Investigating Scaling Behavior of End-to-End Delay

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Abstract—End-to-end delay is an important QoS metric and has received much research attention. Recently, the multifractal detrended fluctuation analysis (MFDFA) is widely used as a robust tool to investigate the scale behavior of non-stationary time series. In this paper, we use MFDFA to analyze the scale behavior of end-to-end delay series. Based on ping series of both home and international paths, we find the 2-order correlation of delay series is analysis scale, path, and (collection) time dependent: may be biscalting or even multiscaling, persistent or anti-persistent. In addition, we observe that delay series may be multifractal at both fine and coarse scales. We also examine the source of multifractality for delay series and find the source is much complex.

I. INTRODUCTION

End-to-end delay is an important QoS metric of the Internet. Studying the properties of the end-to-end delay is useful for network protocol and multimedia application design, delay modeling and network performance evaluation, etc. The Internet delay has received much research attention. The IP performance metric (IPPM) working group of the Internet engineering task force (IETF) has proposed several RFCs (or drafts) for delay (RFC2679, 2330, etc.). In these RFCs (or drafts), the precise definitions of RTT, one-way delay, jitter, etc. are given and the associated measurement technologies are recommended. The skitter project of CAIDA (Cooperative Association for Internet Data Analysis) is aiming to measure the Internet by end-to-end delay. Many researches have been taken based on the collected delay data[1]. Roughly speaking, the study of Internet delay traces could be cataloged into two groups, the space dimension analysis and the time dimension analysis. In space dimension analysis, the distribution of delay among links over a path [2][3] and the estimation technologies are important role in characterizing the end-to-end behavior. M. S. Borella et al.[6] explored the long range dependence (LRD) of RTT time series collected by ping. Ref. [7] found the LRD also exists in RTT series measured by NTP (network time protocol). Q. Li and D. L. Mills[8] used wavelet-based techniques to study the scaling behavior of the measured delay traces by ping and argued that delay properties are complex: may be LRD, self-similar, or even multifractal. In Ref. [9], Wang et al. divided delay into two parts, the normal component and the spike component, and claimed that the normal component behaves strong LRD property. T. Nkashima [10] investigated the scale behavior of jitter series. In Ref.[11], D. P. Pezaros et al. analyzed the LRD of delay series in wireless environment.

Recently, the multifractal detrended fluctuation analysis (MFDFA) is widely used as a robust tool to investigate the scale behavior of non-stationary time series. It is believed that the MFDFA is more creditable than the wavelet based ones[12], which is widely used in traffic and delay series analysis. In this paper, we analyze the scaling behavior of RTT series by MFDFA. We find the 2-order correlation of delay series is analysis scale, path, (collection) time dependent. In addition, we observe that delay series at both fine and coarse scales could be multifractal. We also examine the source of multifractality for RTT series and find the source is complex.

The remainder of this paper is organized as follows. In section 2, we briefly introduce the multifractal detrended fluctuation analysis method. Section 3 describes the delay collection experiment. Then in section 4, we examine the scale behavior delay series and investigate the source of multifractality for delay series. Finally, section 5 concludes the paper.

II. MULTIFRACTAL DETRENDED FLUCTUATION ANALYSIS

Multifractal detrended fluctuation analysis is a generalization of standard DFA by identifying the scaling of the qth-order moments of the time series, which may be non-stationary[13]. Suppose that $x_k$ is a series of length $N$ and the series is of compact support, i.e. $x_k = 0$ for an insignificant fraction of the values only.

- **Step 1:** Determine the 'profile'

  \[ Y(j) = \sum_{i=1}^{j} (x(i) - < x >), j = 1, \ldots, N \quad (1) \]

  where $< \cdot >$ denotes averaging over the time series. The subtraction of the mean $< x >$ is not compulsory, since it would be eliminated by the later detrending in the third step.

- **Step 2:** Divide the profile series $Y(i)$ into $N_s = \lfloor N/s \rfloor$ non-overlapping segments of equal length $s$. Since the length $N$ of the series is often not a multiple of the considered scale $s$, a short part at the end of the profile may remain. In order not to disregard this part of the
series, the same procedure is repeated starting from the opposite end. Therefore, totally $2N_s$ segments are obtained.

- **Step 3:** Calculate the local trend of each of the $2N_s$ segments by a least square fit of the segment. Then determine the variance

$$F^2(s, v) = \frac{1}{s} \sum_{i=1}^{s} \left( Y[(v - 1)s + i] - y_v(i) \right)^2$$

for each segment $v = 1, \cdots, N_s$ and

$$F^2(s, v) = \frac{1}{s} \sum_{i=1}^{s} \left( Y[N - (v - N_s)s + i] - y_v(i) \right)^2$$

for each $v = N_s + 1, \cdots, 2N_s$. Here, $y_v(i)$ is the fitting polynomial of segment $v$. Linear, quadratic, cubic, or higher order polynomials can be used in the fitting procedure. The trend of order $m$ in the profile series or the trend of order $m - 1$ in the original series is eliminated by the $m$th order fitting.

- **Step 4:** Average over all segments to obtain the $q$th-order fluctuation function, defined as

$$F_q(s) = \left\{ \frac{1}{2N_s} \sum_{i=1}^{2N_s} [F^2(s, v)]^{q/2} \right\}^{1/q}$$

where, in general, the index variable $q$ can take any real value except zero. In addition, $F_q(s)$ is only defined for $s \geq m + 2$ by construction. If we consider positive values of $q$, the segments $v$ with large variance $F^2_v(v)$ will dominate the average $F_q(s)$. Thus, for positive $q$, $h(q)$ describes the scaling behavior of segments with large fluctuations. On the contrary, for negative values of $q$, $h(q)$ describes the scaling behavior of the segments with small fluctuations.

- **Step 5:** Determine the scaling behavior of the fluctuation function by analyzing log-log plot of $F_q(s)$ versus $s$ for each value of $q$,

$$F_q(s) \propto s^{h(q)}.$$  

$h(q)$ is called the generalized Hurst exponent. For mono-fractal time series with compact support, $h(q)$ is independent on $q$, while for multifractal time series, $h(q)$ is not. Generally $h(q)$ is a decreasing function and the singularity spectrum of the Hölder exponents $\alpha$ can be obtained by

$$\alpha = h(q) + qh'(q) \quad f(\alpha) = q[\alpha - h(q)] + 1$$

It is reported that in the above procedure, deviation from scaling at very small scales in log-log plot exists. Modified version DFA and MF DFA are proposed to remove the deviation[14], [15].

Generally, two different types of multifractality in time series can be distinguished: (I) the multifractality due to a broad probability density function (pdf) for values of the time series and (II) the multifractality due to different long-range (time-)correlations of the small and large fluctuations.

To distinguish these two types, the shuffled and surrogate series of the original series are often analyzed[13], [15]. In shuffled series, the time correlation of the original series is removed but the pdf is not affected, while in the appropriate surrogate series, the time correlation is retained but the pdf is replaced with one which possesses a regular distribution with finite moments, e.g. Gaussian distribution. Thus, if only type I (or type II) is presented in a time series, the corresponding surrogate (or shuffled) series will behave monofractality. If both kinds exist, the multifractality of the shuffled or surrogate series is weaker than that of the original one.

III. EXPERIMENT

We collect round trip time (RTT) by the ping tool of Linux. RTT series are measured over two different types of path. The first one is home paths which are from a computer in our lab to four popular commercial web sites, google.com, baidu.com, sina.com and sohu.com, denoted by google, baidu, sina and sohu respectively. The other one is international paths which are from a PlanetLab node in Xi’an Jiaotong University to other three nodes located in American, Singapore and Germany. These three pathes are denoted by US, SG and DE respectively. We repeat the experiment twice at idle and busy time. Therefore, there are totally 14 RTT series. The ping interval is 10 milliseconds and the ping packet length is 64 bytes.

In the raw traces, some ping packets are lost. We simply ignore the lost. The length of the series used in our analysis is 65,536.

IV. RESULTS AND DISCUSSION

A. DFA Analysis

We first analyze the 2nd-order scaling behavior of RTT series and estimate the correlation indicated by Hurst exponent. The scales used is $s = 2^j, j = 3, 4, \cdots, N$ while $N = \lfloor \log_2(\text{length-of-series}) \rfloor$, the same as in the wavelet based method analysis. For simplicity, we roughly refer the scales below $s = 256$ (corresponding to 8 in x-axis in figures) as fine scales (FS) while those above as coarse scales (CS). Fig.1 and 2 show the log-log plots of $F_2(s)$ versus $s$ for home and international paths respectively. The detrended order $m$ used is 3. To reduce amount of computation, the original MF DFA is hired instead of the modified version, which increases the amount of calculation several times. Since deviation from fitting may occur at very small scales, the scales below 32 is omitted, namely the first two points in log-log plot are ignored in analysis.

It can be seen from these two figures that the scale behaviors at FS and CS may be different, i.e. biascaling or even multiscaling may exist. The idle series of sina, US, SG and DE and the busy series of DE have approximately the same scale behavior at both FS and CS, but the busy series of sohu has three scale regimes and the rest have two. Table I lists the Hurst exponent of each scale regime estimated over at least four scale points (minimum number requirement in practice).
Only Hurst exponents of the first and the third scale regime of sohu busy series are given.

As Table I shows, for home paths, the idle RTT series are persistent (long-range dependent) at both FS and CS, while the busy series are anti-persistent (short-range dependent) at FS but persistent at CS. Note that the persistent at CS is weaker than that of FS. This can be explained by the fact that the queuing delay is more fluctuant at busy time, thus the busy series at FS is more likely to be anti-persistent. As scale increases, the fluctuation is weakened thus the busy series becomes (weak) persistent. Idle series of international paths are persistent as well. In contrast, busy series of US and SG are persistent at FS but anti-persistent at CS. The busy series of DE is persistent at both FS and CS. These suggest the scale behavior of international path is different from that of home path. In summary, the scale behavior of RTT series is analysis scale, path and (collection) time dependent.

B. MFDFA Analysis

In this section, we analyze the scale behavior of RTT series by multifractal theory. In order to find good fitting scale range, \( q \) belongs to \((0, 6]\) with an increment interval of 0.2. As stated in section II, the fractal behavior of a time series is determined by the dependence of \( h(q) \) on \( q \). If the generalized Hurst exponent \( h(q) \) is independent of \( q \), the series is monofractality, while if \( h(q) \) is not, it is better to consider the series as multifractality. Generally, \( h(q) \) forms a decreasing function of \( q \) and the corresponding multifractal spectrum can be computed according to equation (6).

We show the FS and CS \( h(q) \) of home and international paths through figure 3 to 6. As can be seen, all \( h(q) \) except the FS one of the google busy series obviously depends on \( q \). In fact, the fine scale \( h(q) \) of google busy series slightly increases from 0.2902 to 0.3684. However, several \( h(q) \) are increasing function of \( q \), or even mixing (increasing and decreasing) function. We call the increasing exception. By carefully checking the log-log plot of \( F_q(s) \) versus \( s \) of which the corresponding \( h(q) \) is increasing , we find nearly all linear fittings in the log-log plot are relative good. Thus the increasing is not from computation error. We also find increasing \( h(q) \) in analysis of network traffic series (especially TCP series). RTT fluctuation is mostly from traffic dynamics, therefor we believe the increase of \( h(q) \) is a nature of RTT series. As a result, it is necessary to consider RTT series as multifractality.

We also investigate the source of multifractality for RTT series. The source is indicated by the \( h(q) \) of the corresponding shuffled or surrogate series. The shuffled series is obtained by randomly re-positioning the original series, and the surrogate series is generated with the Fourier transform method, by multiplying the Fourier transform coefficients of original series with a random phase, then transforming back to time domain[16]. The surrogate series generated in such a method can retain the linear correlation of the original series. Due to space limit, we only give out the \( h(q) \) of the shuffled and surrogate series for fine scales through figure 7 to 10. Five shuffled and surrogate series are generated for each RTT series and all \( h(q) \) here are computed from the average \( F_q(s) \) of them. Note that there is no increasing or mixing \( h(q) \) any more. Some \( h(q) \) are decreasing function, while some are nearly independent on \( q \). In order to quantify the dependence degree of \( h(q) \) on \( q \), we compute the corresponding multifractal spectra of the shuffled
and surrogate series. The widths of these spectra are listed through Table II to V. It should be aware that the multifractal spectrum is not complete since we limit the range of $q$ to be $(0, 6]$.

It can be seen from these tables that: (1) both type I and II multifractality may exist in RTT series. For example, the spectrum widths of the shuffled and surrogate idle *baidu* series (coarse scales) are 0.2393 and 0.1206 respectively, significantly greater than 0. (2) either type I or type II appears only. Take the fine scales *US* busy series as an example. The spectrum width of the shuffled series is 0.2266 but that of the surrogate is 0.0091. However, comparing against the width of original series spectrum (fortunately it can be computed), 0.6566, the spectrum span of the shuffled series is much small. (3) neither type I nor type II presents. The *sohu* and the *DE* busy series (coarse scales) are two cases. In contrast, these two series are indeed multifractal and their spectrum widths are 0.2048 and 0.2910 respectively (they also could be computed). The second and the third phenomenons suggest that only the type I and II can not completely explain the source of multifractality for RTT series. The scale nature of RTT series is much more complex.

<table>
<thead>
<tr>
<th>TABLE II</th>
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<tbody>
<tr>
<td>WIDTH OF MULTIFRACTAL SPECTRUM, HOME, FINE SCALE</td>
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<tr>
<td>----------</td>
</tr>
<tr>
<td>google</td>
</tr>
<tr>
<td>idle</td>
</tr>
<tr>
<td>shuffled</td>
</tr>
<tr>
<td>surrogate</td>
</tr>
<tr>
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<tr>
<td>idle</td>
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MFDA method. We find the 2-order correlation of RTT series is analysis scale, path and (collection) time dependent. The biscalping, even multiscaling, phenomena may exist, and the time correlation of a given RTT series at different scale regime may be persistent or anti-persistent. We also examine the numerical evidence of multifractality for RTT series. Interestingly, RTT series at both fine scales and coarse scales could be multifractal. Moreover, by studying the corresponding shuffled and surrogate series, we find the source of multifractality of RTT series is complex and is analysis scale, path and (collection) time dependent as well.

V. CONCLUSION

In this paper, we collect RTT series over both home and international paths, and analyze their scale behaviors by

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TABLE III
WIDTH OF MULTIFRACTAL SPECTRUM, HOME, COARSE SCALE

<table>
<thead>
<tr>
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<tr>
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<td>busy</td>
</tr>
<tr>
<td>shuffled</td>
<td>0.1483</td>
<td>0.0027</td>
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<tr>
<td>surrogate</td>
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TABLE IV
WIDTH OF MULTIFRACTAL SPECTRUM, INTERNATIONAL, FINE SCALE

<table>
<thead>
<tr>
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<th>US</th>
<th>SG</th>
<th>DE</th>
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</thead>
<tbody>
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<td></td>
<td>idle</td>
<td>busy</td>
<td>idle</td>
</tr>
<tr>
<td>shuffled</td>
<td>0.1060</td>
<td>0.0182</td>
<td>0.0607</td>
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<tr>
<td>surrogate</td>
<td>0.0787</td>
<td>0.0852</td>
<td>0.0814</td>
</tr>
</tbody>
</table>

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Fig. 7. $h(q)$ of home paths, shuffled, fine scales.

Fig. 8. $h(q)$ of home paths, surrogate, fine scales.

Fig. 9. $h(q)$ of international paths, shuffled, fine scales.

Fig. 10. $h(q)$ of international paths, surrogate, fine scales.
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REFERENCES


