepted 21 January 2013

Keywords:
Electric powered wheelchairs
Stair-climbing mechanisms
Climbing staircases
Reconfigurable robots

A B S T R A C T

In this study, we propose a new kinematic control for a robotic stair-climbing mechanism which allows successful obstacle surpassing when faced with typical architectural barriers such as curbs, ramps or staircases while maintaining the passenger’s comfort and the seat’s inclination within security margins. The scheme also takes into account perturbations in the system due to the fact that the environment is not perfectly known. The actuated degrees of freedom in charge of the locomotion device and the posture device are controlled in a different way. The locomotion control is achieved with a feedforward term and a standard proportional derivative (PD) control position. The posture control is obtained with a novel variant of multilevel feedback controller design via a suitable combination of a Lyapunov feedback controller design along with a multilevel generalization of the sliding mode based Sigma-Delta (Σ–Δ) modulator coupled with an adequate selection of the trajectory of the center of mass. The main features of this novel kinematic control algorithm are: maintenance of the passenger comfort, high degree of perturbation rejection regarding modeling error, independence of the designed control law on the environment parameters, computational efficiency and minimal sensor requirements. Numerical simulations illustrate the performance of the proposed method when the prototype is to surpass an obstacle with different heights with the value of the height obstacle being unknown. Finally, experimental results validate the behavior of the prototype for both, obstacle surpassing and posture control when the wheelchair prototype ascends a curb of 180 mm.

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1. Introduction

Independent mobility is crucial for the development of physical, cognitive, communicative, and social skills (Galloway, Ryu, & Agrawal, 2008). Electric powered wheelchair (EPW) technology is one of the most favorable technologies to be investigated because it allows people with physical disabilities to complete usual tasks with greater independence and to get on with their activities of daily living (Ceres, Pons, Calderón, Jiménez, & Azevedo, 2005; Chen, Feng, & Hsieh, 2003). However, architectural barriers still exist in our living environments and present major problems to wheelchair users. Roboticians aim to provide a solution of this problem by introducing new stair mobility assistance devices. These equipments are currently rated by FDA as “class III” high-risk devices, defined as “life sustaining or life supporting, implanted in the body, or presenting an unreasonable risk of illness or injury” (Ding & Cooper, 2005). Additionally, the provision of acceptable stability at all times for a stair-climbing mobility device is essential for safety during stair climbing process, and a constant seat angle is usually desired. In Wellman and Krovi (1995) it was demonstrated that the best way to solve the problem of negotiating architectural barriers is by means of mixed systems. There are two ways to exploit this synergy: the first includes articulated-wheeled robots with the wheel attached to the end of the leg. Most of these systems have several types of locomotion: rolling mode, where the systems act like a conventional wheeled vehicle; walking mode, where the wheels are blocked and the vehicles act as legged systems; and a hybrid mode using both legs and wheels (Bourbakis, 1998, Nakajima, Nakano, & Takahashi, 2004). Some have been designed with the objective of providing mobility for people with physical impairments. For example, the stair-climbing mechanism developed in Lawn and Ishimatsu (2003) is able to ascend and descent stairs with a high step height in a forward direction but requires many degrees of freedom and high energy consumption. Also, the device developed in Kamen et al. (2002) presents a simple mechanical design but needs a sophisticated dynamic control to maintain the upright position and there are motion phases during the climbing process that the mechanism is standing on two wheels with...
a common axis. The device presented in Wellman and Krovi (1995) surpasses architectural barriers but presents problems in the step-climbing process as a result of a large variation in the inclination angle of the chair. The second kind of mixed systems consists of robots with independent legs and wheels that operate in coordination. These vehicles are simpler and lighter, and only use the hybrid mode of locomotion. A leg-wheel hybrid stair-climbing wheelchair is presented in Hirose, Yoneda, Arai, and Ibe (1995) with eight independent prismatic joint legs and eight wheels. This prototype can perform safe stable three-dimensional motion but requires a high cost of meeting reasonable safety standards.

The stair-climbing system presented in this paper (see Fig. 1a) maintains the same behavior as the commercial EPWs, with the addition of important properties such as: (a) a capability of adapting to the environment and overcoming typical architectural barriers such as curbs or staircases (profiles characterized by vertical slopes) and (b) a capability to move the system, in a manner which is comfortable for the passenger, over continuous or discontinuous smooth profiles. These advantages are achieved by using a strategy design based on splitting the obstacle-climbing problem into two different problems, using a distinct mechanical device for each. These problems are: (a) controlling the traction of the mechanism and the climbing process; and (b) controlling the posture for the entire mechanism while the device is surpassing the obstacle. The mechanical division between the device in charge to surpass the obstacle and the mechanism that ensures the verticality allows the system to maximize the range of height and width of the steps, permits a good adaptation to the environment when the system faces different kinds of architectural barriers, simplifies the control schemes and the control hardware and reduces the costs.

Nowadays, there is limited literature about modeling stair-climbing mobility systems, and one of the reasons is that these models are not required for a simple control law commonly adopted by commercial EPWs. In the case of stair-climbing devices it is highly important the definition of a mathematical model because it will be necessary to obtain advanced controllers that improve the stair-climbing system performance. Some authors proposed simplified wheelchair models based on a kinematic model (Pavec, Aubin, Aissaoui, Parent, & Dansereau, 2001) and its corresponding kinematic feedback control laws (Yoder, Baumgartner, & Skaar, 1996). These control schemes are justified because the prototype moves at low speeds, high precision is not necessary and the control law is easier to implement (reduction of the amount of computation resources, cost and sensorial system). In this work, we continue with this tendency and we propose a novel kinematic control scheme for a stair-climbing mechanism which takes into consideration the reconfigurable nature of our prototype and the maintenance of the passenger comfort with minimal sensor requirements.

The aim of our kinematic robust control is the successful obstacle avoidance while maintaining the passenger’s comfort and the verticality of the seat when the geometry of the obstacle is not accurately known. Fig. 2 illustrates the different parts that compose the proposed scheme and these parts are now briefly described: (a) The behavior diagram is in charge of the selection of current device configuration and the trajectory generation of the actuated degrees of freedom of the stair-climbing mechanism. The information is provided after processing the sensorial information and taking into account the previous configuration of the stair-climbing mechanism; (b) The locomotion control is in charge of the control of the actuated degrees of freedom related with the traction of the stair-climbing system. It is composed of a feedforward term and a standard proportional derivative (PD) control position. The inputs of the locomotion control are the reference trajectories of these actuated degrees of freedom which are provided by the behavior diagram; and (c) The posture control deals with the actuated degrees of freedom that ensures the verticality of the stair-climbing mechanism. It is accomplished via a suitable combination of a Lyapunov feedback controller design along with a multilevel generalization of the sliding mode control based on Sigma–Delta (Σ–Δ) modulator along with a smart selection of the center of mass reference trajectory. The inputs of the posture control system are the inclination reference angle of the chair provided by the behavior diagram, and the measurements of all the actuated degrees of freedom of the stair-climbing mechanism and the current inclination seat angle. The outputs of this subsystem are the speeds of the motors in charge of the movements of the posture mechanism. This subsystem together with the behavior diagram and the whole conception of the kinematic control scheme state the main results of this work.

The main features of the kinematic control approach are resumed as: (i) maintenance of the passenger comfort at all times; (ii) high degree of perturbation rejection regarding modeling error; (iii) independence of the designed control law on the environment parameters; (iv) computational efficiency and; (v) the obstacle is surpassed with minimal sensor requirements. The following sections treat in detail the aspects of the proposed kinematic control.
The paper is organized as follows: Section 2 details the variables, actuators, sensors, controlled variables and operation modes which include the stair-climbing device. Section 3 is devoted to describe the mathematical model and the implicit posture jacobian models of the robotic mechanism taking into account all the different configurations of the system during a stair climbing/descent process. Section 4 details the proposed control approach and states the main results of this article. Section 5 illustrates the numerical simulations performed when the stair-climbing system overpasses obstacles with a nonaccurate knowledge of their geometry. Section 6 presents the experimental results of the control scheme when the robotic prototype ascends a curb, and Section 7 is devoted to the conclusions and presents some suggestions for further research in this area.

2. Stair-climbing mechanism operation

Brief descriptions of the variables, the different actuators in charge of the movement of the robotic system, the sensorial system, the controlled variables and the operating modes are given in the following subsections.

2.1. Variables

The main variables that will be used during the manuscript are the following (see Fig. 1b): The position of the center of mass of the stair-climbing mechanism is defined as $\mathbf{p}_g = [P_{gx}, P_{gy}]$ where $P_{gx}$ and $P_{gy}$ are the horizontal and vertical components. The inclination angle of the chair frame is defined as $\gamma$. These values will be grouped into the vector $\mathbf{p} = [\mathbf{p}_g, \gamma]^T$. The front and rear joint variables of the positioning mechanism are denoted by $\theta_1$ and $\theta_2$ respectively. The movement of the rear wheels is described by the variable $\theta_3$ and the positions of the front and rear racks are denoted as $z_1$ and $z_2$ respectively. These values can be joined into the vector $\mathbf{q} = [\theta_1, \theta_2, \theta_3, z_1, z_2]^T$. Finally, the reference trajectories for the vectors $\mathbf{p}$ and $\mathbf{q}$ are defined as $\mathbf{p}^* = [P_{gx}^*, \gamma^*]^T$ and $\mathbf{q}^* = [\theta_1^*, \theta_2^*, \theta_3^*, z_1^*, z_2^*]^T$ respectively.

2.2. Actuated variables

The actuated degrees of freedom which compose the stair climbing mechanism (see Fig. 1b) are the following: The actuators in charge of the maintenance of the system verticality produce the change in the variables $\theta_1$ and $\theta_2$ respectively, the motors that move the rear wheels are defined by the variable $\theta_3$, and the actuators that act on the lengths of the racks which define the climbing obstacle mechanical system are denoted as $z_1$ and $z_2$. The signals used to control the locomotion system, i.e. the front and rear racks and the rear wheels are their position reference trajectories denoted by $z_1^*, z_2^*$ and $\theta_3^*$ respectively. On the other hand, the signals used to control the posture of the stair-climbing device are the angular velocity of the actuated degrees of freedom $\theta_1$ and $\theta_2$. Due to the physical properties of the motors which constitute the positioning mechanism of the real prototype, the control inputs $\theta_1$ and $\theta_2$ are restricted to provide three switching levels in the interval $[-\theta_{max}, \theta_{max}]$, due to the prevailing limitations. The magnitude $\theta_{max}$ is a constant value which defines the maximum angular velocities that these actuators achieve. Thus, switched models for the speeds of the motors with three switching values are defined as $\dot{\theta}_1 = u_1 \theta_{max}$ and $\dot{\theta}_2 = u_2 \theta_{max}$, where $u_1$ and $u_2$ are multilevel position functions with discrete values $\{-1, 0, 1\}$. From now on, the variables $u_1$ and $u_2$ will be considered as the control signals for these actuators.
Finally, it is necessary to remark that the dynamic behavior of all the actuators including their individual closed loops can be considered negligible in relation to the entire robotic system because the movements of the prototype, as a whole, are much slower than the time responses of the servo-controlled electrical motors.

2.3. Measured variables

The information achieved from the sensorial system is the following: the ultrasound sensors provide information about the distance between the stair-climbing mechanism and the obstacle to be surpassed; the mechanical switches give information about maximum/minimum positions of the mechanical configurations; the encoders of the motors in charge of the movement of the racks and the rear wheels provide information about the instantaneous length of the racks and the position of the rear wheels, denoted by $z_1$, $z_2$ and $\theta_2$ respectively; two angle sensors illustrate the measurement of the angles of the device in charge of the system verticality, depicted as $\theta_1$ and $\theta_2$ respectively; and an inclination sensor gives the measurement of the chair verticality of the mechanism, depicted as $\gamma$.

2.4. Controlled variables

The controlled variables of the system are the position of the center of mass $P_g = [P_{gx}, P_{gy}]$ and the inclination angle of the chair frame $\gamma$.

2.5. Operating modes

The different operating modes, or configuration modes, that the system performs are illustrated in the following:

(1) Robotic mechanism supported on four wheels. In this configuration the device is supported on four wheels (see Fig. 3a). We assume that the rear and front axles are rolling on flat terrain and that the angles $\mu_i$ are defined to find the geometrical connection between the vectors of which the general kinematic scheme is comprised. This is the configuration used in the movement of the stair-climbing mechanism on flat terrain and when the mechanism is approaching to the obstacle to be solved. When the system works under this configuration, the rear wheels, defined by $\theta_2$, are moving while the actuated degrees of freedom $\theta_1$, $\theta_2$, $z_1$ and $z_2$ remain in a constant position.

(2) Robotic mechanism supported on the rear rack and the front wheels. In this configuration the device is supported on four wheels (see Figs. 3a and 3b). We assume that the front axle is rolling on flat terrain and the rear rack is moving with a slope of $\beta_2$. This configuration is carried out when the obstacle is being surpassed with the rear climbing mechanism. When the system works under this configuration, the rear rack $z_2$ and the joint angle $\theta_1$ are in charge of the movement of the prototype while the actuated degrees of freedom $\theta_2$, $\theta_1$, and $z_1$ remain in a constant position.

(3) Robotic mechanism supported on the front rack and the rear wheels. In this configuration the device is supported on four wheels (see Figs. 3a and 3c). We assume that the rear axle is rolling on flat terrain and the front rack is moving with a slope of $\beta_1$. This configuration is achieved when the obstacle is being surpassed with the front climbing mechanism. When the system works under this configuration, the front rack $z_1$, the joint angle $\theta_1$ and the rear wheels $\theta_3$ are in charge of the movement of the prototype while the actuated degrees of freedom $\theta_2$, and $z_2$ remain in a constant position.

(4) Robotic mechanism supported on two racks. In this configuration the device is supported on four wheels (see Fig. 3d). The rear and front racks are moving with the corresponding slopes $\beta_1$ and $\beta_2$. This configuration is achieved when the system is solving and obstacle with the front and rear climbing mechanisms simultaneously. When the system works under this configuration, the front and rear racks, defined by $z_1$ and $z_2$, are moving while the actuated degrees of freedom $\theta_1$, $\theta_2$ and $\theta_3$ remain in a constant position.

Fig. 3. Operating modes of the robotic stair-climbing mechanism.
3. Kinematics and implicit posture jacobian models

This section is focused on the definition of the kinematics and the implicit posture jacobian models of the different configurations of the chair-climbing mechanism. These subjects are explained in the following subsections.

3.1. Kinematics

The forward kinematic model allows full motion of the degrees of freedom of the whole system, and can be adapted to a continuous smooth profile or a discontinuous profile consisting of a flat floor and staircase. A complete description of the main properties of the prototype can be found in Morales, Feliu, González, and Pintado (2006a, 2006b), González, Morales, Feliu, and Pintado (2007), Morales, González, Feliu, and Pintado (2007), and Morales, González, and Feliu (2010). In this section, we briefly describe the forward kinematic model, since this will clarify the description of the closed loop control developed in Section 4. In a forward kinematic model we know the angles of the joints of the chair structure \( \theta_1 \) and \( \theta_2 \) and the position of the front and rear wheels \( f(\theta_1) \) and \( f(\theta_2) \), respectively, or the instantaneous lengths of the racks \( z_1 \) and \( z_2 \) and their corresponding contact points \( (P_{C1}) \) and \( (P_{C2}) \), depending on the configuration. These data are used to obtain the center of mass position \( (P_{g}) \) and the inclination of the seat of the mechanism \( (\gamma) \). The forward kinematic models of the different chair configurations are now briefly presented in terms of complex equations:

1) Robotic mechanism supported on four wheels. In this configuration the device is supported on four wheels (see Fig. 4a). The initial expressions that define the current position of the mechanism were explained in Morales et al. (2006a), and are the following:

\[
\begin{align*}
\mathbf{P}_g &= f(\theta_1) + l_4 \mathbf{e}^{(\alpha_4)} + \pi/2 + \mu_1 + l_4 \mathbf{e}^{(\alpha_4)} + \pi/2 + \mu_1) \\
\mathbf{P}_2 &= f(\theta_2) + l_1 \mathbf{e}^{(\alpha_1)} + \pi/2 + \mu_1 - l_1 \mathbf{e}^{(\alpha_1)} + \pi/2 + \mu_1) + l_6 \mathbf{e}^{(\alpha_6)} + \pi/2 + \mu_1) 
\end{align*}
\]

(1) and (2)

where \( l_1 \) and \( l_4 \) are the lengths which correspond with the front axle, \( l_4 \) and \( l_6 \) are the lengths which correspond with the rear axle and \( l_5 \) is the length of the frame. Then, equating expressions (1) and (2), taking the imaginary part and rearranging terms, the following implicit expression is yielded:

\[
\begin{align*}
F_1(\mathbf{q}) &= \text{Im}[f(\theta_1) - f(\theta_2)] + \text{Im}[l_6 e^{(\alpha_6)} + \pi/2 + \mu_1] \\
&\quad - \text{Im}[l_1 e^{(\alpha_1)} + \pi/2 + \mu_1] + \text{Im}[l_4 e^{(\alpha_4)} + \pi/2 + \mu_1] + \text{Im}[l_3 e^{(\alpha_3)} + \pi/2 + \mu_1] = 0
\end{align*}
\]

(3)

where the term \( \text{Im}[f(\theta_1) - f(\theta_2)] \) is a constant value but we have no precise knowledge of its true value.

2) Robotic mechanism supported on the rear rack and the front wheels. In this configuration the device is supported on the rear rack and the front wheels (see Fig. 4b). We assume that the front axle is rolling on flat terrain and the rear rack is moving with a slope of \( \beta_2 = \pi/2 - \delta_2 \). The initial expressions that define the current position of the mechanism were explained in Morales et al. (2006a), and are shown here:

\[
\begin{align*}
\mathbf{P}_g &= f(\theta_1) + l_1 \mathbf{e}^{(\alpha_1)} + \pi/2 + \mu_1 - l_1 \mathbf{e}^{(\alpha_1)} + \pi/2 + \mu_1) + l_6 \mathbf{e}^{(\alpha_6)} + \pi/2 + \mu_1) \\
\mathbf{P}_2 &= f(\theta_2) + Z_2 \mathbf{e}^{(\alpha_2)} + \pi/2 + \mu_1 - l_2 \mathbf{e}^{(\alpha_2)} + \pi/2 + \mu_1) + l_6 \mathbf{e}^{(\alpha_6)} + \pi/2 + \mu_1) \\
\end{align*}
\]

(4)

where \( \delta_2 \) is the inclination angle of the rear rack of which the rear climbing mechanism is composed. Then, computing the difference between (4) and (5) and taking the imaginary part, the following implicit expression is obtained:

\[
\begin{align*}
F_2(\mathbf{q}) &= \text{Im}[f(\theta_1) - f(\theta_2)] + \text{Im}[l_6 e^{(\alpha_6)} + \pi/2 + \mu_1] \\
&\quad - \text{Im}[l_1 e^{(\alpha_1)} + \pi/2 + \mu_1] + \text{Im}[l_2 e^{(\alpha_2)} + 3\pi/2 + \delta_2] + \text{Im}[l_3 e^{(\alpha_3)} + \pi/2 + \mu_1] = 0
\end{align*}
\]

(6)

where the term \( \text{Im}[f(\theta_1) - f(\theta_2)] \) is a constant value but we have no precise knowledge of its true value.

3) Robotic mechanism supported on the front rack and the rear wheels. In this case the device is supported on the front rack and the rear wheels (see Fig. 5a). We assume that the rear axle is rolling on flat terrain and the front rack is moving with a slope of \( \beta_1 = \pi/2 - \delta_1 \). The initial expressions that define the current position of the mechanism were explained in Morales et al. (2006a), and are the following:

\[
\begin{align*}
\mathbf{P}_g &= f(\theta_1) + l_4 \mathbf{e}^{(\alpha_4)} + \pi/2 + \mu_1 - l_4 \mathbf{e}^{(\alpha_4)} + \pi/2 + \mu_1) + l_6 \mathbf{e}^{(\alpha_6)} + \pi/2 + \mu_1) \\
\mathbf{P}_2 &= f(\theta_2) + Z_1 \mathbf{e}^{(\alpha_1)} + \pi/2 + \mu_1 - l_2 \mathbf{e}^{(\alpha_2)} + \pi/2 + \mu_1) + l_6 \mathbf{e}^{(\alpha_6)} + \pi/2 + \mu_1) \\
\end{align*}
\]

(7)

(8)

where \( \delta_1 \) is the inclination angle of the front rack of which the front climbing mechanism is composed. Therefore, equating terms (7) and (8), taking the imaginary part and after certain algebraic manipulations, one has the following implicit expression:

\[
\begin{align*}
F_3(\mathbf{q}) &= \text{Im}[f(\theta_1) - f(\theta_2)] - \text{Im}[l_6 e^{(\alpha_6)} + \pi/2 + \mu_1] \\
&\quad - \text{Im}[l_1 e^{(\alpha_1)} + \pi/2 + \mu_1] + \text{Im}[l_2 e^{(\alpha_2)} + 3\pi/2 + \delta_1] + \text{Im}[l_3 e^{(\alpha_3)} + \pi/2 + \mu_1] = 0
\end{align*}
\]

(9)

where the term \( \text{Im}[f(\theta_1) - f(\theta_2)] \) is a constant value but we have no precise knowledge of its true value.

Fig. 4. General kinematic scheme when the robotic mechanism is supported: (a) on four wheels; (b) on rear rack and front wheels.
Robotic mechanism supported on two racks. In this configuration the device is supported on two racks (see Fig. 5b). The rear and front racks are moving with the corresponding slopes $\beta_1 = \pi/2 - \delta_1$ and $\beta_2 = \pi/2 - \delta_2$. The initial expressions that define the current position of the mechanism were explained in Morales et al. (2006a), and are shown here:

$$P_g = P_{C2} + z_2 e^{[\gamma(\beta_2 - \beta_1) + \theta - \theta_0(\beta_2 - \beta_1)]} + l_2 e^{[\gamma(\beta_2 - \beta_1) + \theta - \theta_0(\beta_2 - \beta_1)]} + l_2 e^{[\gamma(\beta_2 - \beta_1) + \theta - \theta_0(\beta_2 - \beta_1)]}$$

(10)

$$P_g = P_{C1} + z_1 e^{[\gamma(\beta_2 - \beta_1) + \theta - \theta_0(\beta_2 - \beta_1)]} + l_2 e^{[\gamma(\beta_2 - \beta_1) + \theta - \theta_0(\beta_2 - \beta_1)]} + l_2 e^{[\gamma(\beta_2 - \beta_1) + \theta - \theta_0(\beta_2 - \beta_1)]}$$

(11)

Again, computing the difference between (10) and (11) and taking the imaginary part, the following implicit expression is obtained:

$$F_k(q) = \operatorname{Im}(P_{C2} - P_{C1}) + \operatorname{Im}(z_2 e^{[\gamma(\beta_2 - \beta_1) + \theta - \theta_0(\beta_2 - \beta_1)]}) - \operatorname{Im}(z_1 e^{[\gamma(\beta_2 - \beta_1) + \theta - \theta_0(\beta_2 - \beta_1)]})$$

$$+ \operatorname{Im}(l_2 e^{[\gamma(\beta_2 - \beta_1) + \theta - \theta_0(\beta_2 - \beta_1)]}) - \operatorname{Im}(l_2 e^{[\gamma(\beta_2 - \beta_1) + \theta - \theta_0(\beta_2 - \beta_1)]}) = 0$$

(12)

3.2. Implicit posture Jacobian models

Eqs. (3), (6), (9) and (12) can be expressed in a compact form as

$$F_k(q) = 0, \quad 1 \leq k \leq 4$$

(13)

where index $k$ indicates the chair configuration. In this section, we develop the implicit posture Jacobian models of the stair-climbing device that will be used in the control law in order to maintain the seat angle at all time within the comfort margins. In these models, the input control variables are the velocities of the motors in charge of moving the actuated degrees of freedom of the current behavior of the robotic mechanism $q = [\theta_1, \theta_2, \theta_3, z_1, z_2]^T$ and the output is the derivative of the inclination angle of the seat $\dot{\gamma}$. Then the differential relationship between the output $\dot{\gamma}$ and the variable $q$ is given by

$$\frac{\partial F_k}{\partial \dot{\gamma}} + \frac{\partial F_k}{\partial q} \dot{q} = 0, \quad 1 \leq k \leq 4$$

(14)

As it was described in Section 2, the posture control is achieved by actuating on joints $\theta_1$ and $\theta_2$. However, using the trajectory generation method presented in Morales et al. (2007), the entire responsibility of the posture control lies on the actuator in charge of the movement of the joint $\theta_1$, while the joint $\theta_2$ is kept constant. Then the relationship between the derivative of the controlled variable $\dot{\gamma}$ and the control signal $\theta_1$ is

$$\frac{\partial F_k}{\partial \dot{\gamma}} + \frac{\partial F_k}{\partial \theta_1} \dot{\theta}_1 + \mathbf{g}_k(q, q) = 0, \quad 1 \leq k \leq 4$$

(15)

Table 1 Terms of differential equations of the implicit Jacobian for the four configurations.

<table>
<thead>
<tr>
<th>$k$</th>
<th>$\frac{\partial F_k}{\partial \dot{\gamma}}$</th>
<th>$\frac{\partial F_k}{\partial \theta_1}$</th>
<th>$\mathbf{g}_k(q, q)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$l_2 \sin(\gamma + \mu_1)$</td>
<td>$-l_2 \sin(\gamma + \theta_1)$</td>
<td>$-l_2 \sin(\gamma + \theta_2)$</td>
</tr>
<tr>
<td>2</td>
<td>$l_2 \sin(\gamma - \theta_2)$</td>
<td>$-l_2 \sin(\gamma + \theta_1)$</td>
<td>$+ \cos(\gamma + \theta_2) l_2$</td>
</tr>
<tr>
<td>3</td>
<td>$l_2 \sin(\gamma + \mu_1)$</td>
<td>$-l_2 \sin(\gamma + \theta_1)$</td>
<td>$-l_2 \sin(\gamma + \theta_2)$</td>
</tr>
<tr>
<td>4</td>
<td>$l_2 \sin(\gamma - \theta_2)$</td>
<td>$-l_2 \sin(\gamma + \theta_1)$</td>
<td>$-l_2 \sin(\gamma + \theta_2)$</td>
</tr>
</tbody>
</table>

(16)

where it has been taken into account that $\theta_2$ remains constant in the four operating modes and then its time derivative is zero (i.e. $\dot{\theta}_2 = 0$). Thus, Table 1 expresses the terms of differential equations (15) for the four configurations:

Finally, expression (15) can be written in terms of the control signal $\theta_1$ as

$$\frac{\partial F_k}{\partial \dot{\gamma}} + \frac{\partial F_k}{\partial \theta_1} \dot{\theta}_1 + \mathbf{g}_k(q, q) = 0, \quad 1 \leq k \leq 4$$

(17)

4. Kinematic control

In this section we present a detailed description of the different parts which compose the proposed control system of the stair-climbing mechanism. Our objectives are: (i) maintenance of the passenger comfort at all times; (ii) high degree of perturbation rejection regarding modeling error; (iii) independence of the designed control law on the environment parameters; (iv) computational efficiency and; (v) the control is achieved with minimal sensor requirements. The following subsections will deal with this.

4.1. General scheme

The robotic driven degrees of freedom of the stair-climbing mechanism in charge of the locomotion (traction and step ascent) and concerning the posture (verticality of the chair frame) will be
dealt with together in the control scheme. Moreover, in the particular case of stair-climbing mechanisms, we have a critical additional restriction that these vehicles will carry people with physical disabilities. Then, the provision of acceptable comfort margins, the obtention of a constant seat angle and stability at all times for a stair-climbing mobility device is essential for safety during obstacle surpassing. Furthermore, the type of obstacles that these particular robotic vehicles have to overpass are those which typically appear in cities and buildings such as curbs, ramps or staircases. The specific geometry of curbs and staircases (discontinuous profiles) increase the difficulty of solving the problem. Therefore, we adopt the following kinematic controller design strategy which is summarized in Fig. 2 and can be decomposed into three different subsystems. The first subsystem, called behavior diagram, is responsible for the selection of the mechanical configuration and the trajectory generation of the motors in charge of the movement of the whole system. The second subsystem, called locomotion control, is in charge of the movement of the degrees of freedom in charge of the traction and step ascent, i.e. $\theta_3$, $z_1$ and $z_2$. The locomotion control is composed of a feedforward term and proportional derivative (PD) control loops. On the other hand, the degrees of freedom concerning the positioning mechanism, i.e. $\theta_1$ and $\theta_2$, compose posture control of the stair-climbing mechanism. The motors in charge of the movement of these degrees of freedom exhibit excellent mechanical properties but present physical limitations that impair the implementation of typical control laws. We found a solution which takes into account these motor physical restrictions whilst maintaining a high perturbation rejection regarding environment uncertainties. The posture control is developed with a novel variant of multilevel feedback controller design via a suitable combination of a Lyapunov feedback controller design along with a multilevel generalization of the sliding mode based Sigma–Delta ($\Sigma$–$\Delta$) modulator coupled with an adequate selection of the trajectory of the center of mass.

Finally, to synthesize the complete kinematic control law it is necessary to prescribe the time evolution of the inclination angle of the robotic mechanism for each possible configuration. After that, the trajectory generation block produces a set of reference trajectories that are used in the closed loop control of all the actuated degrees of freedom which are relevant in the tracking of the reference trajectories. Then, the locomotion and posture controllers work to achieve a successful obstacle surpassing and the regulation of the seat’s inclination within the established comfort margins. The following sections treat the components of the control system in detail.

### 4.2. Behavior diagram

The behavior diagram is in charge of the trajectory generation of the actuated degrees of freedom of the stair-climbing mechanism based on the current mechanical configuration and the sensorial information. The behavior diagram is shown in Fig. 6 and it is explained in detail next. At each instant of navigation, the control architecture of the stair-climbing device extracts the sensor information from the local robot environment. The configuration selector, which is explained in detail below, activates the appropriate behavior of the prototype that is necessary to surpass an architectural barrier. Based on the current behavior of the robotic mechanism, the trajectory generator presented in Morales et al. (2007) is used to develop the center of mass trajectories and the null inclination of the chair frame $p^*$. Then, by using the inverse kinematics model presented in Morales et al. (2006a), we finally achieve the reference trajectories that control the angles of the motors in charge of moving the actuated degrees of freedom of the current behavior of the robotic mechanism $q^*$.

In order to solve the selection of the appropriate mechanism configuration, based on the knowledge of the current configuration and the information that comes from the sensor system, we have developed a configuration selector which is depicted in Fig. 7. This diagram greatly helps to understand the prototype’s transition from one configuration to the next when the mechanism is working to overcome an architectural barrier. The diagram is similar to an addressed state-transition diagram with additional information. The nodes show the different prototype behaviors or configurations and are used to point out the current behavior of the prototype. The diagram arrows represent the behavior transitions. If one of the transitions is activated and this transition is one of the transitions from the current state, the behavior will change to the new behavior which is pointed out by the end of the transition arrow. We shall define the meaning of all the behaviors and all the transitions that appear during the climbing/descent process.

![Behavior diagram](image_url)
Behaviors. These correspond to the different prototype configurations that may appear during the stair-case climbing/descent process.

- I: Robotic mechanism supported on four wheels.
- II: Robotic mechanism supported on the rear rack and the front wheels.
- III: Robotic mechanism supported on the front rack and the rear wheels.
- IV: Robotic mechanism supported on both racks.

Transitions. The information which comes from the internal sensorial system (switches that indicate the end position of the wheels) and the external sensorial system (ultrasound sensors):

- A: The distance between the front wheel axle and the step is lower than a predefined threshold (ultrasound sensors).
- B: The distance between the rear wheel axle and the step is lower than a predefined threshold (ultrasound sensors).
- C: Front wheels completely overcome the obstacle (switches).
- D: Rear wheels completely overcome the obstacle (switches).

4.3. Locomotion control

The DC motors which compose the locomotion control (two for the rear wheels and two for the racks) are commanded based on the pulsedwidth modulation technique. The motor positions are controlled via proportional derivative (PD) control loops. The PWM servo amplifiers control the current to the motors, thus obtaining estimates of the torques at the motor shaft as long as the motors are not saturated. A detailed description of the locomotion control was presented in the previous works (see Morales et al., 2006a, 2007).

4.4. Posture control

Regarding Section 3, we observe that the regulation of the inclination angle of the seat is not an straightforward problem. The inclination angle, γ, is commanded from several control inputs. This implies that the election of the control inputs should be obtained after solving an optimization problem and further problems could appear when the control algorithm is implemented in a real-time system. In this section, we avoid all the previous problems by selecting a new variant of multilevel feedback controller design via a suitable combination of Lyapunov feedback controller design along with a generalization of the sliding mode based Sigma–Delta (Σ–Δ) modulator coupled with a smart selection of the trajectory of the center of mass (see Appendices A and B for a complete description of the theoretical foundations). We proceed to explain next:

Let m be an integer and consider the following partition of the interval I = [−1, 1] of the real line:

$$U_m = \left\{ 1, -\frac{m-1}{m}, -\frac{m-2}{m}, \ldots, 0, \frac{1}{m}, \frac{2}{m}, \ldots, m-1, 1 \right\}$$

expressed as a finite discrete set of rational values in I. The family of closed subintervals of the real line formed by any two consecutive elements of this set will be said to constitute the switching levels. The limits of the switching levels will be called switching values. The set of switching values coincides with Ulm.

The fundamental task of this multi-valued Σ–Δ modulation is that of translating average control input values into switched signals taking values in the discrete set given by U_m, in such a way that the input signal, ζ(t), precisely coincides with the averaged value of the switched output, u, in an ideal sliding mode sense, i.e. such that the equivalent values, under ideal sliding mode conditions, of the signals u, denoted by ueq, coincide, precisely, with ζ(t). This is accomplished via the following sliding mode dynamics:

$$\dot{e} = \zeta - u$$

$$u = \frac{1}{4m} \sum_{k=-m+1}^{m} (2k-1 + \text{sign}(e)) \times \left( \text{sign}\left(\frac{k-1}{m}\right) - \text{sign}\left(\frac{k}{m}\right) \right)$$

Proof. It is easy to show that, for a fixed value of k, say, k = j ≥ 0, with j ≤ m, only one summand is active in (19) with all other summands being identically zero. It follows that whenever ζ ∈ [(j−1)/m,j/m], the following discontinuous dynamics is valid:

$$\dot{e} = \zeta - u$$

$$u = \frac{1}{2m} (2j-1 + \text{sign}(e))$$

When e > 0 then u = j/m and ˙e < 0. Also, when e < 0, then u = (j−1)/m and ˙e > 0. Clearly, ˙e < 0 and a sliding regime exist on e = 0. As a consequence, the average value of the switching signal u, designed by ueq, ideally coincides with ζ. A similar reasoning is valid for j < 0.

Now, according to the previous section, the signal θ1 is in charge of the posture control of the stair-climbing mechanism. Due to the physical properties of the motors which constitute the positioning mechanism, the control input θ1, is restricted to provide 2m + 1 switching levels in the interval [−θmax,θmax], due to the prevailing physical limitations. The switched model for the speed of the motor with 2m + 1, m ≥ 1, switching values, is obtained from the extreme values of adjacent uniformly distributed intervals within the set [−θmax,θmax] for every one of the possible configurations. Let $u_1 \in U_m$ be multilevel position functions. Then, the switched models obtained from the implicit Jacobian model (15) for the four configurations are yielded by expression (17). After that, let $\zeta_1$ denotes the average control input, continuously taking values in the closed interval $I = [−1,1]$. Finally, the state average implicit Jacobian models achieved from the differential Eq. (15) for the four configurations, that will be taken into account in the posture control design stage, are readily given by the following expressions.

$$\frac{\partial F_k}{\partial \theta} \dot{\gamma} + \frac{\partial F_k}{\partial \theta} \theta \dot{\gamma} + \tilde{F}_k(q,Q) = 0, \quad 1 \leq k \leq 4$$

(21)
where the terms \( \partial F_k / \partial y \), \( \partial F_k / \partial \theta_1 \), and \( \mathbf{g}_k(q, \dot{q}, \ddot{q}) \) for all the possible configurations were reported in Table 1.

### 4.4.2. Posture control law synthesis

In order to carry out the control of the seat angle, based on the averaged implicit posture Jacobian model of the mechanism, we first specify a continuous feedback controller, of the Lyapunov type, for the average control inputs \( \hat{\theta}_1 = \hat{\theta}_{1\text{max}} \) where \( \hat{\theta}_1 \in [-1,1] \).

Once the continuous average control input is obtained, as feedback law \( \zeta = \zeta(\hat{y}) \), with \( \hat{y} = y^* - y \) and \( y^* = 0 \), we devise a multilevel sliding mode \( \Sigma-\Delta \) modulator which properly processes the average feedback input signal, \( \zeta(t) \), producing a switched output to be delivered to the actual system input port, \( u_1 \). This input actively takes values from the required adjacent velocity levels of the uniform partition induced on the overall input velocity interval, so that the average output reference trajectory tracking is properly guaranteed. The proposed control algorithm results in a perfect stabilization of the inclination angle of the seat (for all possible configurations) without actually resorting to optimization algorithms, nor iterative calculation procedures. Moreover, the implemented control algorithm is simple enough, computationally efficient, and it allows an easy implementation for real time control purposes.

In the design of the posture control developed, the following assumptions are valid throughout all the different configurations:

- We have no precise knowledge of the true value of the obstacle to be solved.
- Based on the geometry of the robotic prototype, we obtain the following relations: \( l_0 = l_1, l_2 = l_3 \) and \( \mu_1 = \mu_4 = \mu_5 = 0 \).
- The actuated degrees of freedom in charge of moving the front and rear position mechanisms (\( \theta_1 \) and \( \theta_2 \)) are limited, respectively, to satisfy \( \theta_{\text{min}} \leq \theta_1 \leq \theta_{\text{max}} \) and \( \theta_{\text{min}} \leq \theta_2 \leq \theta_{\text{max}} \), with \( \theta_{\text{max}} = \theta_{\text{max}} \).
- The rear position mechanism \( \theta_2 \) remains constant throughout the entire climbing process (\( \zeta_2 = 0 \)) and the maintenance of verticality of the robotic mechanism is entirely supported by the front position mechanism \( \theta_1 \).

Next, we proceed to explain the synthesis of the control law for all the configurations of the reconfigurable stair-climbing device.

**Proposition 1.** Given the previous assumptions, consider the state average nonlinear implicit Jacobian model of the stair-climbing device represented by (21). Assuming that the measurements of the seat inclination, \( y \), and the applied average control input, \( \zeta_1 \), are available. Thus, applying the reference trajectory generation \( \text{P}_{\theta_1} \) in Table 2 and described in Morales et al. (2007), defining \( \Gamma_k \) as positive escalar constants and taking into account the values of \( \partial F_k / \partial y \), \( \partial F_k / \partial \theta_1 \), and \( \mathbf{g}_k(q, \dot{q}) \) illustrated in Table 1 for each particular configuration, then, the feedback controller:

\[
\hat{y} = -
\begin{array}{l}
\left( \partial F_k / \partial \theta_1 \right)^{-1} \hat{\theta}_{\text{max}} \zeta_1 - \left( \partial F_k / \partial y \right)^{-1} \mathbf{g}_k(q, \dot{q})
\end{array}
\]

\( \text{for } k \leq 4 \)

produces the desired average output trajectory tracking, rendering the origin of the tracking error space into an exponentially asymptotic equilibrium point.

### Table 2

<table>
<thead>
<tr>
<th>( k )</th>
<th>( \text{P}_{\theta_k} )</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>( \text{P}_{\theta_1} =</td>
</tr>
<tr>
<td>2</td>
<td>( \text{P}_{\theta_2} =</td>
</tr>
<tr>
<td>3</td>
<td>( \text{P}_{\theta_3} =</td>
</tr>
<tr>
<td>4</td>
<td>( \text{P}_{\theta_4} =</td>
</tr>
</tbody>
</table>

**Proof.** Using the proposed reference trajectory generation \( \text{P}_{\theta_k} \), depicted in Table 2 for each particular configuration, it was demonstrated in Morales et al. (2007) that the angle \( \theta_2 \) remains constant at all times. Therefore, \( \zeta_2 = 0 \). This selection of the center of mass reference trajectory substantially simplifies the control law and avoids the need for solving an optimization problem.

Then, the state average implicit Jacobian model of the stair-climbing mechanism is expressed from (21) as

\[
\hat{y} = -
\begin{array}{l}
\left( \partial F_k / \partial y \right)^{-1} \left( \partial F_k / \partial \theta_1 \right) \hat{\theta}_{\text{max}} \zeta_1 - \left( \partial F_k / \partial y \right)^{-1} \mathbf{g}_k(q, \dot{q})
\end{array}
\]

(23)

On the other hand, defining a Lyapunov function candidate, \( \gamma_k = \frac{1}{2} \hat{y}^2 \) and \( 1 \leq k \leq 4 \), which represents a certain instantaneous energy of the sliding surface coordinate function, \( \hat{y} \), with regard to its zero value defining, in turn, a smooth manifold. A plausible policy through which to attain the desired condition: \( \dot{\gamma}_k(\hat{y}) = 0 \), from any open vicinity of the smooth manifold, is to adopt an action for the control input, \( \hat{\theta}_1 = \hat{\theta}_{1\text{max}} \), which results in a strict decrease of the function \( \dot{\gamma}_k(\hat{y}) \). This can be achieved by influencing the system in such a manner, that the speed of variation of \( \dot{\gamma}_k(\hat{y}) \) will be strictly negative, i.e., \( \dot{\gamma}_k(\hat{y}) < 0 \). Taking into account that \( \hat{y} = y^* - y \), using the state average implicit Jacobian model, given by expression (21), and after certain straightforward algebraic manipulations, we obtain the following result:

\[
\dot{\gamma}_k = \hat{\Gamma}_k \hat{y}^2 + \left( \partial F_k / \partial \theta_1 \right) \hat{\theta}_{\text{max}} \zeta_1 + \left( \partial F_k / \partial y \right) \mathbf{g}_k(q, \dot{q})
\]

(24)

where \( \Gamma_k \), \( 1 \leq k \leq 4 \), are positive escalar constants. Therefore, selecting the average control input \( \zeta_1 \) and the multilevel \( \Sigma-\Delta \) modulator as

\[
\dot{\gamma}_k = - \left( \partial F_k / \partial \theta_1 \right) \hat{\theta}_{\text{max}} \zeta_1 + \left( \partial F_k / \partial y \right) \mathbf{g}_k(q, \dot{q})
\]

(25)

\[ e_1 = \zeta_1 - u_1 \]

(26)

\[ u_1 = \frac{1}{4m} \sum_{l = -m+1}^{m} (2l+1 + \text{sign}(e_1)) \times \left[ \text{sign}(\zeta_1 - \frac{l-1}{m}) - \text{sign}(\zeta_1 - \frac{l}{m}) \right] \]

\( \Gamma_k > 0 \)

(27)

results in \( \dot{\gamma}_k = - \Gamma_k \hat{y}^2 < 0 \), for \( 1 \leq k \leq 4 \). Moreover, in the mechanical configurations where the system is supported on rack and wheels simultaneously, i.e. \( k = 2,3 \), we have additional restrictions related to the maximum velocities achievable by the front and rear climbing mechanisms, \( \dot{z}_1 \) and \( \dot{z}_2 \). These maximum velocity restrictions are defined under the invariance conditions:

\[
\dot{\gamma} = 0, \quad \dot{\gamma} = 0 \quad \text{as}
\]

\[ |z_2| < l_2 \left| \hat{\theta}_{\text{max}} \frac{\sin \theta_1}{\cos \theta_2} \right| \quad \text{for } k = 2 \]

(28)
and

\[ |\dot{z}_1| < \dot{l}_3 \theta_{\text{max}} \sin \theta_1 \cos \theta_1 \text{ for } k = 3 \quad (29) \]

The only parameter that can change in the previous expressions is the angle of the front positioning mechanism \( \theta_1 \). As the range of variation of the variable \( \theta_1 \) is perfectly known due to the mechanical restrictions of the mechanism, we therefore select the value of \( \theta_1 \) which gives us the more restricted values of \( |\dot{z}_1| \) and \( |\dot{z}_2| \) and, additionally, we include a security coefficient to limit these velocities as

\[ |\dot{z}_{1_{\text{max}}}| = |\dot{z}_{2_{\text{max}}}| < \dot{q}_{13} \theta_{\text{max}} \frac{(\sin \theta_1)_{\text{min}}}{\cos \theta_1} \quad (30) \]

where \( \delta_1 = \delta_2 \) and \( \varphi = 0.85 \). Finally, taking into account all the previous considerations, the designed feedback controller produces the desired average output trajectory tracking error, \( \tilde{\gamma} = \gamma^* - \gamma \), so as to render the origin of the tracking error space into an exponentially asymptotic equilibrium point. □

4.5. Control architecture

The whole control architecture has been divided into the sensorial system, the actuator system and the control hardware. Next subsections will treat with those subjects.

4.5.1. Sensorial system

Eight ultrasound sensors are needed to measure the distances between the wheels and the steps and to obtain information about the climbing/descent process. They are placed two per wheel in horizontal and vertical positions. An inclinometer is placed on the frame to measure the verticality of the chair and to detect the instant at which the climbing mechanism touches the floor when being deployed. To measure the positions of the different actuated degrees of freedom, there are four encoders – two in the rear wheels and two in the racks of the climbing mechanism – and two angle sensors on linear actuators. Finally, there are four switches (one per wheel) to indicate the end positions of the six four-bar mechanisms, and eight switches (two per linear actuator and two per rack) to indicate the maximum and minimum positions for the two joints of the chair structure for the racks.

4.5.2. Actuator system

The movement of the reconfigurable stair-climbing mechanism is driven by two DC motors connected to the rear wheels, two DC motors on the front and rear racks, and two linear actuators located on the joints of the chair structure. MATRIX_3A linear power actuators were chosen for their low cost, small size, light weight, linear power and mechanic-electrical characteristics (120 W, maximum 8000 N, 57 mm/s, 24 V, weight 4.5 kg). These linear actuators are driven by an MD22 Advanced Motion Control PWM servo amplifier. The racks and the wheels are actuated by

![Fig. 8. Control architecture of the stair-climbing mechanism.](image-url)
Maxon 148867 DC Motors with planetary gearbox (Maxon motor 203.129, reduction 156:1), chosen as having the same features as the linear drives (150 W, 24 V, 8200 rpm, weight 2.2 kg). These are driven by an Advanced Motor Control EPOS 24/5 servo amplifier. Finally, four electromagnetic solenoids (one per wheel) are used to lock or unlock the mechanisms that connect the wheels to the axles (see Morales et al., 2006a for details of the mechanical design).

4.5.3. Control hardware

The control architecture is a system consisting of a Real-Time PowerPC Controller CompactRio CRIO-9022 533 MHz, a digital I/O board NI 9403, an analog input board NI 9201 and a CAN-Bus communication module NI 9853. The digital output board is used to control the electromagnetic solenoids. The analog input board is used to acquire sensorial data from the different sensors. The CAN-Bus communication module is used to acquire sensorial data from the ultrasound sensors and to command six DC motors (two for the rear wheels, one for the rack and one for the front and rear joints of the positioning mechanisms). The system operates at 24 V nominal voltage provided by two batteries, and a conditioning circuit that provides regulated voltages at 5 V, 18 V, and 24 V. The CompactRio, with the I/O and CAN-Bus boards, gathers data from the sensors and performs the computations to implement the various control schemes. The sampling time used is 10⁻¹ s. Finally, the control architecture of the system is shown diagrammatically in Fig. 8.

5. Simulation results

Numerical simulations were carried out in order to evaluate the performance of the proposed control algorithm for the stair-climbing system as it overpasses unstructured environments in which the geometry of the obstacles was not accurately known. Simulation results show that the mechanical device has to overpass a curb as architectural barrier. Different curb dimensions were used (170 mm, 180 mm, 190 mm and 200 mm). In this study, the height of the curb is unknown in order to test the performance of the proposed controller presented in the previous section with respect to these uncertainty sources. At the same time, both, passenger comfort and seat verticality must be kept within the comfort margins. On the one hand, to achieve passenger comfort the following conditions need to be taken into consideration:

- The inertial forces have to be small relative to gravity forces.
- The mass center accelerations and velocities of the whole system (human-robotic mechanism-obstacle) must be less than the maximum comfort acceleration and velocity, respectively.
- The inclination of the seat of the stair-climbing device must have the acceptable comfort ranges, ± 0.1 m/s² (or ± 0.174 rad/s).

Additionally, in order to fulfill previous comfort conditions, to obtain a successful controller behavior, and an appropriate system response, several additional assumptions have been considered:

- The high frequency commutations (chattering) of the control signal is a characteristic of systems were sliding control is applied. This phenomenon is not desirable because it can damage the electrical and mechanical components of the system. To temper this effect, we added a high-gain band of ± 0.02 rad to the sliding surface $\tilde{\gamma} = 0$, which completely maintains the passenger comfort (Slotine & Li, 1991).
- The maximum angular velocities to the actuated degrees of freedom of the positioning mechanism are $\dot{\theta}_1 = \dot{\theta}_2 = 0.04$ rad/s.
- The arranged switched values which are used to provide two equally spaced subintervals of the interval $[-1,1]$ thus providing the following three normalized switching values $U_k = (-1,0,1)$.
- The posture control of the joint $\theta_j$ is obtained from (27) while the joint $\theta_i$ is kept constant.
- The control parameter value $\Gamma_k$, used in (27) is the same for $1 \leq k \leq 4$ taking a value of 0.8.
- When the prototype is supported on four wheels:
  - The velocity of the rear wheels is maintained constant ($\dot{\theta}_3 = \dot{\theta}_4$) where $C_1$ is defined by the user. In this trial $C_1 = 0.2$ rad/s.
  - The position of the racks is maintained in a constant value, i.e. $\dot{z}_1 = 0$, and $\dot{z}_2 = 0$.
- When the prototype is supported on the rear rack and the front wheels:
  - The position of the rear wheels remains constant ($\dot{\theta}_3 = 0$).
  - The position of the front rack is maintained, $\dot{z}_1 = 0$, and the velocity of the rear rack is a constant value, $\dot{z}_2 = C_2$, where $C_2$ is defined by the user in order to fulfill the condition shown in (28). In this trial $C_2 = 0.01$ m/s.
- When the prototype is supported on the front rack and the rear wheels:
  - The movement of the rear rack is not considered, i.e. $\dot{z}_2 = 0$, and the movement of the front rack is a constant, $\dot{z}_1 = C_3$, defined by the user to fulfill the condition in (29). In this trial $C_3 = 0.01$ m/s.
- When the prototype is supported on both racks:
  - The movement of the racks are considered to be $\dot{z}_1 = \dot{z}_2 = C_4$ where $C_4$ is a constant defined by the user which value is $C_4 = 0.01$ m/s.

Fig. 9a and b depict the angle trajectories of the positioning mechanism. According to the control scheme developed in Section 4, shown in Fig. 9a, the responsibility for the climbing process and the capability to keep the chair verticality is entirely ascribed to the linear actuator connected to the front positioning mechanism, $\theta_1$. The angle of the rear positioning mechanism, $\theta_2$, remains constant in every simulation (see Fig. 9b). Fig. 9a, illustrates that when the curb height increases, the necessary front positioning angle, needed to overpass the obstacle, becomes larger. These findings are in accordance with the theoretical results described in Morales et al. (2007).

Fig. 10a and b show the trajectories of the front and rear climbing mechanisms, $\dot{z}_1$ and $\dot{z}_2$ respectively, depicting the deployment and backward movement of each climbing mechanism when they individually confront obstacles. As it can be noticed in these figures, when the height of the curb increases the length of the positioning mechanisms also increases to overcome an obstacle. Furthermore, the rack velocities have been chosen in order to fulfill the different restrictions in the control law (see Section 4.4.2 for details).

Fig. 11a illustrates the evolution of the rear wheels angle (in charge of the evolution of the angle $\theta_3$) and Fig. 11b shows the
evolution of the seat inclination of the stair-climbing device as a result of the control scheme implementation. It can be observed that the inclination is nearly null in every simulation, which is in accordance with the comfort intervals exposed previously. Finally, Fig. 12a depicts the evolution of the normalized switched control input system based on the multilevel $\Sigma \Delta$ modulator ($u_1$) and Fig. 12b illustrates the trajectory of the mass center as the robotic mechanism climbs the different tested curbs.

It can be seen from the simulations that the robotic mechanism overcomes different curbs maintaining the passenger comfort. From
the previous results, we can affirm the following: (a) a correct posture selection has been achieved from the behavior diagram; (b) a smooth transition between different configurations has been obtained which implies an improvement to the system comfort response; and (c) the proposed controller works satisfactorily to every possible configuration of the stair-climbing device maintaining always the seat verticality of the robotic mechanism within the comfort margins.

6. Experimental results

In this section, we carry out an experimental implementation to assess the performance of the proposed controller in a real robotic prototype. In the experiments reported in this section, we used a curb, 180 mm high, as an architectural barrier and we used a volunteer person on the prototype, to obtain a real interaction with the whole system (human-prototype-obstacle). The climbing obstacle sequence must be comfortable for the passenger. Then, all the assumptions described in the numerical simulations are applied to the experimental platform. Data on the movement of the chair was obtained using the commercial Optotrack system. This system consists of three infrared cameras which can obtain the measurements (6D) of several infrared markers. In our experiment, the Optotrack motion analysis system used two infrared markers to record the mechanism trajectories. One was placed at the center of mass of the stair-climbing device and the other on a horizontal surface to measure the deviation of the seat from verticality. We also used the internal hardware of the prototype to acquire data from the encoders of the wheels and the racks, the sensors that measure the angles of the joints of the structure, and the inclination of the seat. The real-time movement of the prototype was thus appropriately recorded throughout the test. The problem of the synchronization of all the data on the prototype's movement was solved by using a trigger signal.

Fig. 13a and b show the simulated and experimental trajectories of the joint angles connecting the chair structure (positioning mechanism). We notice that the responsibility for the climbing process and the maintenance of verticality of the chair is entirely supported by the linear actuator connected to the chair structure in

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**Fig. 13.** Experimental evolution of the angle of the positioning mechanism when the prototype climbs a curb of 180 mm height: (a) front joint, (θ₁); (b) rear joint, (θ₂).

---

**Fig. 14.** Experimental evolution of length of the racks of the climbing mechanism when the prototype climbs a curb of 180 mm height: (a) front rack, (z₁); (b) rear rack, (z₂).

---

**Fig. 15.** Experimental evolution of: (a) rear wheels angle, (θ₃) and (b) inclination of the seat, γ, when the prototype climbs a curb of 180 mm height.
charge of the evolution of $\theta_1$ (see Fig. 13a). Fig. 13b shows that the linear actuator in charge of the evolution of the rear positioning mechanism (responsible for the angle $\theta_2$) remains constant throughout the entire experiment. These results are in accordance with the simulation results presented in Section 5.

Fig. 14a and b depict the simulated and experimental trajectories of the racks ($z_1$ and $z_2$), illustrating the deployment and backward movement of each climbing mechanism when they individually confront obstacles. Fig. 15a depicts the simulated and experimental evolution of the rear wheels angle (in charge of the evolution of the angle $\theta_3$). Fig. 15b shows the evolution of the seat inclination as a result of the implementation of the control scheme. We plotted the results obtained by the Optotrack system and the measurements obtained with the inclinometer. Observe that although the two sets of measurements are similar and that the inclination is in complete accordance with the comfort intervals, the most visible discrepancy between simulated and real data is produced in this variable. The main reasons are the small movements that the passenger performs during the climbing process and the small geometry differences between the proposed kinematic model and the real prototype. This fact shows that the proposed control system is robust regarding these sources of uncertainty and, even in this case, the inclination angle of the seat is barely deteriorated (maximum errors around 0.02 rad or 1.15°, regarding to the results achieved with an exact kinematic model in the numerical simulations).

The experimental evolution of the normalized switched control input system based on the multilevel $\Sigma-\Delta$ modulator ($u_1$) and the simulated and experimental trajectory of the center of mass ($P_z$) are, respectively, shown in Fig. 16a and b. We can assess the agreement between simulations and experiments illustrating a good control posture and a successful curb avoidance of the real prototype. We notice only a small deviation in the trajectory, which is barely perceptible for the passenger.

7. Conclusions

In this paper, we have presented a new design concept in the control of stair-climbing mechanisms. The main features of the real prototype shown here are: (a) automatic adaptation to steps of different heights; (b) maintenance of the seat verticality; (c) reduced weight and energy consumption; and (d) guaranteed prototype stability at every moment since its weight is always transferred to horizontal surfaces and the support polygon is always greater than or equal to the support polygon of conventionally EPWs. In previous works, all the experiments were developed under complete knowledge of the obstacle to be solved. In order to increase the autonomy of the stair-climbing prototype, when the knowledge of the environment is vague, the sensor system plays an important role and a solution to the problem of the obstacle surpassing has been proposed. The main components of the proposed kinematic control are: (a) by taking advantage of the reconfigurable nature of our stair-climbing device, we have developed a mechanism behavior diagram which selects the correct configuration of the prototype based on its current configuration and the information gathered from the sensor system; (b) Development of a feedforward term and a standard proportional derivative (PD) control for the actuated degrees of freedom responsible for the locomotion control (traction and step ascent); and (c) development of an output posture feedback controller synthesized via a suitable combination of a Lyapunov feedback controller design along with a multilevel generalization of sliding mode control based on Sigma–Delta ($\Sigma-\Delta$) modulation coupled with an adequate selection of the center of mass reference trajectory generation. The proposed kinematic control scheme maintains the inclination of the prototype’s seat within the passenger comfort intervals and allows to overcome obstacles with unknown heights. Moreover, the proposed control has the following additional properties: (a) High degree of perturbation rejection regarding modeling error; (b) They are independent of each other and, also, independent of the environment parameters; (c) the implemented control law does not require the resolution of optimization algorithms nor iterative calculations; (d) computational efficiency; and (e) easy implementation in a real time system. The experimental results demonstrate a correct posture control and a successful obstacle overcoming when the real prototype is confronted with a curb, thus achieving an excellent overall performance of the selected controller. As a bonus, we obtain passenger stability and comfort with minimal sensor requirements.

In future work, based on the rather satisfactory results obtained with the closed loop control, we will focus on different experimental avenues, namely: (a) to improve the design of the positioning and climbing mechanisms to reduce the prototype’s geometry and to obtain a system which is capable of confronting obstacles with more challenging geometries; (b) to study different robust control strategies combined with advanced trajectory generation schemes (as in Morales et al., 2006b, 2010). We shall seek to take advantage of the additional degrees of freedom of the positioning mechanism and smaller quantization steps implemented in the posture control of the stair-climbing prototype in order to further increase the passenger comfort level.

Acknowledgments

This paper was sponsored by the Spanish Government Research Program with the Project DPI2013-37062-C02-01 (Ministerio de Economía y Competitividad) and by the European Social Fund.
Appendix A. Sigma–Delta (Σ–Δ) modulation

Sigma Delta modulation is an important tool that will allow us to translate continuous (i.e. average) feedback controller design options into implementable switch controlled strategies with practically the same closed loop behavior.

Consider the basic block diagram of Fig. 17, reminiscent of a traditional Σ–Δ-modulator block used in early communications systems theory and analog to digital conversion schemes, but with a binary valued forward nonlinearity, taking values in the discrete set (0, 1). The following theorem summarizes the relation of the considered modulator with sliding mode control while establishing the basic features of its input–output performance.

**Theorem 1.** Consider the Σ–Δ-modulator of Fig. 17. Given a sufficiently smooth, bounded, signal μ(t), then the integral error signal, e(t), converges to zero in a finite time, t₀, and, moreover, from any arbitrary initial value, e(t₀), a sliding motion exists on the perfect encoding condition surface, represented by e = 0, for all t > t₀, provided the following encoding condition is satisfied for all t,

0 < μ(t) < 1

(31)

**Proof.** From the figure, the variables in the Σ–Δ-modulator satisfy the following relations:

\[ \dot{t} = \mu(t) - u \]
\[ u = \frac{1}{2} [1 + \text{sign}(e)] \]

(32)

The quantity e is given by

\[ ee = e[\mu - \frac{1}{2} (1 + \text{sign}(e))] = -e[\mu - \frac{1}{2}] \]

For e > 0 we have ee = e(1 - μ), which, according to the assumption in (31) leads to ee < 0. On the other hand, when e < 0, we have ee = -e(1 - μ). A sliding regime exists then on e = 0 for all time t after the hitting time t₀ (see Utkin, 1978). Under ideal sliding, or encoding, conditions, e = 0, we have that the, so called, equivalent value of the switched output signal, u, denoted by uₘ(t) satisfies uₘ(t) = μ(t).

An estimate of the hitting time t₀ is obtained by examining the modulator system equations with the worst possible bound for the input signal μ in each of the two conditions: e > 0 and e < 0, along with the corresponding value of u. Consider then e(t₀) > 0 at time t₀ = 0. We have for all 0 < t ≤ t₀,

\[ e(t) = e(t₀) + \int_{t₀}^{t} (\mu(σ) - u(σ)) dσ \leq e(t₀) + t \left( \sup_{t \in [0,t]} \mu(t) - 1 \right) \]
\[ < e(t₀) + t₀ \left( \sup_{t \in [0,t]} \mu(t) - 1 \right). \]

(33)

Since e(t₀) = 0, we have:

\[ t₀ \leq \frac{e(0)}{1 - \sup_{t \in [0,t]} \mu(t)} \]

(34)

The average Σ–Δ-modulator output uₘ ideally yields the modulator’s input signal μ(t) in an equivalent control sense (Utkin, 1978).

To illustrate, by means of simulations, the feature just stated about Σ–Δ modulation, we let μ(t) = 0.5(1 + A sin(ωt)) with A = 0.8, ω = 3 rad/s. At the output of the modulator we put a second order low pass filter of the form

\[ y = \frac{\omega_n^2}{s^2 + 2 \zeta \omega_n s + \omega_n^2} \]

(35)

with ζ = 0.81 and ωₙ = 30. We may compare the filter output y with the input signal μ(t): modulo a small delay and the second order filter transient from zero initial conditions, the filtering of the switched output signal, u(t), of the modulator, represented by the variable y(t), reproduces the sinusoidal input to the modulator. Fig. 18 depicts the results:

The role of the above described Σ–Δ-modulator in sliding mode control schemes, avoiding full state measurements, and using average based controllers is clear from the following subsection.

A.1. Average feedbacks and Σ–Δ-modulation

Suppose we have a smooth nonlinear system of the form

\[ \dot{x} = f(x) + g(x)u \]

with u being a (continuous) control input signal that, due to some physical limitations, requires to be strictly bounded by the closed interval [0, 1]. Suppose, furthermore, that we have been able to specify a dynamic output feedback tracking controller of the form

\[ u = u^*(t) - k(y, z, y^*(t)) \]

where y = y(t) with desirable closed loop performance features guaranteeing that y → y*(t) with a bounded stable zero dynamics. Assume, furthermore, that for some reasonable set of initial states of the system (and of the dynamic controller), the values of the generated feedback signal function, u(t), are uniformly strictly bounded by the closed interval [0, 1].

If an additional implementation requirement entitles now that the control input u of the system is no longer allowed to...
continuously takes values within the interval (0,1), but that it may only take values in the discrete set, (0,1], the natural question is: how can we now implement the previously derived continuous output feedback controller, so that we can recover, possibly in an average sense, the desirable trajectory tracking features of the derived dynamic output feedback controller design, in view of the newly imposed actuator restriction?

The answer is clearly given by the features present in the previously considered $\Sigma$–$\Delta$-modulator. Recall, incidentally, that the output signal of such a modulator is restricted to take values, precisely, in the discrete set $\{0,1\}$. The output signal of the modulator reproduces, on the average, the required control input signal $u_{av}$.

Fig. 19 shows the switch based implementation of an output feedback controller, through a $\Sigma$–$\Delta$-modulator, which reproduces, in an average sense, the features of a designed continuous controller.

In view of the previous result in this Appendix, we have the following general result concerning the control of nonlinear systems through sliding modes synthesized on the basis of an average sense, the features of a designed continuous controller.

### Theorem 2
Consider the following smooth nonlinear single input, n-dimensional system: $\dot{x} = f(x) + u_{av}g(x)$, with the smooth scalar output map, $y = \sigma(x)$. Let $u^*(t)$ be the nominal control input corresponding to a desired given output reference trajectory $y^*(t)$. Assume that the dynamic smooth output feedback trajectory tracking controller, $u_{av} = u^*(t) - \kappa(y, \zeta, y^*(t))$, $\zeta = \phi(y, \zeta, y^*(t))$, with $\zeta \in \mathbb{R}^n$, locally (globally, semi-globally) asymptotically stabilizes the system output, $y$, toward a desired reference trajectory, $y^*(t)$ with a bounded stable zero dynamics. Assume, furthermore, that the control signal, $u_{av}$, is uniformly strictly bounded by the closed interval $[0,1]$ of the real line. Then the closed loop switched system:

\[ \dot{x} = f(x) + u_{av}g(x) \]

\[ y = \sigma(x) \]

\[ u_{av} = u^*(t) - \kappa(y, \zeta, y^*(t)) \]

\[ u_{av}(y, \zeta, y^*(t)) = u^*(t) - \kappa(y, \zeta, y^*(t)) \]

\[ \zeta = \phi(y, \zeta, y^*(t)) \]

exhibits an ideal sliding dynamics which is locally (globally, semi-globally) asymptotically stable to the same reference trajectory, $y^*(t)$ of the system.

### Proof
The proof of this theorem is immediate upon realizing that under the hypothesis on the average control input, $u_{av}$, the previous theorem establishes that a sliding regime exists on the manifold $e=0$. Under the invariance conditions, $e = 0$, $\dot{e} = 0$, which characterize ideal sliding motions (See Sira-Ramírez, 1988), the corresponding equivalent control, $u_{eq}$, associated with the system satisfies: $u_{eq}(t) = u_{av}(t)$. The ideal sliding dynamics is then represented by

\[ \dot{x} = f(x) + u_{eq}g(x) \]

\[ y = \sigma(x) \]

\[ u_{eq}(y, \zeta, y^*(t)) = u^*(t) - \kappa(y, \zeta, y^*(t)) \]

which is assumed to be locally (globally, semi-globally) asymptotically stable toward the desired reference trajectory.

### Remark
Note that the $\Sigma$–$\Delta$ modulator state, $e$, can be initialized at the value $e(0) = 0$. This implies that the induced sliding regime exists uniformly for all times after $T_0$. Hence, no reaching time of the sliding surface, $e=0$, is required. This practical feature is adopted throughout this article.

### Appendix B. Multilevel $\Sigma$–$\Delta$ modulation

A further generalization of the $\Sigma$–$\Delta$ modulation encoding technique can be obtained by considering several “levels of coding” or “levels of digital quantization”. Suppose we would like to have $N$ positive levels of discontinuous encoding of a strictly positive signal $\zeta(t)$. In other words, let $W$ be a fixed positive real number representing a quantization, or granularity, level. Assume, moreover that the given positive signal $\zeta(t)$ satisfies the following bound $\max_{t \geq 0} \zeta(t) \leq NW$ for some finite integer $N$. We would like to produce a discontinuous signal taking values on the finite set $\{0, W, 2W, \ldots, (N-1)W, NW\}$ and which switches between two adjacent values $(j-1)W$ and $jW$ when the signal $\zeta(t) = [(j-1)W, jW]$ for every $j$.

The following generalization of the $\Sigma$–$\Delta$ modulator produces an $N$ level quantization, of width $W$, of a strictly positive signal bounded between 0 and $NW$.

\[ \dot{e} = \zeta(t) - y \]

\[ y = W \left\{ \sum_{j=1}^{N} (2j-1 + \text{sign}(e)) \times \text{sign}(\zeta(t) - (j-1)W) - \text{sign}(\zeta(t) - jW) \right\} \]

The idea behind this formula is quite elementary. Consider a signal $f_j$ defined by

\[ f_j = 2W \left( \text{sign}(\zeta(t) - (j-1)W) - \text{sign}(\zeta(t) - jW) \right) \]

This signal takes the value 1, only when the signal $\zeta(t)$ lies in the interval $[(j-1)W, jW]$, otherwise, it takes the value 0. We consider then functions of the form $\sum_j y_j f_j$. Once the proper summand is activated and the rest are inhibited, it is necessary to create a sliding regime on the manifold $e=0$. Notice that $\dot{e} = e (\zeta - \sum_j y_j f_j)$ is guaranteed to be always negative, whenever $\zeta(t) \in [(j-1)W, jW]$, provided we choose the following switching strategy for the signal $y_j$ using as binary inputs the real numerical values in the discrete set, $\{0, (j-1)W, jW\}$:

\[ y_j = \begin{cases} (j-1)W & \text{for } e < 0 \\ jW & \text{for } e \geq 0 \end{cases} \]

This is the multilevel $\Sigma$–$\Delta$ modulation.
which may also be synthesized as:

\[ y_j = \frac{W}{2} (1 + \text{sign}(e)) + (j-1) \frac{W}{2} (1 - \text{sign}(e)) \\
= \frac{W}{2} [(2j-1) - \text{sign}(e)] \]

This last switching policy creates a sliding regime on \( e = 0 \) while the signal \( \xi(t) \) takes values in the interval \( (j-1)W, jW \).

Clearly, the sums of products of \( y_j \) and \( f_j \) in the expression

\[ y = \sum y_j f_j \]

yields the proposed formula.

Fig. 20 depicts in its upper graph a six level sliding mode quantization of a biased sinusoidal signal of amplitude 1.5 centered around 1.5, with levels of quantization of 0.5. The lower graph depicts the low pass filtering \( y(t) \) of the signal \( y(t) \) in comparison with the original signal. The low pass filter is a Butterworth second order filter with a damping factor of 0.8 and a cutoff frequency of 100 [rad/s].

Finally, we leave it to the reader to show that for any signal bounded within the interval \([-NW, NW]\), the following multilevel \( \sigma - \Delta \) modulation scheme renders a complete quantization of the system into \( 2N \) levels of width \( W \) with switchings taking place between the numerical values of the bounding levels of the quantization intervals.

\[ \dot{e} = \xi(t) - y \]

\[ y = \frac{W}{4} \left\{ \sum_{n=1}^{N} [2j-1 - \text{sign}(e)] \times [\text{sign}(\xi(t) - (j-1)W) - \text{sign}(\xi(t) - jW)] \right\} \]

Fig. 21 depicts in its upper graph a six level sliding mode quantization of an unbiased sinusoidal signal of amplitude 1.5, with levels of quantization of 0.5. The lower graph depicts the low pass filtering \( y(t) \) of the signal \( y(t) \) in comparison with the original signal. The low pass filter is a Butterworth second order filter with a damping factor of 0.8 and a cutoff frequency of 100 [rad/s].

References


