A Typical Cooperative MIMO Scheme in Wireless Ad Hoc Networks and Its Channel Capacity

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Abstract—In this paper we proposed a typical cooperative MIMO system grounded on wireless mobile ad hoc networks and brought forward the problem of time efficiency in cooperative MIMO system. And then adopting time efficiency, we analyzed the Shannon capacity limit of the cooperative MIMO system. The analysis shows two implications. First, only when intra-cluster channel is better enough than inter-cluster channel, cooperative MIMO can bring increment of channel capacity; second, there should be an optimal number of cooperative partners in a cooperative MIMO system. For instance it’s optimal to use 3 cooperative partners in the proposed typical cooperative MIMO system, when it can achieve a channel capacity increment of about 2bps/Hz compared with direct transmission.

Keywords—Wireless communication; Cooperative MIMO; Channel capacity; Wireless ad hoc networks

I. INTRODUCTION

The future wireless communication systems will be quite different from the 1G and 2G mobile communication systems. It will provide high data rate multimedia service, data service as well as voice service. The requirements for ever increasing data rates seem to act as one of the major drivers of future communication systems. MIMO (Multiple Input-Multiple Output) techniques were already known to boost the system capacity but, to build up the independent MIMO channel, the distance between the antenna have to be larger than the correlation distance, e.g. half of the wavelength. With limited terminal size, the provision of multiple antennas was a considerable implementation problem. Though, for spectral reasons, future wireless communication system will likely operate at higher frequencies, for instance, the next generation WLANs will operate beyond 5GHz (at 17 or 24GHz), which will make the correlation distance smaller, but the transmission of high frequency signal will be close to LOS (Line of sight) transmission, and with poor scattering. Then we’ll face a poor scattering/rich array situation as opposed to the rich scattering/poor array situation before [1].

Recent work [2]-[5] has shown the potential of cooperative MIMO to overcome this poor scattering/rich array or rich scattering/poor array situation. It allows multiple nodes to emulate antenna arrays and perform transmission or reception cooperatively. And hence resolves the problem that mobile nodes have no antenna arrays. Compared with the physical arrays systems, the cooperative MIMO system can be developed and realized more easily, and that’s why it has attracted so many scholars interests to study on its key techniques and communication protocols[2]-[7]. However, the capacity of cooperative MIMO system is still a theoretically unsolved problem. Though some papers had studied this problem, yet they either took the assumption that the communication between the cooperative partners can be neglected, e.g. [7], which is practically not the case, or use a simple system model that only studied the two cases of cooperative transmit and cooperative receive [8], or only the capacity of relay channel is studied [9] [10]. In this paper we take into account the process of communication between cooperative partners, and proposed the problem of time efficiency in the cooperative MIMO system. Then we try to deduce the Shannon capacity of the cooperative MIMO channel based on Telatar’s [11] and Mohinder’s [12] brilliant work, via which we give some implications on when and how to use cooperative MIMO. This is the main contribution of this paper.

The rest of this paper is as followings, in section II we proposed and modeled a cooperative MIMO system on the ground of wireless mobile ad hoc networks, and then in section III described its procedure. In section IV, we calculated the channel capacity using the Shannon capacity theory and then gave implications on the suitable cases to use cooperative MIMO and the optimal number of cooperative partners by checking some numerical results. Section V gave the conclusion of this paper.

II. SYSTEM MODEL

Wireless mobile ad hoc networks (MANETs) are comprised by a set of devices without infrastructure. With its easy development and realization, the MANETs are potentially widely used in the future wireless communication systems. So in this paper we proposed a typical cooperative MIMO scheme used in the MANETs. Fig. 1 gives the structure model of this cooperative MIMO system. Source nodes $S_0$ and its neighbors formed a transmit cluster, while the destination node $D_0$ and its neighbors formed a receive cluster. The transmit cluster and the receive cluster will help $S_0$ and $D_0$ respectively to transmit and receive data. We assume there are $N$ nodes (including $S_0$) in the transmit cluster and $M$ nodes (including $D_0$) in the receive cluster. For explicitly explore the advantages that brought by the pure cooperative MIMO technique, we adopt the following
assumptions. (i) Every node in this network has only one antenna. (ii) There is only one frequency channel in the network, and the nodes share the media via time division. (iii) For local transmission (both in transmit cluster and in receive cluster), we assume a channel with AWGN (Additive White Gaussian Noise). For long haul transmission, we assume a Rayleigh slow fading channel, that is, the channel coefficients follow the complex Gaussian distribution and change with time slowly. This assumption will be further explained later in this paper. (iv) Every node has two levels of transmit power, one for local transmission and the other for long haul transmission. And there is a total power constraint for long haul transmission. That is to say, the intra-cluster received signal noise ratio and inter-cluster received signal noise ratio only depends on the conditions of intra-cluster channel and inter-cluster channel respectively. For brevity, in the rest of this paper we will use “intra-cluster SNR” and “inter-cluster SNR” in place of “the received signal noise ratio in the intra-cluster transmission” and “the received signal noise ratio in the inter-cluster long haul transmission” respectively.

![Diagram of a typical cooperative MIMO system](image)

**Figure 1.** Structure model of a typical cooperative MIMO system

### III. PROCEDURE OF COOPERATIVE MIMO

Cooperative MIMO takes advantage of the thought of MIMO, and different from the IEEE 802.11 scheme of collision and back off, the nodes in cooperative MIMO system receive the signal they heard without detecting collision. The signal received by different cooperative nodes will be used in the joint detection to obtain the original information from source. The detailed procedure of our proposed cooperative MIMO can be divided into three phases:

1) **Data distribution in the transmit cluster.** This phase is composed by N-1 time slot. In slot $i$, node $S_i$ will transmit package $i$ to node $S_j (i=1,2,...,N-1)$, see fig. 1. Each node will encode the package it received according to its role in the system [6]-[8]. Package $i$ and package $k$ ($i \neq k$) can be identical (increasing reliability via diversity) or different (increasing capacity via multiplexing).

2) **Data transmission from transmit cluster to receive cluster.** In this phase, the cooperative transmit nodes transmit the encoded packages that they received in the first phase to the receive cluster in one time slot.

3) **Data assemblage in the receive cluster.** This phase is composed by M-1 time slot. In slot $j$, cooperative transmit node $D_j$ transmit the signal it received in the second phase to the destination node $D_k (j=1,2,...,M-1)$. The destination may use some detection algorithms, e.g. [13] [14], to attain the original information from the source. Detection algorithms are beyond this paper, the above two papers and their references are suggested to read for more detailed information.

### IV. CAPACITY ANALYSIS

#### A. MIMO channel capacity

We can see that the nodes in transmit cluster and those in the receive cluster form an $N \times M$ MIMO channel. Assuming that the receive side has the channel matrix while the transmit side doesn’t have, we invoke Telatar’s landmark MIMO capacity theorem [11], i.e.

$$C_{N,M} = E_2[m \log_2(1 + \frac{\rho}{N})] = \frac{E_m \log_2(1 + \frac{\rho}{N})}{\text{pdf}(\lambda)d\lambda}$$

where $m=\min \{M,N\}$ ; $\rho = \frac{S}{N_0}$ represents the signal-to-noise ratio, $S$ is the average transmit power, $N_0$ is the average noise power in the receive node. $\lambda$ is the unordered eigenvalue of the associated Wishart matrix $W$, which is $HH^H$ when $N > M$ , and $H^H$ when $N \leq M$ . $H \in C^{N \times M}$ is the MIMO channel matrix. pdf$(\lambda)$ is the probability density function of $\lambda$.

#### B. Time efficiency

The major difference between MIMO and cooperative MIMO lies that the cooperative MIMO needs additional intra-cluster information exchanges, which will take some additional time and decrease the system capacity. Here we adopt the term time efficiency to denote this problem. And the definition of time efficiency is as follows.

$$\eta = \frac{T_{\text{MIMO}}}{T_{\text{Coop}}} = \frac{C_{\text{Coop}}}{C_{N,M}}$$

Where $T_{\text{MIMO}}$ and $T_{\text{Coop}}$ is the time consumed by the exact MIMO system and the cooperative MIMO system respectively when the same amount of data are sent. $C_{\text{Coop}}$ is the channel capacity of the cooperative MIMO system, which is our target in this paper.

Given the average length of each package is $L$ bits and the bandwidth in this network is $W$ Hz, to transmit $N$ packages, the time taken by the $N \times M$ MIMO system is:

$$T_{\text{MIMO}} = \frac{NL}{C_{N,M}W}$$

while that time consumed by cooperative MIMO system composes of the time used in the three phases, which is denoted as $T_1$, $T_2$ and $T_3$ respectively, thus we have:

$$T_{\text{Coop}} = T_1 + T_2 + T_3 = \frac{(N-1)L}{C_{W}} + \frac{NL}{C_{N,M}W} + \frac{(M-1)L}{C_{W}}$$

where $C_s$ is the capacity of the intra-cluster channel. In the continuous transmission the information exchange in the
transmit cluster and receive cluster can be operated at the same time without interfering because the lower transmit power is used in local transmission. So $T_i$ can overlay $T_j$ when $N>M$, or vice versa, then we have

$$T_{coop} = \frac{(n-1)L}{C'_W} + \frac{NL}{C_{N,M}'}, \quad \text{where} \ n = \max\{M,N\}$$

So the time efficiency can be expressed as

$$\eta = \frac{T_{coop}}{T_{coop}} = \frac{1}{1 + \frac{n-1}{C_{N,M}'} C_s}$$

(3)

And the actually channel capacity of the cooperative MIMO system can be written as

$$C_{coop} = \eta C_{N,M}$$

(4)

From (3) we can see that when $C_s \gg C_{N,M}'$, $\eta$ will approach to 1, which means that when the intra-cluster channel is far better than inter-cluster channel the channel capacity of cooperative MIMO approach to that of the exact MIMO system.

C. The closed-form expression of the cooperative MIMO channel capacity

In this part we will deduce the closed-form expression of the (4). Referring [11] and [15], the idea is to calculated the closed-form of $pdf(\lambda)$ first and then get that of $C_{N,M}'$, thus get the closed-form expression of $C_{coop}$ according to (4).

Because the average distance between the same cooperative cluster nodes is small compared to the distance between the transmit cluster and receive cluster, which allows us to assume that each sub-channel obeys approximately the same statistics. We will further assume that this statistics is complex Gaussian, i.e., Rayleigh. Under these assumptions, the definition of $pdf(\lambda)$ is as follows [11]:

$$pdf(\lambda) = \frac{1}{m} \sum_{k=0}^{m-1} \frac{k!}{(k+n-m)!} \left(\frac{2}{k+n-m+1}\right)^{n-m} e^{-\lambda}$$

(5)

where $L_2^{n-m}(\lambda)$ is the associated Laguerre polynomial of order $k$:

$$L_2^{n-m}(\lambda) = \sum_{n=0}^{k} \frac{(-1)^n}{(k-n)!} \frac{(k+n-m)!}{(k-n)!} \lambda^n$$

(6)

Let $d = n-m$, and insert (6) to (5), we’ll get (7), where

$$\Psi_1(\alpha) = \frac{(k+d)!}{(k-1)!(d+1)!}$$

(7)

And Inserting (7) to (1) gives (8), where $a = \frac{1}{N} \rho C_s$

$$\Psi(\alpha) = \frac{1}{\ln(2)} \int_0^\infty \Psi_1(\alpha) e^{-a \lambda} d\lambda$$

(8)

The remaining problem is to calculate $\Psi_1(\alpha)$. And because

$$\Pi(\alpha) = \int_0^\alpha \Psi_1(\alpha) e^{-a \lambda} d\lambda$$

where $\ln(2) = Ei(-\alpha)$ where real($x$) is the real part of x.

Furthermore, for $\ell \geq 1$

$$\Psi_{\ell+1}(\alpha) = \int_0^\infty \Psi_1(\alpha) e^{-a \lambda} d\lambda$$

Inserting (9) and (11) to (10) gives (12).

$$\Psi_{\ell}(\alpha) = \int_0^\infty \sum_{n=0}^{\infty} \frac{(-1)^n}{(n+\ell)!} e^{-a \lambda} d\lambda$$

(12)

Thus, we’ll get $C_{N,M}'$ by inserting (12) to (8).

Because the intra-cluster channel is with AWGN, supposing the intra-cluster SNR is $X$, the capacity of intra-cluster channel can be calculated by

$$C_i = \log_2(1+X)$$

(13)

The flowchart in fig. 2 summaries the method to obtain $C_{coop}$, namely the channel capacity of cooperative MIMO system.

D. Numerical results

Having got the closed-form expression of the channel capacity, we’ll check the numerical results of the cooperative MIMO channel capacity, based on which we’ll analyze the factors that affect the capacity of cooperative MIMO and give suggestions on when to use the cooperative MIMO and how to use it. For brevity and without any loss of generality, we only consider the case of $M=\infty$.

Firstly, we’ll check the effect on the channel capacity brought by the states of the intra-cluster channel and inter-cluster channel.
Fig. 3 gives the channel capacity variations of the cooperative MIMO system with different inter-cluster SNRs and intra-cluster SNRs. It shows that the cooperative MIMO can’t bring increment of channel capacity all the time over the direct transmission (the case of \(M=N=1\)). The channel capacity of cooperative MIMO equals that of the direct transmission when the intra-cluster SNR approximates the inter-cluster SNR. If intra-cluster SNR is bigger than inter-cluster SNR the cooperative MIMO will bring increment of channel capacity, and the more cooperative partners are used the more increment is got. On the other hand, if the intra-cluster SNR is smaller than the inter-cluster SNR, the cooperative MIMO will bring decrement of channel capacity, and more cooperative partners will bring more decrement. In this figure we only draw the cases that the number of cooperative partners is 2 and 3. In fact, it is also true when the number of cooperative partner increase. Fig. 4 can show this problem more explicitly, which is got by cutting fig. 3 at the line of inter-cluster SNR equals 20dB.

So whether to use the cooperative MIMO will depend on the conditions of inter-cluster channel and intra-cluster channel. More specifically, when the intra-cluster SNR is bigger than inter-cluster SNR cooperative MIMO can be used to increase channel capacity. Transmit powers for intra-cluster transmission and inter-cluster transmission being fixed, the SNR mainly depend on the transmission distance for wireless ad hoc networks that operate on open field. Under these assumptions, we can theoretically draw the conclusion that when the distance between cooperative nodes is shorter than the distance between clusters, cooperative MIMO can bring increment of channel capacity.
appears optimal to use 3 cooperative partners. We will further explain this problem in our coming work.

![Figure 5. Channel capacity vs. inter-cluster SNR](image)

![Figure 6. Capacity-power efficiencies](image)

V. CONCLUSIONS

In this paper we considered the intra-cluster information exchange and explored the channel capacity of a typical cooperative MIMO system. Then we deduced a closed-form expression of the channel capacity of cooperative MIMO systems. From the former analysis, we can draw the following conclusions. First, cooperative MIMO can increase the channel capacity, even if no extra spectral band is employed. And the capacity is mainly affected by the number of cooperation partners, inter-cluster channel state and intra-cluster channel state. Second, cooperative MIMO won’t increase the channel capacity under all conditions. Whether the capacity can be increased depends on the relation between inter-cluster channel and intra-cluster channel. Generally speaking, the better the intra-cluster channel compared with the inter-cluster channel, the greater increment of channel capacity will be brought by the cooperative MIMO. Third, it’s true that more cooperative nodes bring more increment of channel capacity when the intra-cluster SNR is better enough than inter-cluster SNR, but it is not pro rata. And considering the energy consumption, there will be an optimal number of cooperative partners. For instance, under the scenario in this paper, the optimal number of cooperative partner is 3. Using the cooperative MIMO technique properly can increase the channel capacity, which provides us a new way to realize high-speed communication in poor wireless scenarios.

The cooperative MIMO is a relative novel research area, and its application will face many challenges, such as effective partner selection algorithm, cooperative space-time coding, joint detection of received signal, link-adaptive cooperative MIMO protocols, and so on. However, thanks to that many fruits in the research of MIMO can be used in the cooperative MIMO system, it will hopefully have a rapid development in the next decade.

REFERENCES