The nonholonomic nature of human locomotion: a modeling study

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Abstract—This work presents a differential system which accurately describes the geometry of human locomotor trajectories of humans walking on the ground level, in absence of obstacles. Our approach emphasizes the close relationship between the shape of the locomotor paths in goal-directed movements and the simplified kinematic model of a wheeled mobile robot. This kind of system has been extensively studied in robotics community. From a kinematic perspective, the characteristic of this wheeled robot is the nonholonomic constraint of the wheels on the floor, which forces the vehicle to move tangentially to its main axis. Humans do not walk sideways. This obvious observation indicates that some constraints (mechanical, anatomical...) act on human bodies restricting the way humans generate locomotor trajectories. To model this, we propose a differential system that respects nonholonomic constraints. We validate this model by comparing simulated trajectories with actual (recorded) trajectories produced during goal-oriented locomotion in humans. Subjects had to start from a pre-defined position and direction to cross over a distant porch (position and orientation of the porch were the two manipulated factors). Such comparative analysis is undertaken by making use of numerical methods to compute the control inputs from actual trajectories. Three body frames have been considered: head, pelvis and trunk. It appears that the trunk can be considered as a kind of a steering wheel that steers the human body with a delay of around 0.2 second. This model has been validated on a database of 1,560 trajectories recorded from seven subjects. It opens a promising route to better understand the human locomotion via differential geometry tools successfully experienced in mobile robotics.

Index Terms—Human locomotion, nonholonomic mobile robots, path integration

I. PROBLEM STATEMENT, RELATED WORK AND CONTRIBUTION

This paper deals with human locomotion. It investigates a simple statement: the human beings usually walk forward and the direction of their body is tangent to the trajectories they perform (neglecting fluctuations induced by steps alternation). This coupling between the direction $\theta$ and the position $(x, y)$ of the body can be summarized by the following differential equation: $\tan \theta = \frac{x}{y}$. It is known that this differential equation defines a non integrable 2-dimensional distribution in the 3-dimensional manifold $R^2 \times S^1$ gathering all the configurations $(x, y, \theta)$: the coupling between the direction and the position is said to be a nonholonomic constraint. A basis of the distribution is done by the two following vector fields:

$$
\begin{pmatrix}
\cos \theta \\
\sin \theta \\
0
\end{pmatrix}
\quad \text{and} \quad
\begin{pmatrix}
0 \\
0 \\
1
\end{pmatrix}
$$

supporting the linear velocity and the angular one respectively. Both linear and angular velocities appear as the only two controls that perfectly define the shape of the paths in the 3-dimensional manifold $R^2 \times S^1$. This rough analysis is done at a macroscopic level. The purpose of this paper is to refine the analysis and to answer the following question: what is the body frame that better accounts for the nonholonomic nature of the human locomotion? Goal-oriented locomotion has mainly been investigated with respect to how different sensory inputs are dynamically integrated, facilitating the elaboration of locomotor commands that allow reaching a desired body position in space [1]. Visual, vestibular and proprioceptive inputs were analyzed during both normal and blindfolded locomotion in order to study how humans could continuously control their trajectories (see [6] and for a review, see [3]). However, which principles govern the generation (or planning) of whole body trajectories has received little attention. Recently, it was shown that common principles govern generation of hand trajectories and whole body trajectories [4], [5]. In particular, a strong coupling between path geometry (curvature profile) and body kinematics (walking speed) was observed with some quantitative differences between the two types of movements [5].

It can be argued that geometric configurations of human bodies are constrained, at the joint level, by anatomical parameters that limit a given rotation of a body segment within a certain space. For example, abduction/adduction movements of a given leg cannot cover a wide range of spatial configurations as it can be the case for the shoulders segment. Ground reaction forces also act first at the legs level and constraint indirectly the center of mass trajectory. Such a mechanical point of view has been investigated in biomechanics for the study of the human locomotion (see for instance [12]), in computer animation (see for instance [9]) and in robotics for the study of the humanoid robots locomotion (see for instance the pioneering work [10] or the more recent worked out example of HRP robot [7]).
The point of view addressed in this paper differs from the previous ones. We do not consider neither the sensory inputs nor the complexity of mechanical system modeling the human body. The point of view is complementary and more macroscopic than the standard biomechanics approaches. We want to take advantage of the observation of the shape of the locomotion trajectories in the simple 3-dimensional space of both the position and the orientation of the body. We show that the shape of the human trajectories can be described by a simple differential system. The differential model we propose opens an original bridge between the researches performed in the human physiology and the mathematical background developed on the nonholonomic systems in mobile robotics. This point of view constitutes the first contribution of the paper. The most popular nonholonomic system is a rolling vehicle. This vehicle rolls without sliding. This non-sliding constraint defines the distribution of Eq. (1). Motion planning and control for rolling vehicles is an active research area in mobile robotics [2], [8]. The controls of a vehicle are usually the linear velocity (via the accelerator and the brake) and the angular velocity (via the steering wheel). The question addressed in this paper can be roughly formulated as: where is the “steering wheel” of the human body located? Several body frames have been considered on the human skeleton (head, pelvis and trunk). The conclusion of our experimental study is to show, first that there exists a body frame that accounts for the nonholonomic nature of the human locomotion and second that the trunk is the best “steering wheel” compared to the head and the pelvis.

The following section presents the experimental protocol. This protocol is original. This is the first one that considers the problem of the shape of the human locomotion trajectories just defined by a goal to be reached in both position and orientation. Then we present the data analysis and processing in Section III. A comparative study involving head, pelvis and trunk frames is presented in Section IV. The differential model of Eq. (1) is instantiated with the trunk frame in Section V. By integrating such a differential model we show that the simulated trajectories fit with the real ones from a statistical study including 1,560 trajectories performed by seven subjects. The conclusion develops the interest of the proposed differential model for future research directions.

II. APPARATUS AND PROTOCOL

We used motion capture technology to record the trajectories of body movements. Subjects were equipped with 34 light reflective markers located on their head and bodies. The sampling frequency of the markers was 120 Hz using an optoelectronic Vicon motion device system (Vicon V8, Oxford metrics) composed of 24 cameras. It is important to mention that we do not apply any kind of filter to raw data in our analysis (see Fig. 1).

To examine the geometrical properties of human locomotor paths, actual trajectories were recorded, in a large gymnasium in seven normal healthy males who volunteered for this study. Their ages and heights ranged from 25 to 30 years and from 1.60 to 1.80 m respectively.

In order to specify the position of the subject on the plane we established a relationship between the laboratory’s fixed reference frame and the trajectory’s reference frame which can be computed using either head, trunk or pelvis markers as we explain in Section III. Hence, the configuration A of the subject is described as a 3-vector \((x_a, y_a, \theta_a)\).
In the experiment, subjects walked from the same initial configuration $A_{\text{init}}$ to a randomly selected final configuration $A_{\text{final}}$. The target consisted in a porch which could be rotated around a fixed point to indicate the desired final orientation (see Fig. 2). The subjects were instructed to freely cross over this porch (from $A_{\text{init}}$ to $A_{\text{final}}$) without any spatial constraints relative to the path they might take. Subjects were allowed to choose their natural walking speed in order to perform the task.

The final orientation varies from $-\pi$ to $\pi$ in intervals of $\frac{\pi}{6}$ at each final position. In order to exclude the positive and negative acceleration effects at the beginning and at the end of trajectories, the subjects started to walk straight ahead one meter before the initial configuration $A_{\text{init}}$ and stopped two meters after passing through the porch (see Fig. 3.a). Thus, the first and the last steps are not considered in this study.

The experiment was carried out in seven sessions. The first subject was asked to perform 480 different trajectories in two sessions. The starting point $A_{\text{init}}$ was always the same while the target $A_{\text{final}}$ was randomly selected at each trial (see Fig 3.b).

The 6 other subjects were asked to perform 180 different trajectories during the 6 sessions. Each subject performed 3 trials for a given configuration of the porch $A_{\text{final}}$. Therefore, they walked 180 trajectories with only 60 different final configurations.

### III. Frames and Data Analysis

#### A. Global, head, trunk and pelvis coordinate frames

While walking, the body generated trajectories in the space relative to the laboratory’s reference frame $LRF$. To describe the movement of the body, a local reference frame was defined (see Fig. 4). Three body coordinate frames were used for the head $RF_H$, the trunk $RF_T$ and the pelvis $RF_P$ respectively. The origins of $RF_H$, $RF_T$ and $RF_P$ and their orientations have been determined from the markers’ coordinates.

To represent the origin $x_H,y_H$ of $RF_H$, the markers located on the back and the forehead are used. The orientation $\varphi_H$ of $RF_H$ is easily identified according to the segment whose endpoints are the back and the forehead markers. Therefore, the desired orientation is merely the rigid body transformation of $RF_H$ onto $LRF$.

The midpoint of the shoulder markers and the direction orthogonal to the shoulder axis are considered to represent the origin $x_T,y_T$ and the orientation $\varphi_T$ of $RF_T$ respectively. Finally, to find the origin $x_P,y_P$ and the orientation $\varphi_P$ of $RF_P$, four markers are used, left and right-front, left and right-back. These markers are located on the bony prominences of the pelvis.

#### B. Data processing

Numerical computation is performed to obtain the walking velocity profile. Each recorded trajectory is represented as a sequence of discrete points on the plane. We computed the linear $v$ and angular $\omega$ velocities at each point such that

$$v(t) \leftarrow \left( \frac{x(t+\Delta t) - x(t-\Delta t)}{2\Delta t}, \frac{y(t+\Delta t) - y(t-\Delta t)}{2\Delta t} \right)$$

(2)
Fig. 4. Definition of the local frames and their trajectories projected on the ground. All of them correspond to the same motion. (a) shows a 3D reconstruction of human body from the markers. (b) shows the trajectory followed by the head reference frame and its orientations. (c) shows the trajectory followed by the trunk reference frame and its orientations. (d) shows the trajectory followed by the pelvis reference frame and its orientations.

\[
\omega(t) = \frac{\varphi(t + \Delta t) - \varphi(t - \Delta t)}{2\Delta t} \tag{3}
\]

where \(x(t), y(t)\) and \(\varphi(t)\) are the configuration parameters of the body along the trajectory. Therefore, these parameters describe the motion of any of the three \(RF_H\), \(RF_T\) or \(RF_P\) local frames. We computed the desired tangential direction \(\theta(t)\) along the path as

\[
\theta(t) = \frac{y(t + \Delta t) - y(t - \Delta t)}{x(t + \Delta t) - x(t - \Delta t)} \tag{4}
\]

It is important to note that \(\varphi(t)\) has been calculated from the markers while \(\theta(t)\) is computed from the sequence of discrete points \(x(t), y(t)\). We used Eq. (3) to obtain the instantaneous variation of \(\theta(t)\) replacing \(\omega(t)\) and \(\varphi(t)\) with \(\dot{\theta}(t)\) and \(\theta(t)\) respectively.

IV. COMPARISON BETWEEN HEAD, TRUNK AND PELVIS DIRECTIONS

The purpose of this section is to analyze the timing of the three different orientation parameters \(\varphi_H(t), \varphi_T(t)\) and \(\varphi_P(t)\). This quantitative and qualitative analysis is done to determine which of them better approximates \(\theta(t)\).

To accomplish such evaluation, we performed some tests and measurements for the different reference frames \(RF_H\), \(RF_T\) and \(RF_P\): the direction of the head, trunk and pelvis versus \(\theta(t)\) while steering along a path.

A. Head direction profile

Defining \(RF_H\) as the local coordinate frame it is noted that \(\varphi_H(t)\) points most of the time towards the direction of the target as it is illustrated in Fig. 4.b. Furthermore, there are some cases where \(\varphi_H(t)\) is pointing to the opposite half-plane with respect to \(\theta(t)\). For instance, analyzing the behavior of \(\varphi_H(t)\) and \(\theta(t)\) in the trajectory of Fig. 4.b, it can be shown that \(\varphi_H(t)\) and \(\theta(t)\) follow a similar trace until 0.55s and just after that both directions start to diverge until 1.4s (see Fig 5).

B. Trunk direction profile

Choosing \(RF_T\) rather than \(RF_H\), we observed that for every trajectory the curves traced by \(\varphi_T(t)\) and \(\theta(t)\) had a similar form. However, comparing \(\varphi_T(t)\) and \(\theta(t)\) in time, it is noted that \(\varphi_T(t)\) is shifted between \(\frac{1}{4}\) and \(\frac{1}{4}\) s backward (see Fig. 6). It means that the trunk as well as the head anticipates the orientation relative to the current walking direction. In other terms

\[
\varphi_T(t + \epsilon) \simeq \dot{\theta}(t)
\]

Fig. 5. Head direction profile with respect to the tangential direction respectively. Both of them correspond to the same motion.
C. Pelvis direction profile

Examining $\varphi_P(t)$ relative to $\theta(t)$ while steering along a path, we observed that $\varphi_P(t)$ oscillates with amplitude close to 15 degrees even along a curve (see Fig. 7). These instantaneous variations reflect the significant influence of the gait cycle at each step. To fit the curves of $\varphi_P(t)$ in agreement with the shape of $\theta(t)$, these oscillations were averaged out by further filtering $\varphi_P(t)$ using a fourth-order low-pass filter algorithm with a cut-off frequency of 0.5 Hz.

where $\epsilon$ represents the time shifted backwards.

V. MODEL

To measure the error of the approximation $\dot{\varphi}_T(t + \epsilon)$ we defined $RF_T$ as the local reference frame of the body to perform numerical integration using the following kinematic model:

$$
\begin{bmatrix}
\dot{x}_T \\
\dot{y}_T \\
\dot{\varphi}_T
\end{bmatrix} = 
\begin{bmatrix}
\cos \varphi_T \\
\sin \varphi_T \\
0
\end{bmatrix} v + 
\begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix} \omega
$$

(5)

The control inputs $v$ and $\omega$ are the linear and angular velocities respectively. The nonholonomic constraint imposed by the control system (5) is due to the wheels on the floor, which force the mobile robot to move tangentially to its main axis. Such constraint is expressed by the following equation:

$$
y_T \cos \varphi_T - x_T \sin \varphi_T = 0
$$

To validate the model, we computed $v(t)$ and $\omega(t)$ from Eqs. (2) and (3) to obtain the control inputs of the actual locomotor trajectory expressed by $RF_T$. Then, we integrated the differential system (see Eq. (5)) using the control inputs and considering $\epsilon$. Finally, we calculated the distance error point by point between the actual and the simulated trajectories. Then, we computed the mean distance error dividing the sum of the errors by the number of points (see Fig. 8).

This procedure has been executed for 1,560 trajectories performed by seven subjects. The length of the trajectories ranged between 3 and 9 meters. The walking speed of the subjects was equal to 1.26 ±0.3 meters/seconds (m/s).

We represented the mean distance errors as a point. It is interesting to note that the model approximates 87 percent of trajectories with a precision error $<10$cm. Consequently, $\dot{\varphi}_T(t + \epsilon)$ satisfies the model of the vehicle.
VI. CONCLUSION

This model shows that human locomotion can be approximated by the motion of a nonholonomic system. Indeed, we were able to predict more than 87 percent of the 1560 trajectories recorded in 7 subjects during walking tasks with a <10 cm accuracy. Thus, nonholonomic constraints, similar to that described in wheeled robots, seem to be at work during human locomotion. Nevertheless, choosing different body reference frames yields different results. We obtained the best results using the shoulder’s segment. It appears that yaw oscillations induced by step alternation differently affect the head, trunk or pelvis movements so that only the shoulders’ midpoint trajectory was good fitted by our model’s predictions. Further investigation is required to account for these differences.

The present model will be the starting point of the next stage of our work where we plan to provide further evidence and details about how nonholonomic constraints are exerted during the generation of human locomotor trajectories. Our current model does not explain the geometric shape of the locomotion trajectories. Why in some cases (see Figure 4) we are turning first on the right to finally reach a goal whose position is on the left of the our starting configuration? Such a difficult question is related to optimal control theory (e.g. [11]) already successfully applied to mobile robotics (e.g. [8]). The application of these tools to the understanding of human locomotion opens an original promising route which is currently under development.

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