Kinematic and dynamic analysis of a hexapod walking–running–bounding gaits robot and control actions

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A B S T R A C T

Kinematic and dynamic analysis, and control actions of a hexapod robot were realized for walking, running and bounding gaits in this study. If biological inspiration can be used to build robots that deal robustly with complex environments, it should be possible to demonstrate that legged biorobots can function in natural environments. Firstly, we tried to report on theoretic work with a six legged robot designed to emulate spider behavior like walking, running and bounding. We demonstrated theoretically that it can successfully walk, run and bound like a spider over natural terrain. Secondly, limitations in its capability were evaluated, and many biologically based important improvements were obtained for future experimental work. Thirdly, the hexapod robot with bounding gait was controlled by proportional-derivative control algorithm and was carried out by using spring loaded inverted pendulum model. Consequently, the developed kinematic and dynamic methods, and control action method makes both the system control easy and the system performance is improved by decreasing the run time for each loop.

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1. Introduction

During the last decade there has been growing interest in the applications of legged system in robotics because the mobility of legged robots in the widest variety of terrain conditions is better than that of most existing mobile robots such as wheeled, tracked or railed robots. The most important desired features of mobile robots in today’s technology are the high capacity of mobility and multi-functionalities [1–3]. Due to its superiority over wheeled and tracked systems on loose-rough-uneven terrains, legged locomotion has attracted attention of robotics researchers. The advantages of legged locomotion are mostly due to the fact that it makes use of isolated footholds [4–6]. Considering the manufacturing and control of walking, the inspirations derived from the biological systems, namely legged animals from cockroaches to camels, are most important for multi-legged robot researchers [7].

Our purpose is to use the legged robots in abrupt indoor (laboratory, train and airplane stations, stores, etc.) and outdoor areas (forests, volcanoes, mountains, etc.) that are the kind of environments where legged robots result in advantages with over wheeled ones. Following a classical approach, the task of legged–robot walking is confronted as an optimization process. The gait of the robot is selected optimizing, for instance, the number of footholds necessary to achieve a given position [8,9], the size of the stability margin [10,11], the desired trajectory and speed [12], the mobility of the legs [13], the energy consumed [14] or reducing the number of actuators to simplify the control problem [1,2]. The ability of animals to deal flexibly with complex environments is often advanced as a reason to adopt a biology-based approach to robotics. Besides, the mobility of animals, including many insects and spiders, is typically superior to current legged robots. This fact recommends the use of animal designs in robots. However, the reality of current technology often encourages engineers to use different

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designs for legged robots than those found in nature. Some robots use different mechanisms. In previous studies, we carried out the designed and manufactured six legged robot named ROBOTURK SA-1 [1] and eight legged robot named ROBOTURK SA-2 [2] by reducing the number of actuators to simplify the control problem. ROBOTURK SA-1 has been driven by using only two actuators while ROBOTURK SA-2 has been driven by using only one actuator. Moreover, in recent years, numerous studies on legged robots have been reported. Scientists and engineers have taken an interest in constructing legged robots with biologically-based designs [15–22].

The goal of our work is to realize a dynamic stable walking, running and bounding gaits of a hexapod robot. First of all, the kinematic and dynamic model of the hexapod robot with bounding gait has been formed and then, the simulation of the system has been realized. To obtain the numerical simulation of the equation of motion for bounding gait, the various dynamic structures have been solved in a sequential closed loop (stance, flight phase). Spring loaded inverted pendulum

<table>
<thead>
<tr>
<th>Nomenclature</th>
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<tbody>
<tr>
<td>$\alpha$</td>
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<td>$\phi$</td>
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<td>$\tau_f$</td>
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<td>$\tau_s$</td>
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Fig. 1. SLIP model for dynamic locomotion.
(SLIP) has been used as a dynamic model to simplify the simulation of the system and ease of understanding the bounding gait. In order to maintain the stability of the legged robot, the location of the center-of-mass (COM) must be kept within the polygon of support excited by the stance legs [23]. Fig. 1 shows the structure of trajectory or polygon followed by COM.

Different from the studies in the literature [24–26], the number of the performed control actions during the phases of bounding gait in this study is smaller than those of the studies in the literature in the limit of authors' knowledge. Therefore, the number of the performed control actions in this study makes the system control quite easy. In addition, the manufacturing of robot was done for future experimental works. Fig. 2 shows the manufactured hexapod robot ROBOTURK SA-3. It is used as both the quadrupedal and hexapod robot. The fixed middle legs hexapod robot is used as quadrupedal robot.

2. Planar kinematic and dynamic model of a hexapod robot

2.1. Problem definition

Fig. 3 indicates a planar hexapod robot. There are three reference cartesian frames in the model, where \( W \) represents the fixed world cartesian frame, \( V \) is the virtual toe frame located at foot of a virtual leg and \( B \) is the body frame connected to the center of mass of the robot.

The system has a rigid body with six flexible legs mounted. \( a_i \) represents the displacement of \( i \) joint with respect to the \( B \) body frame. Furthermore, \( m \) and \( I \) define mass and inertia, respectively. \( \alpha \) denotes the orientation of the body with respect to \( W \) cartesian frame and this angle is used to obtain rotation matrix expressed by \( W^B R \). Finally, \( b \) defines the position of the body with respect to \( V \) frame.

The leg masses \( m_l \) are very small, comparing with the body mass \( m \) (the ratio 1/100). The reason why the leg masses are introduced for each toe is to form the flight dynamics of legs. In addition, each leg possesses a radial spring, denoted \( V_i(\rho) \), as well as potential and radial viscous damper denoted by \( d_i \). In order to actuate the system, independent DC motors with torque \( \tau_i \) are connected to each hip.

The leg-ground interaction model is very important to capture the existing phase for each leg because of selecting true dynamic model for simulation of the system. The system consists of two alternative phases, stance and flight. The contact configuration of the legs determines alternative phases of the system. The proposed control algorithm decides which dynamic structures are evaluated according to \( s_i \in \{0,1\} \) contact states. For instance, 0 states that \( i \)th leg touch the ground whereas 1 states that \( i \)th leg is in flight.

2.2. The coordinate transformations of the kinematic equations

The only way acting external force on the body is the six legs by means of their interactions with the ground. That’s why; the position and orientation of the legs with respect to the body frame are the most important parameters in this manner [27].

The toe position vector \( \vec{c} \), the variable distance between the toe and the center of mass of the body, is defined with respect to \( V \) frame:

\[
\vec{f}_i = \vec{b}_i + \vec{c} \\
\vec{c} = \vec{f}_i - \vec{b}_i
\]

if \( \vec{c} \) is defined with respect to the moving frame located within the main body:

\[
\vec{c} = \vec{u}_i + \vec{I}_i
\]
Then, Eq. (16) is taken derivative with respect to time, we get

\[ \dot{q} = \frac{l_y l_y + l_z l_z}{\tilde{l}_i} \]

If \( \tilde{l}_i \) vector is transformed to polar coordinates, then we have

\[ \rho \dot{\phi} = \frac{l_y l_y + l_z l_z}{2l_y l_z} \]

The differential coefficients of variables versus time in Eq. (7) are described by

\[ \frac{d}{dt} \left( \begin{array}{c} \mathbf{I}_i \\ \mathbf{R}^{T}(x) \end{array} \right) = \mathbf{D} \mathbf{R}^{T}(x), \quad \frac{d}{dt} \mathbf{x} = \dot{\mathbf{x}} \]

The above equations can be rearranged as the following form:

\[ \dot{\mathbf{I}}_i = \mathbf{R}^{T}(x) \cdot \left( \dot{\mathbf{f}}_i - \dot{\mathbf{b}} \right) + \mathbf{R}^{T}(x) \left( \mathbf{f}_i - \mathbf{b}_i \right) - \dot{\mathbf{a}}_i \]

\[ \dot{\mathbf{f}}_i = \mathbf{D} \mathbf{f}_i \]

Transformation matrix can be formed because of stating the vector \( \mathbf{e} \) as function of \( \mathbf{f}_i \) and \( \mathbf{b} \):

\[ \mathbf{I}_i = \tilde{\mathbf{c}} - \tilde{\mathbf{a}}_i \]

\[ \frac{w}{b} \mathbf{R}^{T}(z) = \left[ \begin{array}{cc} \cos(z) & -\sin(z) \\ \sin(z) & \cos(z) \end{array} \right] \]

\[ \mathbf{R}^{T}(z) = \left( \frac{w}{b} \mathbf{R}^{T}(z) \right)^{T} = \left[ \begin{array}{cc} \cos(z) & \sin(z) \\ -\sin(z) & \cos(z) \end{array} \right] \]

According to \( B \) body frame, it can be defined:

\[ \mathbf{I}_i = \mathbf{R}^{T}(z) \cdot (\mathbf{f}_i - \mathbf{b}) - \dot{\mathbf{a}}_i \]

\[ \frac{d}{dt} \mathbf{R}^{T}(z) = \mathbf{D} \mathbf{R}^{T}(z), \quad \frac{d}{dt} \mathbf{x} = \dot{\mathbf{x}} \]

The above equations can be rearranged as the following form:

\[ \dot{\mathbf{I}}_i = \mathbf{R}^{T}(z) \cdot \left( \dot{\mathbf{f}}_i - \dot{\mathbf{b}} \right) + \mathbf{R}^{T}(z) \left( \mathbf{f}_i - \mathbf{b}_i \right) - \dot{\mathbf{a}}_i \]

\[ \dot{\mathbf{f}}_i = \mathbf{D} \mathbf{f}_i \]

If \( \tilde{l}_i \) vector is transformed to polar coordinates, then we have

\[ \rho \dot{\phi} = \frac{l_y l_y + l_z l_z}{2l_y l_z} \]

Then, Eq. (16) is taken derivative with respect to time, we get

\[ \dot{i}_i = \frac{l_y l_y + l_z l_z}{\tilde{l}_i} \]

\[ \phi_i = \frac{a \tan \left( \frac{l_y}{l_z} \right)}{\tilde{l}_i} \]
As the same way, the time derivative of Eq. (18):
\[
\dot{\phi}_i = \frac{l_y l_z - l_y l_z}{l^2}
\]  
(19)

In order to obtain the equation of motion of the system, all forces and torques acted on the system must be determined. These are the spring, damping forces and hip torques. The radial forces existing on the legs in the planar hexapod model shown in Fig. 3 can be formulated as follows:
\[
F_{ri} = -k_i V_i (l_0 - \rho_i) - d_i \dot{\rho}_i
\]  
(20)

As a result, ith leg force acted on the ground can be written as,
\[
F_i = R(\alpha + \phi_i) \left[ \tau_i/\rho_i \right] - F_{ri}
\]  
(21)

Then, \( R(\alpha + \phi_i) \) is formed in the following form:
\[
R(\alpha + \phi_i) = \begin{bmatrix} \cos(\alpha + \phi_i) & -\sin(\alpha + \phi_i) \\ \sin(\alpha + \phi_i) & \cos(\alpha + \phi_i) \end{bmatrix}
\]  
(22)

If the radial leg forces are rearranged, we get
\[
F_{iy} = \cos(\alpha + \phi_i) \cdot (\tau_i/\rho_i) + \sin(\alpha + \phi_i) \cdot F_{ri}
\]  
(23)

\[
F_{iz} = \sin(\alpha + \phi_i) \cdot (\tau_i/\rho_i) - \cos(\alpha + \phi_i) \cdot F_{ri}
\]  
(24)

Furthermore, the power consumption for all virtual leg can be written as,
\[
P_{hexapod\_total} = \phi_{front\_leg} \cdot T_{front\_leg} + \phi_{middle\_leg} \cdot T_{middle\_leg} + \phi_{back\_leg} \cdot T_{back\_leg}
\]  
(25)

One considers that the ground acts on the system and these forces equal to the negative direction of the related leg forces in the stance phase. Whereas in the flight phase, the inertia forces of the toe masses only exist and the ground reaction forces do not exist [27,28].

Consequently, the leg forces and hip torques are represented as vector forms:
\[
\tau := [\tau_1, \ldots, \tau_6]
\]  
(26)

\[
F_r := [F_{r1}, \ldots, F_{rn}]
\]  
(27)

2.3. Equations of motion of the system

Equations of motion the system are formed in this section. The dynamics of the planar hexapod robot are obtained based on a planar rigid body affected by the leg forces. Therefore, the equations of motion can be obtained as follows:

If Newton’s 2nd law is used for rigid body, we have
\[
m(\ddot{b} + g) = \sum_{i=1}^{6} s_i F_i
\]  
(28)

\[
m \ddot{b}_y = s_{1y} F_{1y} + s_{2y} F_{2y} + s_{3y} F_{3y}
\]  
(29)

\[
m \ddot{b}_z = s_{1z} F_{1z} + s_{2z} F_{2z} + s_{3z} F_{3z}
\]  
(30)

Again from Newton’s second law, the following equations are obtained:
\[
l \ddot{\alpha} = \sum_{i=1}^{6} s_i (f_i - b) \times F_i
\]  
(31)

\[
\ddot{f}_i = f_0 j + f_a \ddot{k}
\]  
(32)

\[
\ddot{b} = b_0 j + b_a \ddot{k}
\]  
(33)

\[
l \ddot{\phi}_i = s_i \left[ (f_{iy} - b_{iy}) j + (f_{iz} - b_{iz}) k \right] \times \left[ F_{iy} j + F_{iz} k \right]
\]  
(34)

If leg masses are considered, the equations of motion can be expressed as follow:
\[
m_i \cdot \ddot{f}_i = (1 - s_i) F_i
\]  
(35)

\[
m_0 \cdot \ddot{f}_i = (1 - s_i) \cdot R(\alpha + \phi_i) \cdot \left[ \tau_i/\rho_i \right] - F_{ri}
\]  
(36)
The stance legs only are considered in the rigid body motion. In addition, a plastic collision of toe masses is assumed therefore the initial velocities of toe masses are zero [27]. That’s why, stance toes will be fixed during the all stance duration.

2.4. Phase transitions

Since there are discrete changes in leg contact states, we have to formally specify the points in the trajectory where such changes occur. Therefore, we should define initial functions for each mode pair. Furthermore, the touchdown and liftoff conditions for all the legs can be independently determined as follows:

\[ h_t^i = b_x - \rho_i \cos(x + \phi_i) \]

\[ h_l^i = -\pi_z(F_i) = -\sin(x + \phi_i)(\tau_i/\rho_i) - \cos(x + \phi_i) \cdot F_{\text{int}} \] (38)

(39)

Fig. 4. The virtual legs for the bounding gait of hexapod robot.

Fig. 5. The stance and flight states of the virtual legs.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>The variables of robot.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbols</td>
<td>Descriptions</td>
</tr>
<tr>
<td>(z), (y)</td>
<td>The center-of-mass cartesian coordinate (COM)</td>
</tr>
<tr>
<td>(x)</td>
<td>Body pitch angle</td>
</tr>
<tr>
<td>(\phi)</td>
<td>Leg angle</td>
</tr>
<tr>
<td>(\rho)</td>
<td>Leg length</td>
</tr>
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<thead>
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<th>Table 2</th>
<th>The design parameters of robot.</th>
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<td>Descriptions</td>
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<td>Body mass</td>
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<tr>
<td>(l)</td>
<td>Body inertia</td>
</tr>
<tr>
<td>(k_i)</td>
<td>(i) Spring constant</td>
</tr>
<tr>
<td>(d_i)</td>
<td>(i) Damping constant</td>
</tr>
<tr>
<td>(l_o)</td>
<td>Leg length</td>
</tr>
<tr>
<td>(L)</td>
<td>Body length</td>
</tr>
<tr>
<td>(m_i)</td>
<td>Leg mass</td>
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Table 3
The initial conditions of the control parameters.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>The front virtual leg</th>
<th>The middle virtual leg</th>
<th>The back virtual leg</th>
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<tr>
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<td>18</td>
<td>15</td>
</tr>
<tr>
<td>Sweep limit angles (°)</td>
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<td>0–4</td>
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Table 4
The initial conditions.

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<th>Parameters</th>
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<td>Horizontal position ($b_y$)</td>
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</tr>
<tr>
<td>Vertical speed ($b_z$)</td>
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<td>m/s</td>
</tr>
<tr>
<td>Vertical position ($b_z$)</td>
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<td>m</td>
</tr>
<tr>
<td>Front virtual leg</td>
<td>18</td>
<td>(°)</td>
</tr>
<tr>
<td>Middle virtual leg</td>
<td>18</td>
<td>(°)</td>
</tr>
<tr>
<td>Back virtual leg</td>
<td>15</td>
<td>(°)</td>
</tr>
</tbody>
</table>

(No Controlled Coupling)

BACK_LEG STATE MACHINE

MIDDLE_LEG STATE MACHINE

FRONT_LEG STATE MACHINE

Fig. 6. The bounding controller states for hexapod robot.

\[\tau_r = k_p (\phi^{id} - \phi) + k_d \dot{\phi}\]

\[\tau_s = k_p (\phi^{swl} - \phi) + k_d \dot{\phi}\]

Fig. 7. Control actions during the phases of the bounding gait.
The above conditions ensure that flying legs go into stance when their toes come in contact with the ground [27]. In addition, the transition from stance to flight is determined by the $z$ component of the ground reaction force on the toe. The initial func-

**Fig. 8.** The block diagram of the proposed control algorithm.

**Fig. 9.** The flow-chart of the proposed PD control algorithm for the system.


tion of touchdown phase \( h_t \) is the difference between vertical z-axis position of each leg and vertical z-axis of the system’s center of gravity. If this difference is zero, the leg is in contact with the ground and is within touchdown phase. The initial function of liftoff phase \( h_l \) is the sum of the radial forces in vertical z-axis of each leg. If the total value of these forces is zero, the leg no longer cut off contacting with ground and is within liftoff phase. As a result, the initial function for a particular mode pair can be constructed by combining the appropriate touchdown or liftoff conditions for all the legs.

3. Bounding gait model for hexapod robot

In this study, we only consider the bounding gait. It is also assumed that bounding motion is planar. That’s why, our model can be composed of the body and the front, middle and back leg pairs, also called the *virtual* legs are shown in Fig. 4. A linear spring-damper system represents the model of the leg flexibility during the stance phase [29].

Fig. 5 shows the states of the virtual legs. All the variables and the sign conventions are shown in Fig. 3 and are summarized in Table 1. The design parameters of the robot are indicated in Table 2.

![Simulation of the bounding gait of the hexapod robot in the sagittal plane](image)

**Fig. 10.** Simulation of the bounding gait of the hexapod robot in the sagittal plane: (a) the simulation of the bounding gait of the planar hexapod robot. (b) The oscillation of the body \( z \) versus time. (c) The cyclic trajectories \( b_z \) versus time in the vertical direction. (d) The phase plot of the center of mass of the body (vertical axis: \( b_z \) (m)-horizontal axis: \( b_z \) (m/s)).
4. Bounding gait control

In this section, the bounding controller are introduced based on the control parameters. There are three control parameters, namely $\phi^{td}$, $\phi^{swl}$, and $\tau$, representing the touchdown angle, the sweep limit angle and the maximum torque, respectively, for each virtual legs. The front, middle and back virtual legs are controlled independently from each other based on their states in the designed controller. Each of the virtual legs independently realize phases and events as mentioned before. In other words, the overall body is controlled by only using leg-controllers.

It is generally desired that the center of mass of the main body follows a cyclic trajectory in shown literatures [15,24,27,29,30]. This trajectory can be represented as the pitch angle and the vertical velocity of the main body. For this reason, the cyclic trajectory formed by the vertical body oscillations should be realized by controlling the legs.

Table 3 indicates the control parameters used for each of the virtual legs. Furthermore, Table 4 illustrates the initial conditions for simulations. There are two-main control states for the front, middle and back virtual legs in the proposed control algorithm. The flight and stance-brake states are separated by touchdown, sweep limit and lift-off events as shown in Fig. 6.

4.1. Touchdown and sweep angle control

The touchdown angles were directly considered for the controls of Scout II's cyclic motion [25,26]. Furthermore, most of the control action is applied during the flight phase since the actuator torques are limited during the stance phase [24,31].

Fig. 11. Simulation of the bounding gait of the hexapod robot in the sagittal plane; (a) the forward displacement of the body ($b_f$) versus time. (b) The forward velocity of the body ($b_f$) versus time. (c) The instantaneous leg-lengths of front-middle-back virtual legs ($l_f$) versus time. (d) The instantaneous leg-length of front virtual legs ($l_{f_{front}}$) versus time.
Fig. 7 indicates the control action methods in two-phases of the system. At least, one of the virtual legs must not contact the ground during the flight phase. In this case, $\phi^{ed}$ angle is always controlled. Moreover, the leg should have the desired angle when the virtual legs touch the ground after the flight phase. This action is generally performed during the flight phase. proportional-derivative (PD) control algorithm is assigned to obtain the desired touchdown angle by moving the legs to the desired positions. The required torque is obtained by the actuators mounted on the hips of each virtual leg. As a result, $\phi^{ed}$ angle can be taken as a control variable of the flight phase [32].

$\phi^{swl}$ Represents the sweep angle, the control variable of each virtual leg during the stance phase until each virtual leg stops contacting the ground. After the virtual leg touches the ground, it forces the main body forward while itself is moving backward. That’s why; the sweep angle must be controlled in order to successfully perform the bounding gait of the robot.

4.2. PD control algorithm

The control and numerical simulation of the hexapod robot system have been realized by using MATLAB package programmer. The proposed algorithm consists of one main and five subroutine programs. Fig. 8 indicates the block diagram of the proposed control algorithm. In this block diagram, the main program contains the inputs such as the initial conditions, the robot parameters and the graphical functions of simulation. Slip_Events forms the dynamical loop by determining the

![Graphs and Diagrams]

Fig. 12. Simulation of the bounding gait of the hexapod robot in the sagittal plane; (a) the instantaneous leg-length of middle virtual legs ($l_{middle}$) versus time. (b) The instantaneous leg-length of back virtual legs ($l_{back}$) versus time. (c) The angular positions of front-middle-back virtual legs $z(\cdot)$ with respect to the body versus time. (d) The angular positions of front virtual leg $\phi(\cdot)$ with respect to the body versus time.
phases and states of the robot. Moreover, Slip_Vf contains the equations of motion of the system. The kinematic equations of the system are solved in To_Radical subroutine program. Flight_Control and Stance_Control contain the control algorithms.

The outputs of the control algorithm are the appropriate torque values generated by PD control algorithm in the hip actuators of the hexapod robot. As explained before, the system consists of two-closed dynamic loops. Bounding gait contains various dynamic structures in a sequential closed loop. Stable bounding gait of the hexapod robot has been realized to have control in each phase, separately. Therefore, the system is controlled by obtaining appropriate torque values generated by PD control algorithm so that the touchdown angles of the front, middle and back virtual legs ($\phi^{vt}$) in the flight phase and the sweep angles of the front, middle and back virtual legs ($\phi^{sw}$) have the desired values. In the control algorithm, the inputs of the system are the error ($e$) and the rate of change of error $\dot{e}$, where error is difference between instantaneous $\phi$ and the desired $\phi$ of all virtual legs in the flight and stance phase. On the other hand, the outputs of the system are the appropriate torque values generated by the hip actuators in order to minimize the error ($e$) and the rate of change of error $\dot{e}$. Fig. 8 and Fig. 9 show the inputs and the outputs of the system. In this study, proportional coefficient ($k_p$) and derivative coefficient ($k_v$) values were determined by using the trial and error method.

5. Simulation results

In this study, numerous simulation walking, running and bounding gait of the hexapod robot have been performed. Fig. 10 shows the simulation of the bounding gait of the hexapod robot in the sagittal plane (Fig. 10a). Furthermore, this figure illustrates the oscillation of the body ($z$) versus time (Fig. 10b), the cyclic trajectories ($b_z$) versus time in the vertical direction (Fig. 10c) and the phase plot of the center of mass of the body (Fig. 10d). Fig. 11 shows the forward displacement of the body ($b_x$) versus time (Fig. 11a), the forward velocity of the body ($\dot{b}_x$) versus time (Fig. 11b) and the instantaneous leg-lengths of front-middle-back virtual legs versus time (Fig. 11c). Fig. 11d shows the instantaneous leg-length of front virtual legs versus time. Besides, Fig. 12a shows the instantaneous leg-length of middle virtual legs versus time, and Fig. 12b shows the instantaneous leg-length of back virtual legs versus time. Then, Figs. 12c, d and 13b show the angular positions of front-middle-back virtual legs with respect to the body versus time. Fig. 13c shows the instantaneous error changes of front-middle-back virtual leg angles $\phi$ with respect to the versus time. Moreover, Fig. 14 shows the torques applied the virtual legs ($\tau$) versus time. Finally, Table 5 shows the locomotion performances of the bounding gait of the hexapod robot.

It is considered that the lengths of the legs only change after they touch the ground. The virtual legs behave as if they were linear springs during the stance phase because of the mass of the robot and the flexibility of the legs. First, they compress during early stage of the stance phase then, they decompress to form the bounding state. The lengths of the legs are constant.

![Fig. 14](image-url). Simulation of the bounding gait of the hexapod robot in the sagittal plane: (a) the torques applied the front-middle-back virtual legs \( \tau \) (Nm) versus time. (b) The torques applied the front virtual legs \( \tau_{\text{front}} \) (Nm) versus time. (c) The torques applied the middle virtual legs \( \tau_{\text{middle}} \) (Nm) versus time. (d) The torques applied the back virtual legs \( \tau_{\text{back}} \) (Nm) versus time.

**Table 5**

<table>
<thead>
<tr>
<th>Locomotion performances of the bounding gait of the hexapod robot.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Hexapod robot</strong></td>
</tr>
<tr>
<td><strong>Six legged</strong></td>
</tr>
<tr>
<td><strong>The initial condition values</strong></td>
</tr>
<tr>
<td><strong>Height of the robot’s left leg</strong> ( b_z ) (cm)</td>
</tr>
<tr>
<td><strong>The angle with the horizontal axis of robot</strong> ( \alpha ) (°)</td>
</tr>
<tr>
<td><strong>Horizontal speed of the robot</strong> ( \dot{d}_{by} ) (m/s)</td>
</tr>
<tr>
<td>50</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>2.5</td>
</tr>
<tr>
<td><strong>Power consumed by actuators</strong></td>
</tr>
<tr>
<td><strong>Time ( t ) (s)</strong></td>
</tr>
<tr>
<td><strong>Horizontal displacement ( b_z ) (m)</strong></td>
</tr>
<tr>
<td><strong>Power consumed for front leg ( P_{\text{front}} ) (W)</strong></td>
</tr>
<tr>
<td><strong>Power consumed for middle leg ( P_{\text{middle}} ) (W)</strong></td>
</tr>
<tr>
<td><strong>Power consumed for back leg ( P_{\text{back}} ) (W)</strong></td>
</tr>
<tr>
<td><strong>Total consumed power ( P_{\text{total}} ) (W)</strong></td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>1.4</td>
</tr>
<tr>
<td>175</td>
</tr>
<tr>
<td>156</td>
</tr>
<tr>
<td>176</td>
</tr>
<tr>
<td>509</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>1.82</td>
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<td>382</td>
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<td>374</td>
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<td>354</td>
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<td>455</td>
</tr>
<tr>
<td>450</td>
</tr>
<tr>
<td>1440</td>
</tr>
</tbody>
</table>

It is considered that the lengths of the legs only change after they touch the ground. The virtual legs behave as if they were linear springs during the stance phase because of the mass of the robot and the flexibility of the legs. First, they compress during early stage of the stance phase then, they decompress to form the bounding state. The lengths of the legs are constant.
during the flight phase. The unimportant vibrations only occur because of the toe masses $m_t$. Moreover, the instantaneous change of the leg-length is represented by $\phi$. In Fig. 10c, $\phi$ defines the angular displacement of the virtual legs with respect to the body. Then, the phase plot is formed based on $(b_2)$ and $(b_4)$, and gives information about the system stability (Fig. 10d). During the stance phase, the trajectory is a cyclic in phase plane because of the mass-spring model assumption whereas trajectory is a parabolic because of gravitational effects in the flight phase [28,29]. Finally, the next figure illustrates the applied torques calculated by PD control algorithm to all virtual legs to obtain the desired angles.

6. Conclusion

In this study, the proposed controllers are simpler than those of Raibert proposals because the body-state feedback and leg-length control are not necessary. The idea of the proposed controller has been successfully used in the studies of different robots with bounding gait [33–35]. In addition to that, the number of the performed control action during the phases of bounding gait is smaller than those of the studies in the literature in the limit of authors knowledge [24–26,31,32,36]. This showed that, the proposed control action method in this study makes both the system control easy and the system performance increase by decreasing the run time for each loop [37,38]. It has been seen that the simulation results of the hexapod robot controlled by PD control algorithms are as well as much as those of the existing studies in the literatures [32] in terms of energy consumption and stability of motion. Besides, the SLIP model has been extensively used as the simplest model to obtain the dynamic behaviors of legged-animals in the biomechanics and robotics literatures in the last years. Moreover, this model leads easily understanding of complex motion mechanisms of legged-animals and humans. Consequently, the SLIP model has been successfully used as a dynamic model of bounding gait of the hexapod robot in this study as also those of walking [20,31], running [36], flipping [39] and climbing gaits [32] in the literatures.

References


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