A NOVEL WAVELET DOMAIN STATISTICAL APPROACH FOR DENOISING SAR IMAGES

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ABSTRACT
In this paper, we present a novel Bayesian-based speckle suppression method for Synthetic Aperture Radar (SAR) images within the framework of wavelet analysis. We introduce two-dimensional Generalized Autoregressive Conditional Heteroscedasticity Mixture (2D-GARCH-M) model as an extension of two-dimensional GARCH (2D-GARCH) model and use it for statistical modeling of SAR images subbands. Similar to 2D-GARCH model, this new model can capture heavy tailed marginal distribution and the intrascale dependencies of wavelet coefficients. Also, 2D-GARCH-M model introduces additional flexibility in the model formulation in comparison with 2D-GARCH model, which can result in better characterization of SAR images’ subbands and improved restoration in noisy environments. Then, we design a Bayesian estimator for estimating the clean image wavelet coefficients. Finally, we compare our proposed method with various speckle suppression methods applied on actual and synthetic SAR images and verify the performance improvement in utilizing the new strategy.

Keywords– 2D-GARCH-M model, statistical modeling, Synthetic Aperture Radar (SAR) images, speckle.

1. INTRODUCTION
Image speckle is an inherent property of all coherent imaging systems, including synthetic aperture radar (SAR). The presence of speckle is undesirable since it degrades image quality and it affects the tasks of human interpretation and diagnosis. Many methods for speckle reduction have been proposed. There are some adaptive filtering techniques, such as Lee filter [1], which are based on computing the local statistics of the image in a fixed window. One of the major problems of these techniques is the trade-off between the image resolution and the speckle removal, arising from the selection of the window size. There has been considerable interest in using the wavelet transform as a powerful tool for recovering images from noisy data [2, 3]. Wavelet denoising procedures consist of three main steps: First, Calculate the wavelet transform of the noisy image. Second, Manipulating the wavelet coefficients. Third, Compute the inverse transform using the modified coefficients. When multiplicative contamination is concerned, multiscale methods involve a preprocessing step consisting of a logarithmic transform to separate the noise from the original image. Loosely speaking, two major denoising techniques used in this context are the thresholding technique and the Bayesian estimation shrinkage technique. However, thresholding methods have two main drawbacks: i) the choice of the threshold, arguably the most important denoising parameter, is made in an ad-hoc manner; and ii) the specific distribution of the signal and noise may not be well matched at different scales [4]. As far as Bayesian estimation is concerned, it is necessary to assume an a priori distribution $p(x)$ associated with the wavelet coefficients of the noise-free image. For a Bayesian estimation process to be successful, the correct choice of priors for wavelet coefficients is certainly a very important factor. Several different priors have been considered for the wavelet coefficients [4, 5, 6] that each of these models has some disadvantages [7]. In the recent works [7, 8], we have shown that the subband decompositions of SAR and ultrasound images have significantly non-Gaussian statistics that are best described by two-dimensional GARCH model. The one-dimensional GARCH model [9] is widely used for modeling financial time series. In this paper, we introduce two-dimensional GARCH Mixture (2D-GARCH-M) model that is more flexible and more general than 2D-GARCH model and includes the 2D-GARCH as a special case. This new model preserves the appropriate properties of 2D-GARCH for modeling the wavelet coefficients of SAR images such as capturing heavy tailed marginal distribution and the intrascale dependencies of wavelet coefficients. The 2D-GARCH-M model extends the dynamic formulation of 2D-GARCH model and enables a better fit for a process with a more complex volatility structure such as SAR images subbands. We demonstrate that 2D-GARCH-M is superior to GARCH in modeling SAR images subbands. Consequently we design a Bayesian estimator for estimating the clean wavelet coefficients of SAR images based on 2D-GARCH-M modeling. It should be mentioned that using 2D-GARCH-M model, all the formulations are different from the 2D-GARCH model. The performance of the proposed method is compared with some of the widely used denoising methods. Experimental results demonstrate the efficacy of the proposed method. This paper is organized as follows: In Section 2, we introduce the 2D-GARCH-M model. The results on modeling the subband coefficients of SAR images indicating their compatibility with 2D-GARCH-M model is discussed in section 3. Section 4 is dedicated to describing new image denoising method based on 2D-GARCH-M modeling of the image wavelet coefficients. The experimental results are presented in section 5. Finally, concluding remarks are given in section 6.

2. MODEL DEFINITION
GARCH processes are a class of stochastic processes. These are zero mean, serially uncorrelated processes with non-constant variances conditioned on the past [9]. Extending the one-dimensional GARCH model into two-dimensions has been discussed in [10]. In this section, we briefly review the 2D-GARCH model, and introduce the new 2D-GARCH-M model, which allows further flexibility in the formulation of the variation of conditional variance.

2.1. 2D-GARCH Model
Let’s consider $z_{ij}$ to represent a two-dimensional zero mean stochastic process. $z_{ij}$ follows a pure 2D-GARCH($p_1$, $p_2$, $q_1$, $q_2$) model if
\[ z_{ij} = \sqrt{h_{ij}} \varepsilon_{ij} \]  
\[ h_{ij} = \alpha_0 + \sum_{k \in \Lambda_1} \alpha_{k} z_{i-k,j-l}^2 + \sum_{k \in \Lambda_2} \beta_{k} h_{i-k,j-l}, \]  
where \( \Lambda_1 = \{k \mid 0 \leq k \leq q_1, 0 \leq l \leq q_2, (k, l) \neq (0, 0)\} \), \( \Lambda_2 = \{k \mid 0 \leq k \leq p_1, 0 \leq l \leq p_2, (k, l) \neq (0, 0)\} \), \( h_{ij} \) denotes the conditional variance of \( z_{ij} \) and \( \varepsilon_{ij} \sim N(0, 1) \) is an iid two dimensional stochastic process. Let the information set \( \psi_{ij} \) be defined as 
\[ \psi_{ij} = \{z_{i-k-j-l}, h_{i-k,j-l}, k, l \in \Lambda_1, \Lambda_2\}. \]
\[ z_{ij} \] is therefore conditionally distributed as 
\[ z_{ij} | \psi_{ij} \sim N(0, h_{ij}). \]

The model parameters, \( \Gamma = \{\alpha_0, \alpha_{q_1}, \cdots, \alpha_{q_2}, \beta_{q_1}, \cdots, \beta_{p_2}\} \), can be found using maximum likelihood estimation (MLE). From (2) it is obvious that at every spatial location, both the neighboring sample variances and the neighboring conditional variances play a role in the current conditional variance. The 2D-GARCH has been used for modeling SAR and ultrasound images subbands in [7, 8].

2.2. 2D-GARCH-M Model

2D-GARCH-M model is a mixture of some 2D-GARCH models. Extending 2D-GARCH model in such a way introduces additional flexibility in the model formulation, which may result in better characterization of SAR images’ subbands and improved restoration in noisy environments. Assuming the number of 2D-GARCH models in the mixture is equal to \( m \), we represent them by 2D-GARCH1, 2D-GARCH2, ..., 2D-GARCHm, when 
\[ \Gamma_i = \{\alpha_{q_1}, \alpha_{q_2}, \cdots, \alpha_{q_2}, \beta_{q_1}, \cdots, \beta_{p_2}\} \]
is the set of 2D-GARCH parameters. Let \( \{w_1, w_2, \ldots, w_m\} \) denote the weights of 2D-GARCH models in the mixture such that \( \sum_{i=1}^{m} w_i = 1 \), hence, the parameters of 2D-GARCH-M model are 
\[ \{\Gamma_1, \Gamma_2, \ldots, \Gamma_m, \{w_1, w_2, \ldots, w_m\}\}. \]

Now, we assume that \( z_{ij} \), a two-dimensional zero mean stochastic process, is generated by a 2D-GARCH-M model of order \( (m, p_1, q_1, p_2, q_2) \), denoted by \( z_{ij} \sim 2D-GARCH-M(m, p_1, q_1, p_2, q_2) \), which follows:
\[ z_{ij} | \psi_{ij} \sim \sum_{n=1}^{m} w_n \mathcal{N}(0, h_{ijn}) \]  
where 
\[ h_{ijn} = \alpha_{q_1} + \sum_{k \in \Lambda_1} \alpha_{q_2} z_{i-k,j-l}^2 + \sum_{k \in \Lambda_2} \beta_{p_2} h_{i-k,j-l}. \]  
where \( \psi_{ij}, \Lambda_1 \) and \( \Lambda_2 \) are as defined in section 2.1. In the right side of the above equation, we have \( h_{i-k,j-l} \), hence, we should define \( h_{ij} \) and consequently \( h_{i-k,j-l} \) : 
\[ h_{ij} = \sum_{n=1}^{m} w_n z_{ij} h_{ijn} \Rightarrow h_{i-k,j-l} = \sum_{n=1}^{m} w_n z_{i-k,j-l} h_{ijn} \]
where \( w_n z_{ij} \) denotes the weight or the probability of the nth 2D-GARCH model in the mixture conditioned on the value of \( z_{ij} \) and can be computed using:
\[ w_n z_{ij} = \frac{w_n \times \mathcal{N}(0, h_{ijn})}{\sum_{n=1}^{m} w_n \times \mathcal{N}(0, h_{ijn})} \]

Fig. 1. Modeling results of H2 subband of “piperiver” image

The above definition for \( h_{i-k,j-l} \) and using it in the formulas of \( h_{ijn} \) prevents the problem of path dependence, because each conditional variance doesn’t depend on the entire past history of the process, since the conditional variances are recombining in \( h_{ij} \). If we work with a stochastic process, such as \( y_{ij} \) that is not zero mean, first, we define \( z_{ij} \) as the innovation in a linear regression
\[ z_{ij} = y_{ij} - r_{ij} b \]
where \( r_{ij} \) is a vector of explanatory variables and \( b \) is a vector of unknown parameters. Then, \( z_{ij} \) follows a 2D-GARCH-M model and
\[ y_{ij} | \psi_{ij} \sim \sum_{n=1}^{m} w_n \mathcal{N}(\mathbf{z}_{ij}^T b, h_{ijn}), \]
where \( h_{ijn} \) can be computed using (4). The unknown parameters that should be estimated are 
\[ \Gamma = \{\Gamma_1, \Gamma_2, \ldots, \Gamma_m, \{w_1, w_2, \ldots, w_m\}\}. \]

We find the parameter set (\( \Gamma \)) using MLE.

3. 2D-GARCH-M MODELING OF SAR IMAGE SUBBANDS

Research on statistical properties of images wavelet coefficients have shown that the marginal distributions of wavelet coefficients are highly kurtotic, and can be described using suitable heavy-tailed distributions [4]. Here, we study whether the 2D-GARCH-M model provides a flexible and appropriate tool for modeling the coefficients within the framework of multiscale wavelet analysis of logarithmically transformed SAR images. We employ normalized histograms and compare the fits with that provided by the generalized Gaussian density, used in many papers for wavelet coefficients [5], and also 2D-GARCH model [7, 8]. Fig. 1 shows the histograms of wavelet coefficients of a logarithmically transformed filtered image (Ku-band SAR image over the Rio Grande River acquired by the Sandia National Laboratories twin otter aircraft despeckled using Lee filter “piperiver”). The figure shows that 2D-GARCH-M model is superior to the generalized Gaussian and the 2D-GARCH modeling because it provides a better fit to the actual data. Recent work has investigated the dependencies between coefficients [5] that these dependencies have been captured in 2D-GARCH and 2D-GARCH-M models.
4. A MAP ESTIMATOR FOR SAR SPECKLE REMOVAL

In this section, our goal is the design of a MAP estimator that recovers the signal component of the wavelet coefficients in noisy images by using a 2D-GARCH-M model. Let \( y \) and \( x \) represent a noisy observation and corresponding noise-free SAR image. Also, let \( n_{mul} \) represents the corrupting multiplicative noise component (speckle). We can write \( y = x n_{mul} \). A functional block diagram of the proposed denoising method is shown in Fig. 2. Our procedure for image denoising consists of six steps. First, we apply logarithmic transformation to the speckled SAR image \( \log(y) = \log(x) + \log(n_{mul}) \). Then, we apply DWT to the logarithm of the speckled image \( \log(y) \) up to arbitrary levels to obtain the subbands at different scales and orientation. For an arbitrary subband indicated by \( m \), we represent the DWT of \( \log(x) \), \( \log(n_{mul}) \), and \( \log(y) \) by \( X^m \), \( N^m \), and \( Y^m \) respectively; then we have

\[
Y_{ab}^m = X_{ab}^m + N_{ab}^m. \tag{5}
\]

To simplify the notation, in following parts we ignore the index \( m \). It has been shown that when the image intensity is logarithmically transformed, the speckle noise is approximately Gaussian additive noise \([4]\). Also, our processor employs the wavelet transform which, through the central limit theorem, drives the noise wavelet coefficients to approximate a Gaussian distribution, hence, in the following, we use this distribution for \( N_{ab} \). At this step, we desppecle each subband except for the lowpass residual band based on 2D-GARCH-M models. We model each subband by a 2D-GARCH-M model. Let \( \mathcal{M} \) be such a model, for each subband. Then, we express:

\[
z_{ij} = Y_{ij} - \sum_{n=1}^{m} w_n N(\varepsilon_{ij}^n, \sigma_{y_{ij}}^2)
\]

\[
Y_{ij} | \psi_{ij} \sim \sum_{n=1}^{m} w_n N(\varepsilon_{ij}^n, \sigma_{y_{ij}}^2)
\]

where \( h_{ij} = \sigma_{y_{ij}}^2 \) for \( n = 1, 2, ..., m \) are to denote the conditional variance of 2D-GARCH models in 2D-GARCH-M model of \( Y_{ij} \) and can be computed using (4). As mentioned above, we assume that the noise \( N_{ij} \) is zero mean white Gaussian noise, hence, according to (5), the conditional distribution of \( X_{ij} \) is:

\[
X_{ij} | \psi_{ij} \sim \sum_{n=1}^{m} w_n N(\varepsilon_{ij}^n, \sigma_{x_{ij}}^2) \tag{6}
\]

where \( \sigma_{x_{ij}}^2 = \sigma_{y_{ij}}^2 \) for \( n = 1, 2, ..., m \) and \( \sigma_{y_{ij}}^2 \) is the variance of noise. In some applications, the input noise variance is known, otherwise, we use the recommendation by Dohono \([2]\) for computing the input noise variance. Then, we consider the MAP estimator for estimating \( X_{ij} \) given the noisy observation, \( Y_{ij} \), that is:

\[
\hat{X}_{ij} = \max_{X_{ij}} P(X_{ij} | Y_{ij}, \psi_{ij}) \tag{7}
\]

\[
= \max_{X_{ij}} P(X_{ij} | \psi_{ij}) P(Y_{ij} | X_{ij}, \psi_{ij}) \tag{8}
\]

therefore, we need to compute \( P(X_{ij} | \psi_{ij}) \) and \( P(Y_{ij} | X_{ij}, \psi_{ij}) \).

In order to be able to quantify the improvement achieved by our method, we have first degraded an aerial "noiseless" image with synthetic speckle. Finally, for qualitative visual evaluation, we processed real SAR images with proposed method. We have simulated radar textured images by degrading aerial photographs with various levels of log-normal multiplicative noise that is a realistic speckle noise model in SAR images \([4]\). An aerial test image, corresponding simulated radar textured image, and the despeckled image using the proposed method have been shown in Fig. 3. For this experiment, we chose to compare the proposed filter with a local variance adaptive method in the pixel domain \([1]\)(Lee filter), BayesShrink \([3]\), adaptive thresholding method proposed in \([11]\), and 2D-GARCH based denoising \([7, 8]\). In order to evaluate the result of filters quantitatively, we use three parameters \( MSE \), signal-to-noise \( (S/MSE) \) ratio and \( \beta \) as defined in \([2, 7, 8]\) to evaluate the denoising methods. We use \( S/MSE \) because the \( MSE \) is not adequate to evaluate the noise suppression in the case of multiplicative noise. \( S/MSE \) corresponds to the classical SNR in the case of additive noise. In

![Fig. 2. Block diagram of the proposed algorithm for speckle suppression.](image)

![Fig. 3. Results of speckle suppressing. original, noisy, proposed method filtered images.(left to right).](image)
SAR imaging one is interested in speckle noise suppression while at the same time preserving the edges and linear structures of the original image. Therefore, another measure, $\beta$, is also considered for edge preservation that should be close to unity for an optimal effect of edge preservation. The results for the aerial image shown in Fig. 3.a are summarized in Table 1. In this table, the best value in each row is bold. It is obvious from Table 1 that the proposed image denoising method outperforms other mentioned methods. Because of limited space, we describe the results of one representative image, but denoising results of different image are similar. Finally, we apply our proposed method on actual SAR images and visually evaluate the denoised images. The test image, is a Ku-band SAR image over RFK stadium Washington, D.C., acquired by the Sandia National Laboratories twin otter aircraft. The despeckled images for the adaptive soft thresholding scheme, 2D-GARCH based denoising and the proposed method are shown in Fig. 4. Although qualitative evaluation in these cases is highly subjective, i.e., no universal quality measure for filtered SAR data exists, the results of this experiment seem to be consistent with the simulation results. As can be observed, the proposed method has performed better in reducing the speckle noise while preserving edge information.

6. Conclusion

Previously, it has been shown that 2D-GARCH is an efficient model for images subbands [7, 8]. We introduced an extension of 2D-GARCH model called 2D-GARCH-M. This new model that includes the 2D-GARCH model as a special case, extends the dynamic formulation of 2D-GARCH model and enables a better fit for SAR images subbands as a process with a more complex volatility structure. We apply this new statistical model for the wavelet coefficients of SAR images. Consequently, to denoise SAR images, we designed and tested a MAP estimator that relies on this statistical model. Experiments are also conducted on different images corrupted by various multiplicative noise levels to access the performance of proposed image denoising method in comparison with different denoising methods. Experimental results illustrate the efficacy of the proposed method.

Table 1. Image enhancement measure obtained by several denoising methods at different levels of multiplicative noise.

<table>
<thead>
<tr>
<th></th>
<th>Noisy</th>
<th>Lee</th>
<th>Bayes</th>
<th>Adap.</th>
<th>GARCH based</th>
<th>Propo.</th>
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<tbody>
<tr>
<td>MSE</td>
<td>98.740</td>
<td>42.234</td>
<td>44.452</td>
<td>40.128</td>
<td>38.835</td>
<td>36.234</td>
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<tr>
<td>S/MSE</td>
<td>13.690</td>
<td>17.378</td>
<td>17.156</td>
<td>17.600</td>
<td>17.743</td>
<td>18.044</td>
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<tr>
<td>$\beta$</td>
<td>0.6097</td>
<td>0.6346</td>
<td>0.6456</td>
<td>0.7020</td>
<td>0.7001</td>
<td>0.7282</td>
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<tr>
<td>MSE</td>
<td>75.934</td>
<td>39.358</td>
<td>39.388</td>
<td>35.269</td>
<td>33.320</td>
<td>31.541</td>
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<tr>
<td>S/MSE</td>
<td>14.830</td>
<td>17.685</td>
<td>17.685</td>
<td>18.161</td>
<td>18.408</td>
<td>18.646</td>
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<tr>
<td>$\beta$</td>
<td>0.6649</td>
<td>0.6546</td>
<td>0.6816</td>
<td>0.7308</td>
<td>0.7360</td>
<td>0.7598</td>
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<tr>
<td>MSE</td>
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<td>37.289</td>
<td>34.726</td>
<td>30.617</td>
<td>29.491</td>
<td>27.547</td>
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<td>S/MSE</td>
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<td>17.919</td>
<td>18.228</td>
<td>18.775</td>
<td>18.938</td>
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<td>$\beta$</td>
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7. References