Full length article

Static hybrid amplify and forward (AF) and decode and forward (DF) relaying for cooperative systems

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ABSTRACT

In this paper, we propose a static hybrid amplify and forward (AF) and decode and forward (DF) relaying protocol for cooperative systems. In such a scheme, relays close to the source amplify the received signal whereas the remaining relays transmit only if they decode correctly. We consider two subclasses of the proposed hybrid AF–DF relaying protocol. In the first one, all AF relays and DF relays that have decoded correctly transmit using orthogonal channels. The second protocol, called opportunistic hybrid AF–DF relaying, consists in activating only the relay offering the highest instantaneous signal to noise ratio (SNR) among AF relays and the relays that have decoded correctly. The outage SNR probability, and the exact and asymptotic bit error probability (BEP) values of both all-participating and opportunistic hybrid AF–DF relaying protocols are derived and compared to conventional AF and DF relaying. The proposed protocol offers better performance than AF relaying and similar performance to DF relaying with a lower computational complexity. Simulation results are also provided to verify the tightness of the derived results.

1. Introduction

Amplify and forward (AF) and decode and forward (DF) relaying are usually used in cooperative systems in order to benefit from spatial diversity without requiring multiple antennas at the transmitter [1–8]. In the AF relaying protocol, each relay amplifies the received signal from the source and forwards it to the destination. In the DF relaying protocol, only the relays that have decoded correctly regenerate and transmit the original signal to the destination. AF relaying is less complex than DF relaying since decoding is not performed at the relays. AF and DF relaying offer the same diversity order equal to the number of transmitters (the relays and the source). However, AF relaying may offer worse performance than DF relaying since the noise is amplified along with the useful signal. The bit error probability (BEP) and signal to noise ratio (SNR) outage probability of AF and DF relaying have been intensively studied in the literature [1–8].

The outage probability of millimeter wave dual-hop systems has been derived in [9]. The optimum placement of radio relays in dual-hop networks has been investigated in [10]. [11] derived the outage performance of DF selection cooperation with maximum ratio combining at the destination in Nakagami-m fading channels.

In order to improve the system spectral efficiency, opportunistic AF relaying has been proposed in [12]. In opportunistic AF relaying, only the relay offering the highest instantaneous SNR of the relaying link (source–relay–destination) is activated. In opportunistic DF relaying, the selected relay offers the highest SNR of the relay to destination link among the relays that have decoded correctly. It has been shown that opportunistic AF and DF relaying achieve a full spatial diversity. The performance of opportunistic relaying has been investigated in [8,12–16]. Cooperative hybrid automatic repeat request (HARQ) protocols using opportunistic AF or DF relaying have been proposed in [17].

Some adaptive AF and DF relaying protocols have been proposed in the literature [18–24]. In [18–20,23], an adaptive DF–AF relaying protocol has been proposed in...
which the relay first tries to decode the received signal. If the decoding succeeds, it transmits the decoded signal as in the DF protocol. If the decoding fails, the relay simply amplifies the received signal. The computational complexity of this adaptive protocol is high since decoding is always performed at the relays. In [21], the relay estimates the BEP using log-likelihood ratios. If the estimated BEP is above a given threshold, the DF strategy is used. Otherwise, the relay amplifies the received signal since it contains no or only few errors. In [22], a decode–amplify–forward protocol is proposed, in which a relay amplifies the soft output of the channel decoder output. In [24], a threshold-based hybrid relay selection (THS) protocol has been proposed, in which relays with an SNR higher than the threshold \( T \) are included in the DF set. The rest of the relays are included in the AF set and a single relay is activated based on the instantaneous SNR. In this paper, we propose a static hybrid AF and DF relaying protocol in which relays close to the source amplify the received signal whereas the remaining relays transmit only if they have decoded correctly. In fact, when the SNR of the first hop is much greater than that of the second hop, the SNR of AF relaying is almost equal to the SNR of the second hop, which is that of DF relaying. Therefore, when a relay is close to the source, AF and DF relaying offer similar performance. We propose an “all-participating” and “opportunistic” hybrid AF and DF relaying protocol. In all-participating hybrid AF–DF relaying, all AF relays and the relays that have decoded correctly transmit using orthogonal channels. In opportunistic hybrid AF and DF relaying, the relay offering the highest instantaneous SNR is activated among the set of AF relays and the DF relays that have decoded correctly.

The proposed hybrid protocol can be used in practical networks since it offers a good compromise between computational complexity and performance. It is obviously less complex than DF relaying and the adaptive protocol proposed in [18–20], since decoding is not performed at AF relays. Therefore, it decreases the energy consumption and delay at the relays. It offers better performance than AF relaying and close performance to that of DF relaying with a lower computational complexity. We provide closed-form expressions of the SNR outage probability, a tight lower bound of the BEP, and the asymptotic BEP of all-participating and opportunistic hybrid AF–DF relaying.

The paper is organized as follows. The next section describes the system model. Sections 3 and 4 derive respectively the performance of the all-participating and opportunistic hybrid AF and DF relaying protocols. Section 5 provides some numerical and simulation results. Finally, Section 6 draws some conclusions.

2. System model

Fig. 1 shows the studied wireless communication system with a source \( S \), a destination \( D \), \( N \) AF relays \( \{R_i\}_{i=1}^{N} \) and \( K \) DF relays \( \{R_i\}_{i=N+1}^{N+K} \). The AF relaying set is composed by the relays that are close to the source and the DF relaying set contains the remaining relays. In the first phase of the transmission, \( S \) broadcasts a symbol \( x \) to \( D \) and all the relays. The \( K \) DF relays estimate the transmitted symbol. In the second phase, and in all-participating relaying, the \( N \) AF relays and the relays in the DF set that have decoded correctly transmit using orthogonal channels (in time, frequency, or spreading codes). In opportunistic relaying, only the relay offering the highest instantaneous SNR is activated. This relay is chosen among the \( N \) AF relays and the relays that have decoded correctly. The destination combines all received signals using a maximum ratio combining (MRC) strategy. Perfect channel estimation is assumed at the different nodes.

The received signal at the destination from the source is modeled as follows:

\[
y_{S,D} = \sqrt{E_S} h_{S,D} x + n_{S,D},
\]

(1)
where $h_{X,Y}$ and $n_{X,Y}$ are respectively the channel coefficient and noise on the $X-Y$ link, and $E_X$ is the transmitted energy per symbol by $X$. The noise $n_{X,Y}$ is modeled by a zero-mean Gaussian noise with variance $N_{X,Y}$.

The received signal at relay $R_i$ from the source is given by

$$y_{S,R_i} = \sqrt{E_i}h_{S,R_i}x + n_{S,R_i}, \quad 1 \leq i \leq N + K.$$  

(2)

The received signal at $D$ from the $j$-th AF relay is given by

$$y_{R_j,D} = h_{R_j,D}y_{S,R_j} + n_{R_j,D}, \quad 1 \leq j \leq N,$$  

(3)

where $G_j$ is the amplification factor used by $R_j$:

$$G_j = \sqrt{E_j |h_{R_j,D}|^2 + N_{R_j,D}}.$$  

(4)

The decision variable used by DF relay $R_k$ is given by

$$\tilde{x}_k = \sqrt{E_k}h_{R_k,D}^*x, \quad N + 1 \leq k \leq N + K.$$  

(5)

The decision variable is used to obtain an estimate, $\hat{x}_{RS_k}$, of the transmitted symbol. For example, we have $\hat{x}_{RS_k} = \text{sign}(\tilde{x}_k)$ for Binary Phase Shift Keying (BPSK) modulations. If relay $R_k$ has decoded correctly (i.e. $x_{RS_k} = x$), it transmits symbol $x$ using a symbol energy equal to $E_k$. Otherwise, it remains idle. Similarly to [2–5], DF relaying is made symbol by symbol.

The received signal at $D$ from DF relay $R_k$ is given by

$$y_{R_k,D} = \sqrt{E_k}h_{R_k,D}x + n_{R_k,D}, \quad N + 1 \leq k \leq N + K,$$  

(6)

where $\hat{E}_{R_k} = E_k$ if relay $R_k$ has decoded correctly, and $\hat{E}_{R_k} = 0$ otherwise.

3. Performance of all-participating hybrid AF–DF relaying

In all-participating hybrid AF and DF relaying, the destination combines the received signals from the source $S$, AF relays, and DF relays that have decoded correctly using an MRC strategy:

$$\tilde{x}_D = \frac{\sqrt{E_S}h_{S,D}^*y_{S,D}}{N_{S,D}} + \sum_{j=1}^{N} \frac{\sqrt{E_j}G_j h_{S,R_j}^*h_{R_j,D}y_{R_j,D}}{|G_j|^2 |h_{R_j,D}|^2 N_{S,R_j} + N_{R_j,D}} + \sum_{k=N+1}^{N+K} \frac{\sqrt{E_k}h_{R_k,D}^*y_{R_k,D}}{N_{R_k,D}}.$$  

(7)

Using (1)–(7), we deduce the expression of the instantaneous SNR at $D$ for a given decoding set $\Theta$:

$$\Gamma_{D(\Theta)} = \Gamma_{S,D} + \sum_{j=1}^{N} \Gamma_{S,R_j,D} + \sum_{k \in \Theta} \Gamma_{R_k,D}.$$  

(8)

where $\Theta \subset \{N + 1, \ldots, N + K\}$ is the set of relay indices that have decoded correctly, $\Gamma_{X,Y} = \frac{E_{X|h_{X,Y}|^2}}{N_{X,Y}}$ is the instantaneous SNR of the $X-Y$ link, and $\Gamma_{S,R_j,D}$ is the instantaneous SNR of the relaying AF link between $S$, $R_j$ and $D$ [1]:

$$\Gamma_{S,R_j,D} = \frac{G_j^2}{1 + G_j^2 + |G_j|^2}.$$  

(9)

Using the SNR expression (8), we derive the performance of the proposed hybrid protocol in the following subsections.

3.1. Lower bound of the BEP

The BEP at the destination $D$ can be written as

$$p_{e,D} = \sum_{\Theta} P_{e,D(\Theta)} p(\Theta),$$  

(10)

where the above sum is made over all possible sets $\Theta$ of relays that have decoded correctly, and $p(\Theta)$ is the probability that $\Theta$ is the set of relays that have decoded correctly.

$$p(\Theta) = \prod_{i \in \Theta} (1 - P_{e,R_i}) \prod_{j \notin \Theta} P_{e,R_j}.$$  

(11)

Therefore, it is assumed that each relay perfectly judges whether the received signal is decoded correctly or not, i.e. no error propagation is allowed. $P_{e,R_i}$ is the average BEP at relay $R_i$:

$$P_{e,R_i} = \int AQ(\sqrt{Bx})p_{SR_i}(x)dx,$$  

(12)

where $T_{S,R_i}$ is the instantaneous SNR at relay $R_i$, $A$ and $B$ depend on the considered modulation (for example, $A = 1$, $B = 2$ for BPSK), $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{+\infty} e^{-t^2/2} dt$, and $p_x(x)$ is the probability density function (PDF) of $X$.

For a given decoding set $\Theta$, the BEP at the destination can be written as

$$P_{e,D(\Theta)} = \int AQ(\sqrt{Bx})p_{D(\Theta)}(x)dx,$$  

(13)

where $T_{D(\Theta)}$ is the instantaneous SNR at $D$ when the decoding set is $\Theta$.

3.1.1. BEP at the DF relays

Since Rayleigh fading channels are considered, the SNR at relay $R_i$ is exponentially distributed, i.e.

$$p_{T_{S,R_i}}(y) = \frac{1}{T_{S,R_i}} e^{-\frac{y}{T_{S,R_i}}} U(y),$$  

(14)

where $T_{S,R_i} = E(T_{S,R_i})$ is the average SNR at relay $R_i$, $E(\cdot)$ is the expectation operator, and $U(y)$ is the unit step function.

Using (12) and (14), we deduce the expression of the BEP at the DF relay $R_i$ [25]:

$$P_{e,R_i} = \Psi(T_{S,R_i}), \quad N + 1 \leq i \leq N + K,$$  

(15)

where

$$\Psi(\alpha) = \frac{\lambda}{2} \left[ 1 - \sqrt{\frac{\alpha}{\alpha + \frac{\lambda}{2}}} \right].$$  

(16)
3.1.2. BEP at the destination

The moment-generating function (MGF) of the SNR of the AF relaying link \( \Gamma_{S,R_i,D} \) [9] has been recently derived in [14]. Using these results, we can derive the exact MGF of the SNR at D for a given decoding set and deduce the BEP at the destination. However, this approach leads to a non-closed-form expression, and the BEP is expressed in terms of an integral that can be evaluated numerically. In this paper, we propose using the following tight upper bound of the SNR at D:

\[
\Gamma_{D|\Theta} \leq \Gamma_{D|\Theta}^{up} = \Gamma_S + \sum_{j=1}^{N} \Gamma_{S,R_j,D}^{up} + \sum_{k \in \Theta^c} \Gamma_{R_k,D},
\]

where

\[
\Gamma_{S,R_j,D}^{up} = \min(\Gamma_{S,R_j}, \Gamma_{R_j,D}).
\]

Since Rayleigh fading channels are considered, \( \Gamma_{X,Y} \) is exponentially distributed, and we can easily show that \( \Gamma_{S,R_j,D}^{up} \) is also exponentially distributed with mean

\[
\overline{\Gamma}_{S,R_j,D}^{up} = \frac{\overline{T}_{S,R_j}^{up} \overline{T}_{R_j,D}}{\overline{T}_{S,R_j} + \overline{T}_{R_j,D}}.
\]

Assuming that the channel gains of the different links are independent, the MGF of \( \Gamma_{D|\Theta}^{up} \) is given by

\[
M_{\Gamma_{D|\Theta}^{up}}(s) = E(e^{-s \Gamma_{D|\Theta}^{up}}) = M_{13,S}(s) \prod_{j=1}^{N} M_{\Gamma_{S,R_j,D}^{up}}(s) \prod_{k \in \Theta^c} M_{\Gamma_{R_k,D}}(s)
\]

\[
= \frac{1}{1 + s \overline{T}_{S,D}} \prod_{j=1}^{N} \frac{1}{1 + s \overline{T}_{S,R_j,D}} \prod_{k \in \Theta^c} \frac{1}{1 + s \overline{T}_{R_k,D}}.
\]

Using a fractional decomposition, the MGF of the SNR at D is expressed as follows:

\[
M_{\Gamma_{D|\Theta}^{up}}(s) = \frac{\overline{T}_{S,D}}{1 + s \overline{T}_{S,D}} + \sum_{j=1}^{N} \frac{\overline{T}_{S,R_j,D}}{1 + s \overline{T}_{S,R_j,D}} + \sum_{k \in \Theta^c} \frac{\overline{T}_{R_k,D}}{1 + s \overline{T}_{R_k,D}}.
\]

where the residues are given by

\[
\pi_{S,D}(\Theta) = \sum_{j=1}^{N} \frac{\overline{T}_{S,D}}{\overline{T}_{S,D} - \overline{T}_{S,R_j,D}} \prod_{k \in \Theta^c} \frac{\overline{T}_{S,D}}{\overline{T}_{S,D} - \overline{T}_{R_k,D}},
\]

\[
\pi_{S,R_j,D}(\Theta) = \frac{\overline{T}_{S,R_j,D}}{\overline{T}_{S,R_j,D} - \overline{T}_{S,D}} \prod_{k \in \Theta^c} \frac{\overline{T}_{S,D}}{\overline{T}_{S,D} - \overline{T}_{R_k,D}} \prod_{j=1}^{N} \frac{\overline{T}_{S,D}}{\overline{T}_{S,R_j,D} - \overline{T}_{R_j,D}},
\]

\[
\pi_{R_k,D}(\Theta) = \frac{\overline{T}_{R_k,D}}{\overline{T}_{R_k,D} - \overline{T}_{S,D}} \prod_{j=1}^{N} \frac{\overline{T}_{R_k,D}}{\overline{T}_{R_k,D} - \overline{T}_{S,R_j,D}} \prod_{k \in \Theta^c} \frac{\overline{T}_{S,D}}{\overline{T}_{S,D} - \overline{T}_{R_k,D}}.
\]

Using the inverse Laplace transform (LT), we deduce the expression of the PDF of the SNR at D:

\[
P_{\Gamma_{D|\Theta}^{up}}(\gamma) = \frac{\pi_{S,D}(\Theta)}{\overline{\Gamma}_{S,D}} e^{-\overline{\Gamma}_{S,D} \gamma} + \sum_{j=1}^{N} \frac{\pi_{S,R_j,D}(\Theta)}{\overline{\Gamma}_{S,D}} e^{-\overline{\Gamma}_{S,R_j,D} \gamma} \sum_{k \in \Theta^c} \frac{\pi_{R_k,D}(\Theta)}{\overline{\Gamma}_{S,D}} e^{-\overline{\Gamma}_{R_k,D} \gamma}.
\]

Using (13) and (25), we deduce the BEP at the destination for a given decoding set \( \Theta \):

\[
P_{e,D}(\Theta) \geq \pi_{S,D}(\Theta) \Psi(\overline{T}_{S,D}) + \sum_{j=1}^{N} \pi_{S,R_j,D}(\Theta) \Psi(\overline{T}_{S,R_j,D}) + \sum_{k \in \Theta^c} \pi_{R_k,D}(\Theta) \Psi(\overline{T}_{R_k,D}).
\]

We have a lower bound in the above equation because we used an upper bound of the SNR. Using (10), (11), (15) and (26), we deduce the BEP at the destination.

3.2. Asymptotic BEP

At high SNR, all DF relays decode correctly, and the BEP at D is given by [25]

\[
P_{e,D} \approx \frac{A}{(2B)^{N+K+1}} (\sqrt{N} + K + 1) \prod_{j=1}^{N} \frac{\overline{T}_{S,R_j,D}}{\overline{T}_{R_j,D}}.
\]

where

\[
C_k^p = k(k-1) \cdots (k-p+1) / p!.
\]

We verify that the hybrid AF–DF relaying protocol offers a diversity order equal to the total number of nodes, i.e. \( N + K + 1 \) (the source, N AF relays, and K DF relays).

3.3. Outage probability

For a given decoding set \( \Theta \), the SNR outage probability at the destination D is the cumulative distribution function (CDF) of the SNR that can be deduced from the SNR PDF (25) as follows:

\[
P_{\Gamma_{D|\Theta}^{up}}(\gamma) = P(\Gamma_{D|\Theta}^{up} \leq \gamma), \quad \text{if } \gamma \geq 0
\]

\[
= \pi_{S,D}(\Theta) \left[ 1 - e^{-\overline{\Gamma}_{S,D} \gamma} \right] + \sum_{j=1}^{N} \pi_{S,R_j,D}(\Theta) \left[ 1 - e^{-\overline{\Gamma}_{S,R_j,D} \gamma} \right] + \sum_{k \in \Theta^c} \pi_{R_k,D}(\Theta) \left[ 1 - e^{-\overline{\Gamma}_{R_k,D} \gamma} \right].
\]

We deduce the expression of the SNR outage probability at D:

\[
P_{\Gamma_{D|\Theta}^{up}}(\gamma) = \sum_{\Theta} P_{\Gamma_{D|\Theta}^{up}}(\gamma)p(\Theta),
\]

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3.1.2. ... R_k,D \}. (29)

We deduce the expression of the SNR outage probability

\[
P_{\Gamma_{D|\Theta}^{up}}(\gamma) = \pi_{S,D}(\Theta) \left[ 1 - e^{-\overline{\Gamma}_{S,D} \gamma} \right] + \sum_{j=1}^{N} \pi_{S,R_j,D}(\Theta) \left[ 1 - e^{-\overline{\Gamma}_{S,R_j,D} \gamma} \right] + \sum_{k \in \Theta^c} \pi_{R_k,D}(\Theta) \left[ 1 - e^{-\overline{\Gamma}_{R_k,D} \gamma} \right].
\]

We deduce the expression of the SNR outage probability at D:

\[
P_{\Gamma_{D|\Theta}^{up}}(\gamma) = \sum_{\Theta} P_{\Gamma_{D|\Theta}^{up}}(\gamma)p(\Theta),
\]
where \( p(\Theta) \) is defined in (11) and can be computed using (15).

4. Performance of opportunistic hybrid AF–DF relaying

4.1. SNR statistics

In this section, we evaluate the performance of opportunistic hybrid AF–DF relaying in which a single relay \( R_{rel} \) is activated among the set of AF relays and the set \( \Theta \) of relays that have decoded correctly: \( \text{sel} \in \Delta = \{1, \ldots, N\} \cup \Theta \).

Since the selected relay offers the highest instantaneous SNR, we have

\[
\Gamma_{\text{rel}, D} = \max_{1 \leq \ell \leq N, j \in \Theta} \left\{ \Gamma_{S, R_{\text{rel}}, D}, \Gamma_{R_{\ell}, D} \right\}.
\]

Using the upper bound (18), we obtain

\[
\Gamma_{\text{rel}, D} \leq \Gamma_{\text{rel}, D}^{\text{up}} = \max_{1 \leq \ell \leq N, j \in \Theta} \left\{ \Gamma_{S, R_{\text{rel}}, D}, \Gamma_{R_{\ell}, D} \right\}.
\]

As will be shown in the simulation results, this upper bound is very tight at medium and high SNR.

In order to compact the expressions, we introduce the following notation:

\[
\Gamma_i = \left\{ \begin{array}{ll}
\Gamma_{S, R_{\ell}, D} & \text{if } 1 \leq i \leq N \\
\Gamma_{R_{\ell}, D} & \text{if } i \in \Theta.
\end{array} \right.
\]

Therefore, we have

\[
\Gamma_{\text{rel}, D} = \max_{i \in \Delta} \left\{ \Gamma_i \right\}.
\]

**Proposition 1.** The PDF of \( \Gamma_{\text{rel}, D}^{\text{up}} \) is given by

\[
p_{\Gamma_{\text{rel}, D}^{\text{up}}} (\gamma) = \sum_{i \in \Delta} e^{-\gamma \Gamma_i^{\text{up}}} \sum_{n=0}^{2|\Delta|-1-1} (-1)^n \xi(n) \\
\times \exp \left(-\gamma \sum_{p=1}^{\Delta} \epsilon_n (p) \frac{\Gamma_i}{\Gamma_{i(p)}} \right),
\]

where \( \{i(p)\}_{p=1}^{\Delta} \) is the set of relay indices which belong to \( \Delta \) and are different from \( i \). \( |\Delta| \) is the cardinality of \( \Delta \), and \( \epsilon_n = (\epsilon_n(1) \cdots \epsilon_n(|\Delta| - 1)) \) is the binary representation of \( 0 \leq n \leq 2^{|\Delta| - 1} - 1 \),

\[
\xi(n) = \sum_{p=1}^{\Delta} \epsilon_n (p).
\]

The proof is provided in Appendix A.

The destination uses an MRC of the signals received from the source and the selected relay from the set \( \Delta = \{1, \ldots, N\} \cup \Theta \). For a given decoding set \( \Theta \), the instantaneous SNR at the destination is given by

\[
\Gamma_{D(\Theta)} = \Gamma_{S, D} + \Gamma_{\text{rel}, D} \leq \Gamma_{D(\Theta)}^{\text{up}} = \Gamma_{S, D} + \Gamma_{\text{rel}, D}^{\text{up}}.
\]

Assuming that the channel gains of the different links are independent, the MGF of \( \Gamma_{D(\Theta)}^{\text{up}} \) is given by

\[
M_{\Gamma_{D(\Theta)}^{\text{up}}} (s) = M_{\Gamma_{S, D}} (s) M_{\Gamma_{\text{rel}, D}^{\text{up}}} (s).
\]

Taking the LT of (35), we obtain

\[
M_{\Gamma_{D(\Theta)}^{\text{up}}} (s) = \sum_{i \in \Delta} \sum_{n=0}^{2^{|\Delta|}-1} \alpha_{n,i} \frac{(-1)^n}{1 + s\alpha_{n,i}},
\]

where

\[
\frac{1}{\alpha_{n,i}} = \frac{1}{\Gamma_i} + \sum_{p=1}^{\Delta} \varepsilon_n (p) \frac{\Gamma_{i(p)}}{\Gamma_i}.
\]

Using (38), (39), a fraction decomposition, and inverse LT, we obtain the expression of the PDF of \( \Gamma_{D(\Theta)}^{\text{up}} \):

\[
p_{\Gamma_{D(\Theta)}^{\text{up}}} (\gamma) = \sum_{i \in \Delta} \sum_{n=0}^{2^{|\Delta|}-1} \alpha_{n,i} \frac{(-1)^n}{\Gamma_i} \\
\times \left[ \frac{\psi(\alpha_{n,i})}{\alpha_{n,i}} \frac{e^{-\gamma \alpha_{n,i} \Gamma_{S,D}}}{\Gamma_{S,D} - \alpha_{n,i}} + \frac{\Gamma_{S,D}}{\Gamma_{S,D} - \alpha_{n,i}} \right].
\]

4.2. Lower bound of the BEP

The BEP of opportunistic hybrid AF–DF relaying is given by

\[
p_{e,D}^{\text{op}} = \sum_{i \in \Theta} p_{e,D}^{\text{op}} (\Theta).
\]

where \( p_{e,D}^{\text{op}} (\Theta) \) is the BEP at the destination for a given decoding set \( \Theta \):

\[
\psi_{e,D}^{\text{op}} (\Theta) \geq A \int Q(\sqrt{\gamma} T_{S,D} p_{\Gamma_{D(\Theta)}^{\text{up}}} (\gamma) d\gamma.
\]

Using (41), (43), and [25], we obtain

\[
p_{e,D}^{\text{op}} (\Theta) \geq \sum_{i \in \Delta} \sum_{n=0}^{2^{|\Delta|}-1} \alpha_{n,i} \frac{(-1)^n}{\Gamma_i} \\
\times \left[ \frac{\psi(\alpha_{n,i})}{\alpha_{n,i}} \frac{e^{-\gamma \alpha_{n,i} \Gamma_{S,D}}}{\Gamma_{S,D} - \alpha_{n,i}} + \frac{\Gamma_{S,D}}{\Gamma_{S,D} - \alpha_{n,i}} \right],
\]

where \( \psi(x) \) is defined in (16).

4.3. Asymptotic BEP

At high SNR, all DF relays have decoded correctly. The asymptotic BEP of opportunistic hybrid AF and DF relaying is given by

**Proposition 2.**
The proof is provided in Appendix B. We verify that hybrid AF and DF relaying offers a diversity order equal to the number of transmitters, i.e. \( N + K + 1 \). (The source, \( N \) AF relays, and \( K \) DF relays.)

If we allocate the same power to the source and the relays (i.e. \( E_\chi = E_s/(N + K + 1) \)) for all-participating relaying and \( E_\chi = E_s/2 \) for opportunistic relaying, where \( E_s \) is the transmitted energy per symbol, using (27) and (45), we deduce the BEP improvement brought by relay selection:

\[
\frac{p_0}{P_{s,D}} \approx (N + K)! \left( \frac{2}{N + K + 1} \right)^{N+K+1}. \tag{46}
\]

We verify that we obtain an extension of the result presented in [12] (equation 16), which is valid only for AF relaying.

4.4. Outage probability

For a given decoding set \( \Theta \), the outage probability is the probability that the SNR is lower than \( \gamma \). It can be deduced from the PDF of the SNR at \( D, (41) \):

\[
P_{\gamma,D|\Theta}(\gamma) = P(\gamma_{D|\Theta} \leq \gamma) = \sum_{i \in A} \sum_{n=0}^{2|\Delta|-1} \frac{\alpha_n,i}{T_i} (-1)^{\xi(n)} 
\times \left[ \frac{\alpha_{n,i}}{\alpha_{n,i} - T_{SD}} \left( 1 - e^{-\frac{\gamma}{\alpha_{n,i}}} \right) \right.
\left. + \frac{T_{SD}}{T_{SD} - \alpha_{n,i}} \left( 1 - e^{-\frac{\gamma}{T_{SD}}} \right) \right]. \tag{47}
\]

We deduce the expression of the SNR outage probability at \( D \) using (30), (47), (11) and (15).

5. Numerical and simulation results

In this section, we provide some numerical and simulation results for BPSK modulation. A Monte Carlo simulation was performed until 500 symbol errors occurred. The average SNR of the different links is modeled as follows:

\[
\gamma_{XY} = \frac{E_X}{N_0} \frac{\beta}{d_{XY}}, \tag{48}
\]

where \( N_0 \) is the noise variance, \( \alpha \) is the path loss exponent, \( d_{XY} = \frac{d_{\text{eff}}}{d_0} \) is the normalized distance between \( X \) and \( Y \), \( d_{\text{eff}} \) is the effective distance in meters, \( d_0 \) is an arbitrary reference distance, and \( \beta \) is the path loss at the reference distance. In the simulation results, the following parameters were used: \( \alpha = 3 \), \( d_0 = d_{SR} \), and \( \beta = 1 \).

Fig. 2 shows the BEP evolution with respect to the fraction of power allocated to the source \( \alpha_s = \frac{E_s}{E_0} \) (\( E_0 \) is the transmitted energy per bit and \( E_s \) is the transmitted energy per bit by the source) in the presence of a single relay. The fraction of power allocated to the relay is given by \( \alpha_{SR} = \frac{E_s}{E_0} = 1 - \alpha_s \). These theoretical results were obtained using (26), \( N = 1 \), and \( K = 0 \) for AF relaying, and (26), \( N = 0 \), and \( K = 1 \) for DF relaying. They correspond to \( E_0/N_0 = 10 \) dB and two positions of the relay: \( d_{SR} = 1 - d_{SR,0} = 0.3 \) and \( d_{SR} = 1 - d_{SR,0} = 0.7 \). When the relay is close to the source (\( d_{SR} = 0.3 \)), we notice that AF and DF relaying offer almost the same performance at the optimum value of the fraction of power allocated to the source \( \alpha_s \). In fact, in such scenarios, the SNR of AF relaying is almost equal to that of the second hop, which is the SNR of DF relaying. However, we notice that the BEP of DF relaying is lower than that of AF relaying when the relay is far from the source (\( d_{SR} = 0.7 \) in the numerical results). We finally verify that the fraction of power allocated to the source should be increased when the relay is far from it.

Fig. 3 shows the evolution of the optimized BEP in the same context of Fig. 2 for different positions of the relay. The fraction of power allocated to the source is determined by an extensive search so that it yields the lowest BEP.
at the destination. We notice that AF and DF relaying offer the same performance only when the relay is close to the source \(d_{SR_1} \leq 0.3\). This result justifies the use of the proposed hybrid AF and DF relaying protocol in which the relays close to the source amplify the received signal whereas the remaining relays transmit only if they decode correctly. The simulation results are also in good agreement with the theoretical ones.

In the presence of two relays, Fig. 4 shows the evolution of the BEP with respect to the fraction of power allocated to the source \(a_0\) after optimizing the fraction of power allocated to \(R_1\) and \(R_2\) for \(E_b/N_0 = 10\) dB. The first relay is close to the source, \(d_{SR_1} = 1 - d_{R_1,D} = 0.3\), and the second relay is far from the source, \(d_{SR_2} = 1 - d_{R_2,D} = 0.7\). In the proposed hybrid AF–DF relaying protocol, relay \(R_1\) amplifies the received signal and relay \(R_2\) transmits only if it has decoded correctly. We notice that the proposed hybrid AF–DF relaying protocol offers better performance than AF relaying and comparable performance to DF relaying with a lower computational complexity.

Fig. 5 studies the evolution of the optimized BEP of hybrid AF–DF, AF, and DF relaying with respect to \(E_b/N_0\) in the presence of two relays: \(d_{SR_1} = 1 - d_{R_1,D} = 0.3\) and \(d_{SR_2} = 1 - d_{R_2,D} = 0.7\). The fraction of power allocated to the source and relays is determined by an extensive search so that it yields the lowest BEP at the destination.

Two simulation setups were considered for the proposed hybrid AF–DF relaying protocol. The first one corresponds to the model considered in the paper in which the relay transmits only if it has decoded correctly and no error propagation is allowed. In the second setup (hybrid AF–DF–T), a more practical scenario is considered, in which the DF relay transmits only if its SNR is greater than a threshold \(T = 4\) dB. Fig. 5 shows that the BEP performances at the destination for the two simulation setups are very close. This validates our model in the paper, which has been used for mathematical tractability.

Fig. 5 shows that the proposed hybrid AF–DF relaying offers 2.8 dB gain compared to AF relaying for \(E_b = 10^{-3}\). Moreover, hybrid AF–DF relaying offers the same performance as DF relaying. The proposed protocol is less complex than DF relaying, since decoding is not performed at close relays, which reduces power consumption. We also observe a very small gap between the simulation results of the proposed hybrid protocol and theoretical ones: 0.4 dB for \(E_b = 10^{-2}\) and 0.2 dB for \(E_b = 10^{-3}\). The theoretical results were obtained using (26), which provides a lower bound of the BEP of AF and hybrid AF–DF relaying and an exact BEP for DF relaying. The observed gap between the simulation and theoretical results for AF relaying is due to the upper bound of the SNR (17) used. We finally verify that the derived lower bound (26) converges to the asymptotic (27) and exact BEP at high SNR.

Fig. 6 studies the evolution of the BEP of opportunistic hybrid AF–DF relaying, opportunistic AF relaying, and opportunistic DF relaying in the presence of two relays and the same context as Fig. 5. The same power is allocated to the source and the selected relay: \(E_s = E_{Rel} = E_b/2\). We observe that the proposed opportunistic hybrid AF–DF relaying protocol offers better performance than opportunistic AF relaying. The proposed protocol is less complex than opportunistic DF relaying and offers almost the same performance. The simulation results are in good agreement with the theoretical results. Besides, the asymptotic BEP (45) converges to the lower bound (42) for high SNRs.

Figs. 7 and 8 show the BEP of different relaying protocols when a convolutional code with generator polynomials (15) and (17) and memory \(m = 3\) is used. A soft-input–soft-output symbol-by-symbol decoding is performed. The network is composed of three relays, located at \(d_{SR_1} = 1 - d_{R_1,D} = 0.3\), \(d_{SR_2} = 1 - d_{R_2,D} = 0.2\), and \(d_{SR_3} = 1 - d_{R_3,D} = 0.7\). As the number of relays increases, the performance improves, due to spatial diversity.
Fig. 6. Evolution of the BEP with respect to the average SNR in the presence of two relays for opportunistic AF, opportunistic DF, and opportunistic hybrid AF–DF relaying.

Fig. 7. BEP of different all-participating relaying protocols. Fig. 7 compares the performance of the proposed protocol using a threshold $T = 4$ dB, AF, DF, all-participating hybrid forward (APHF) [23], adaptive decode and forward (ADF) with a BEP threshold $P = 10^{-3}$ [21], and decode–amplify–forward (DAF) [22] relaying. In APHF relaying, relays that are not able to decode are included in the AF set. We notice that the proposed protocol offers better performance than APHF and ADF relaying (at medium and high SNR). Also, ADF relaying offers close performance to DF relaying only at low SNR. DAF relaying offers better performance than hybrid AF–DF relaying. However, it requires soft decoding at the relay, which increases the receiver complexity.

Fig. 8. BEP of different opportunistic relaying protocols. Fig. 8 compares the performance of opportunistic hybrid AF–DF–T ($T = 4$ dB), AF, DF, and threshold-based hybrid relay selection (THS) [24] ($T = 4$ dB) relaying. In THS relaying, relays with SNRs greater than $T$ are included in the DF set. The other relays are included in the AF set, and a single relay is selected among AF and DF sets based on the instantaneous SNR. We observe that opportunistic hybrid AF–DF relaying offers better performance than THS relaying.

6. Conclusion

In this paper, we have proposed an all-participating hybrid amplify and forward (AF) and decode and forward (DF) relaying protocol in which relays close to the source amplify the received signal whereas the remaining relays transmit only if they have decoded correctly. The proposed protocol offers better performance than AF, APHF, and ADF relaying, and very close performance to DF relaying, while having a lower implementation complexity since close relays do not have to decode the received signal. In order to improve the system spectral efficiency, we have also proposed an opportunistic static hybrid AF and DF relaying protocol in which a single relay is activated among AF relays and the relays that have decoded correctly. The proposed opportunistic hybrid AF and DF relaying protocol offers better performance than opportunistic AF and THS relaying, and close performance to opportunistic DF relaying. The SNR outage probability, and the lower and asymptotic BEP of both all-participating and opportunistic hybrid AF and DF relaying were derived.

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Appendix A

Assuming that the channel gains of the different links are independent, the CDF of the SNR between the selected relay $R_{sel}$ and the destination is given by

$$P_{r_{sel}}(\gamma) = P\left(\max_{i \in \Delta} \Gamma_i \leq \gamma\right) = \prod_{i \in \Delta} P_r(\gamma).$$

(49)

Taking the derivative of (49), we obtain the PDF expression of the SNR between the selected relay and the destination:

$$P_{r_{sel}}(\gamma) = \sum_{i \in \Delta} P_r(\gamma) \prod_{j \in \Delta, j \neq i} P_r(\gamma).$$

(50)

Since Rayleigh fading channels are considered, $\Gamma_i$ follows an exponential distribution with mean $\xi_i$.

Therefore, we have

$$P_{r_{sel}}(\gamma) = \sum_{i \in \Delta} \exp\left(-\frac{\gamma}{\xi_i}\right) \prod_{j \in \Delta, j \neq i} \left[1 - e^{-\frac{\gamma}{\xi_j}}\right].$$

(52)

Let $\{(i, p)\}_{p=1}^{|\Delta|-1}$ be a set of relay indices which belong to $\Delta$ and are different from $i$, $|\Delta|$ is the cardinality of $\Delta$, i.e. the number of AF relays and the relays that have decoded correctly.

The last term in the above equation can be written as

$$\prod_{j \in \Delta, j \neq i} \left[1 - e^{-\frac{\gamma}{\xi_j}}\right] = \prod_{j=1}^{\Delta-1} \left[1 - \exp\left(-\frac{\gamma}{\xi_{l(i,p)}}\right)\right] = \sum_{n=0}^{2^{\Delta-1}-1} (-1)^{\xi(n)} \exp\left(-\gamma \sum_{p=1}^{\Delta-1} \frac{\xi_n(p)}{\xi_{l(i,p)}}\right),$$

(53)

where $\xi_n(1) \cdots \xi_n(|\Delta|-1)$ is the binary representation of $0 \leq n < 2^{\Delta-1}-1$.

$$\xi(n) = \sum_{p=1}^{\Delta-1} \xi_n(p).$$

(54)

Using (52) and (53), we obtain the desired result, shown in (35).

Appendix B

We can write

$$\Psi(x) = \frac{A}{2} \sum_{p=1}^{+\infty} C_p^2 x^{-\frac{1}{2}} x^{BP_x}.$$ 

(55)

At high SNR, all relays have decoded correctly, i.e. $\Delta = \{1, \ldots, N, N + 1, \ldots, N + K\}$. Using (55), (44) can be written as

$$P_{c,D/\epsilon}\geq \frac{A}{2} \sum_{i=1}^{2^{N+K-1}-1} \frac{(-1)^{\xi(n)}}{\xi_{l(i,p)}} \sum_{p=1}^{+\infty} C_p^2 \frac{2^p}{B^{p-1}} \left[1 - \exp\left(-\frac{\gamma}{\xi_{l(i,p)}}\right)\right].$$

(56)

We deduce

$$P_{c,D/\epsilon}\geq \frac{A}{2} \sum_{i=1}^{2^{N+K-1}-1} \frac{(-1)^{\xi(n)}}{\xi_{l(i,p)}} \sum_{p=2}^{+\infty} \sum_{q=0}^{p-2} C_p^q \frac{2^q}{B^{p-1-q}} \left[1 - \exp\left(-\frac{\gamma}{\xi_{l(i,p)}}\right)\right].$$

(57)

The smallest exponent of $T_{S,D}$ in (57) corresponds to $q = p - 2$. Using (40), we can write

$$\frac{1}{\alpha_{n,i}} = \sum_{k=0}^{p-2} C_p^{k-2-k} N^{k-1} \sum_{p_1=1}^{N+k-1} \sum_{p_2=1}^{N+k-1} \frac{\epsilon_n(p_1) \cdots \epsilon_n(p_k)}{T_{l(i,p_1)} \cdots T_{l(i,p_k)}}.$$

(58)

Therefore, (57) becomes

$$P_{c,D/\epsilon}\geq \frac{A}{2} \sum_{i=1}^{2^{N+K-1}-1} \frac{(-1)^{\xi(n)}}{\xi_{l(i,p)}} \sum_{p=2}^{+\infty} \sum_{q=0}^{p-2} C_p^q \frac{2^q}{B^{p-1-q}} \left[1 - \exp\left(-\frac{\gamma}{\xi_{l(i,p)}}\right)\right].$$

(59)

The lowest exponent of $T_{l(i,p_1)} \cdots T_{l(i,p_k)}$ in the above equation corresponds to $k = p - 2$. The lowest value of $p$ in (59) leading to a non-null result is $p = N + K + 1$. For this value of $k = p - 2 = N + K - 1$, the only indices leading to a non-null result in (59) correspond to $n = 2^{N+K-1} - 1$ and the following term:

$$\frac{1}{\xi_{l(i,p_1)} \cdots T_{l(i,p_k)}}.$$

(60)

This term appears $(N + K - 1)!$ times. Using (59) and (60) and the last statement, we obtain the desired result (45).

References


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