Development of a proportional control method for a mobile robot

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Abstract

Proportional-integral-derivative (PID) control is a control strategy that has been successfully used over many years. Simplicity, robustness, a wide range of applicability and near-optimal performance are some of the reasons that have made PID control popular in the academic institutes and industries. This paper presents a new development of the proportional control method for stable tracking control system for a mobile robot. Proportional control parameters for each wheel are decided by confirmation of the minimal root mean square error (RMSE) of deviation in wheel rotations for each wheel. The accuracy performance was compared with the predictive nonlinear control method and the predictive proportional nonlinear control method. The experiment results demonstrated the feasibility and advantages of the proportional control on a trajectory tracking of a mobile robot.

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1. Introduction

In the field of mobile robot control, many control schemes for stabilization and trajectory tracking problem have been proposed. At the moment, many different methods to find parameters for suitable controllers exist [1]. The methods differ in complexity and flexibility. Depending on the application, there is a need to have several types of tuning method. There are simple and easy methods to use which require little information [2] as well as more sophisticated methods which require more information and more computations. In previous research [3,4] we use proportional controller but the proportional control parameter confirmation is done by a computer simulation. In order to provide consistent, reliable and stabilized movement during running...
course, the proportional control parameter confirmation experiments were performed with the mobile robot in
the real environment.

A new development of a proportional control method for stable tracking control for a mobile robot is
developed in this paper. The RMSE of deviation in wheel rotations is introduced to obtain parameters of
the right wheel and the left wheel for the proportional control. The system of the two wheels of the mobile
robot works concurrently with different proportional control parameters value to perform stable movement
in trajectory straight line tracking. The accuracy performance was compared with the predictive nonlinear
control method and the predictive proportional nonlinear control method [5]. The experiments were demon-
strated through an application by means of experiments carried out on a mobile robot called “Taro” shown in
Fig. 1 [6].

The paper is organized as follows. In Section 2, kinematic model for the mobile robot is presented. In Sec-
tion 3, a proportional control method for stable trajectory tracking is discussed. In Section 4, calculation
method of proportional control for a mobile robot is described. Section 5 contains experimental comparison
and experimental results. Conclusions are shown in Section 6.

2. Kinematic model for a mobile robot

The robot has two wheels which are individually powered and controlled by DC motor in the differential
steering system configuration. The two independently controlled wheels provide both drive and steering. The
front and back castor are purely passive support. The optical quadrate encoders are used to measure rotation
with the accuracy of 80 numbers of encoder pulses per revolution. Table 1 shows specifications of the mobile
robot “Taro”.

It is assumed that the plane of each wheel is perpendicular to the ground. The contact between the wheels
and the ground should satisfy the conditions of pure rolling and nonslipping. The velocity of the center of
the mass of the mobile robot is orthogonal to the wheel axis. Wheel rotations in radian are described by

\[
\phi_R = \frac{x_R \times 2\pi}{\text{number of encoder pulses per revolution}} = \frac{x_R \times 2\pi}{80},
\]

\[
\phi_L = \frac{x_L \times 2\pi}{\text{number of encoder pulses per revolution}} = \frac{x_L \times 2\pi}{80},
\]

where \(\phi_R\) and \(\phi_L\) are wheel rotations in radian, \(x_R\) and \(x_L\) are the numbers of encoder pulses obtained from
the encoders for the right wheel and the left wheel, respectively.

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ematics and Computation (2006), doi:10.1016/j.amc.2006.05.214
3. Proportional control

We control the speed of mobile robot by proportional control using the number of pulses obtained from the encoders. The control rules are given by following equations:

\[ r_{\text{ope}}(k+1) = r_{\text{ope}}(k) + c_R, \]
\[ l_{\text{ope}}(k+1) = l_{\text{ope}}(k) + c_L, \]

where

\[ k + 1 \equiv (k + 1)\Delta t, \]
\[ \Delta t = 0.1 \text{ s}. \]

\[ r_{\text{ope}}(k) \] and \[ l_{\text{ope}}(k) \] are numbers of encoder pulses of the right wheel and the left wheel at the \( k \)th sampling time and the \( r_{\text{ope}}(k+1) \) and \( l_{\text{ope}}(k+1) \) are numbers of encoder pulses of the right wheel and the left wheel at the \((k+1)\)th sampling time:

\[ r_{\text{ope}}(k) = \frac{\Delta \phi_R}{\Delta t} \times \frac{80}{2\pi}, \]
\[ l_{\text{ope}}(k) = \frac{\Delta \phi_L}{\Delta t} \times \frac{80}{2\pi}, \]

where

\[ \Delta \phi_R = \phi_R(k+1) - \phi_R(k), \]
\[ \Delta \phi_L = \phi_L(k+1) - \phi_L(k), \]
\[ \phi_R(k) = x_R(k) \times \frac{2\pi}{80}, \]
\[ \phi_L(k) = x_L(k) \times \frac{2\pi}{80}, \]

\( \phi_R(k) \) and \( \phi_L(k) \) are wheel rotations in radian of the right wheel and the left wheel at the \( k \)th sampling time. \( x_R(k) \) and \( x_L(k) \) are the numbers of encoder pulses of the right wheel and the left wheel at the \( k \)th sampling time.

In Eqs. (3) and (4), \( c_R \) and \( c_L \) are the proportional control factors to be applied to the system and it can be expressed as follows:

\[ c_R = e_R(k) \times K_{p_R}, \]
\[ c_L = e_L(k) \times K_{p_L}. \]

\( K_{p_R} \) and \( K_{p_L} \) are the proportional control parameters of the right wheel and the left wheel to be adjusted for the error of the wheel rotation numbers explained as below. \( e_R(k) \) and \( e_L(k) \) are equal to the differences between the numbers of encoder pulses of targets and the numbers of encoder pulses of the mobile robot “Taro” for the right wheel and the left wheel at the present \( k \)th sampling time, respectively defined by

\[ e_R(k) = \left( \frac{\Delta \phi_R}{\Delta t} \right)_T - r_{\text{ope}}(k), \]
\[ e_L(k) = \left( \frac{\Delta \phi_L}{\Delta t} \right)_T - l_{\text{ope}}(k). \]
where the numbers of encoder pulses of targets $\left(\Delta \phi_R / \Delta t\right)_R$ and $\left(\Delta \phi_L / \Delta t\right)_R$ for the right wheel and the left wheel are specified by
\[
\left(\Delta \phi_R / \Delta t\right)_R = \left(\Delta \phi_L / \Delta t\right)_R = 80 \text{ pulses/s.}
\] (17)

4. Calculation method of proportional control

Fig. 2 shows the calculation method of the proportional control for the right wheel and the left wheel of mobile robot. In Fig. 2, the controller and system are shown. The controller has two inputs and two outputs which are inputs to the system. The two wheels which are individually powered by two DC motors are controlled individually with the same equations of the proportional control given by Eqs. (3) and (4), respectively. To get a good result in trajectory straight line tracking, the rotation speed of each wheel must be the same. The proportional controller uses the error information at each sampling time and the proportional control parameters. The two wheels of mobile robot works concurrently with different values of $K_{p_R}$ and $K_{p_L}$ proportional control parameters to perform stable movement in trajectory straight line tracking. In Fig. 2,
\[
x_R(k)z = x_R(k+1),
\]
\[
\text{r._ope}(k+1)z^{-1} = \text{r._ope}(k),
\] (18) (19)
where $z^{-1}$ is the time delay operation of the $z$ transform.

5. Experimental results

5.1. Comparison of the proportional control method with the predictive nonlinear control method and the predictive proportional nonlinear control method

To make a comparison of the methods mention above, three types of experiments were conducted. The accuracy performance of the proportional control is compared with the accuracy performances of the other two types of control methods previously developed [5]. In our previous research, we developed a new predic-
tive nonlinear control method and a new predictive proportional nonlinear control method to move the mobile robot in stable movements in trajectory straight line tracking.

In [5], to determine the best nonlinear proportional control parameter, the preliminary experiments were conducted by changing the nonlinear proportional control parameters value  \( K_{pr} \) and  \( K_{pl} \) between the ranges of 0.1 and 1.0. The RMSE is a proven measure of control and quality [7]. The minimal RMSE of deviation in wheel rotations is the best nonlinear proportional control parameter. The RMSE of deviation in wheel rotations for the right wheel and the left wheel can be expressed as follows:

\[
\text{RMSE}_R = \sqrt{\frac{1}{m} \sum_{i=1}^{m} \left( \frac{1}{N} \sum_{k=1}^{N} \left( e_R^{(i)}(k) \right)^2 \right)},
\]

\[
\text{RMSE}_L = \sqrt{\frac{1}{m} \sum_{i=1}^{m} \left( \frac{1}{N} \sum_{k=1}^{N} \left( e_L^{(i)}(k) \right)^2 \right)},
\]

where  \( e_R^{(i)}(k) \) and  \( e_L^{(i)}(k) \) were the error functions of  \( i \)th experiment defined by Eqs. (15) and (16).  \( N = 161 \) is amount of sample data for the predictive nonlinear control and the predictive proportional nonlinear control.  \( m = 3 \) is the number of experiments. From the experimental results, we found that the minimal RMSE\(_R\) and RMSE\(_L\) of deviation in wheel rotations for the right wheel and the left wheel were in the range of 0.1 for the nonlinear proportional control parameters defined by  \( K_{pr} \) and  \( K_{pl} \), respectively. Based on these results, we were able to perform the fine tuning experiments. From the fine tuning experimental results, we found that, the minimal RMSE\(_R\) and RMSE\(_L\) of deviation in wheel rotations are 0.06 for the nonlinear proportional control parameters. Therefore, we used  \( K_{pr} = K_{pl} = 0.06 \) as nonlinear proportional control parameters of the mobile robot in Eqs. (13) and (14) for the right wheel and the left wheel, respectively.

After the nonlinear proportional control parameters  \( K_{pr} \) and  \( K_{pl} \) were determined, we compared the accuracy performance of the proportional control with the predictive nonlinear control method and the predictive proportional nonlinear control method for trajectory straight line tracking using  \( K_{pr} = K_{pl} = 0.06 \). We used the optimal parameters  \( K_x = 0.27 \),  \( K_y = 0.003 \) and  \( K_\delta = 0.06 \) as a nonlinear controller parameters in the predictive nonlinear control method and the predictive proportional nonlinear control method given in [5].

The results are shown in Fig. 3. In Fig. 3, the horizontal axis is the traveling distance of mobile robot in centimeters and the vertical axis is the RMSEs’ of deviation in wheel rotations defined by Eqs. (22) and (23). The data shown in the figure provide evidence that the RMSEs’ of deviation in wheel rotations of the proposed proportional control method for the right wheel and the left wheel are increasing in unstable condition. The mobile robot shows high speed from 3.14 cm to 25 cm of movement and does not move ahead to the target point in trajectory straight line tracking. The predictive nonlinear control method with optimal parameters  \( K_x = 0.27 \),  \( K_y = 0.003 \) and  \( K_\delta = 0.06 \) as a nonlinear controller parameters given in [5] shows lower speed at the beginning of 3.14–25 cm of moving distance compared to proposed proportional control method. Meanwhile, the predictive proportional nonlinear control method with optimal parameters  \( K_x = 0.27 \),  \( K_y = 0.003 \),  \( K_\delta = 0.06 \) as a nonlinear controller parameters,  \( K_{pr} = 0.06 \) and  \( K_{pl} = 0.06 \) as a nonlinear proportional controller parameters shows the best movement at the beginning of 3.14–25 cm of moving distance compare to the other two methods.

After 25 cm of movement, the gradient line of the RMSE\(_R\)' and RMSE\(_L\)' of deviation in wheel rotations for the right wheel and the left wheel, respectively, in the predictive proportional nonlinear control method are the lowest gradient compared with other two gradients of line methods. The lowest gradient line of the RMSEs’ of deviation in wheel rotations means that the errors of wheel rotations deviation are lower compared to higher gradient lines.

The best performance in trajectory straight line tracking is the line with lower gradient, and the differences between the right wheel line of the RMSE\(_R\)' of deviation in wheel rotations and the left wheel line of the RMSE\(_L\)' of deviation in wheel rotations is small. The predictive proportional nonlinear control method with optimal parameters shows the best results and the proposed proportional control method with the same non-

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5.2. Determining proportional control parameters

In order to prove the effectiveness of our new developed proportional control method in a practical application, a laboratory experimental setup was employed. The experiment condition is such that the mobile robot tracks the trajectory straight line on the floor. The minimal RMSE of deviation in the wheel rotations for each wheel is determined by changing the proportional control parameters \( K_{pR} \) and \( K_{pL} \). Preliminary experiments were conducted by changing the proportional control parameter values between 0.1 and 1.8 for the right wheel and the left wheel.

The results of the experiment are shown in Fig. 4. In Fig. 4, the horizontal axis is the proportional control parameter values and the vertical axis is the RMSE of deviation in wheel rotations for the right wheel and the left wheel defined by Eqs. (20) and (21). From these results, we found that the minimal RMSE\(_R\) and RMSE\(_L\) of deviation in wheel rotations are 1.1 for \( K_{pR} \) and \( K_{pL} \). From this preliminary experiment results, we performed some fine tuning experiments. These fine tuning experiments were performed by changing the proportional control parameters \( K_{pR} \) and \( K_{pL} \) in the ranges of 1.0–1.2. The findings of these experiments are shown in Figs. 5 and 6. In Figs. 5 and 6, the horizontal axis are the proportional control parameter values and the vertical axis are the RMSE of deviation in wheel rotations for the right wheel and the left wheel defined by Eqs. (20) and (21).

Figs. 5 and 6 show the minimal proportional control parameters are \( K_{pR} = 1.16 \) for the right wheel and \( K_{pL} = 1.14 \) for the left wheel. Figs. 7 and 8 show the results of RMSE\(_R\) and RMSE\(_L\) of deviation in wheel rotations with the proportional controller by using selected proportional control parameters \( K_{pR} = 1.16 \) and \( K_{pL} = 1.14 \).
Fig. 4. Preliminary experiment.

Fig. 5. Fine tuning experiments for the right wheel $K_{pR}$.

Fig. 6. Fine tuning experiments for the left wheel $K_{pL}$.
From Figs. 7 and 8, we can understand that from 3.14 cm to 25 cm of movement the mobile robot becomes high speed because the RMSE\(_{0R}\) and RMSE\(_{0L}\) of deviation in wheel rotations increase very fast for the right wheel and the left wheel.

After 25 cm of moving distance, the gradient line of RMSE\(_{0R}\) and RMSE\(_{0L}\) of deviation in wheel rotations for the right wheel and the left wheel shows the gradient line is almost equal to zero. This means that the errors of deviation of wheel rotations are lower and as a result the mobile robot will moves stably in trajectory straight line tracking.

A close examination of both Figs. 7 and 8, (after 25 cm of movement) shows the differences between the right wheel line of the RMSE\(_{0R}\) of deviation in wheel rotations and the left wheel line of the RMSE\(_{0L}\) of deviation in wheel rotations is small in comparison to the other two methods [5] in Fig. 2. The small differences between the right wheel line of the RMSE\(_{0R}\) of deviation in wheel rotations and the left wheel line of the RMSE\(_{0L}\) of deviation in wheel rotations in Figs. 7 and 8 means that the speed of the wheel rotation between the right wheel and the left wheel are almost the same therefore the mobile robot will moves stably in tracking a straight line until the end of the target point.

6. Conclusions

The nonlinear proportional control parameters \(K^pR = K^pL = 0.06\) yields the stable movement in trajectory straight line tracking by the predictive nonlinear control method and the predictive proportional nonlinear
control method in reference [5]. However, these parameters result in an unstable condition in trajectory straight line tracking by the proportional controller method shown in Fig. 2.

From the experimental studies, the mobile robot shows a stable movement in trajectory straight line tracking with the proportional controller after using the parameters $K_{pR} = 1.16$ for the right wheel and $K_{pL} = 1.14$ for the left wheel as shown in Figs. 5 and 6, respectively. These parameters were determined with the fine tuning experiments explained in Section 5.2. The system of the two wheels of the mobile robot works concurrently with different proportional control parameters value to perform stable movement in trajectory straight line tracking.

The $\text{RMSE}_R$ and $\text{RMSE}_L$ by using the proportional controller with the optimal proportional parameters yields the best results after 25 cm of moving distance shown in Figs. 7 and 8 compared with the predictive nonlinear control method and the predictive proportional nonlinear control method. However, during the first start of movement, the predictive proportional nonlinear control method in Ref. [5] shows better results than the proportional control method.

The findings of this research provide strong evidence that we found how to adjust the parameters for our new development proportional control method for the right wheel and the left wheel of our mobile robot “Taro”, the system of the two wheels of the mobile robot works concurrently with the selected proportional control parameters to perform stable movement in trajectory straight line tracking.

Even though proportional controller produces stable movement in trajectory straight line tracking after 25 cm of moving distance, we have to consider other factors that may explain the high speed during the first start of moving distance. Finally, such factors as nonlinear constraints need to be examined in future researches.

References