Interference-Limited Resource Optimization in Cognitive Femtocells with Fairness and Imperfect Spectrum Sensing

Haijun Zhang, Member, IEEE, Chunxiao Jiang, Member, IEEE, Xiaotao Mao, and Hsiao-Hwa Chen, Fellow, IEEE

Abstract—Cognitive radio enabled femtocell is regarded as a promising technique in wireless communications, and many works have been reported on its resource allocation and interference management. However, fairness and spectrum sensing errors were ignored in most of the existing works. In this paper, we propose a resource allocation scheme for orthogonal frequency division multiple access (OFDMA) based cognitive femtocells. The target is to maximize the total capacity of all femtocell users (FUs) under given QoS and co-tier/cross-tier interference constraints with imperfect channel sensing. To achieve the fairness among FUs, the minimum and maximum numbers of sub-channels occupied by each user are considered. First, the sub-channel and power allocation problem is modeled as a mixed integer programming problem, and then it is transformed into a convex optimization problem by relaxing sub-channel sharing and applying co-tier interference constraints, which is finally solved using dual decomposition method. Based on the obtained solution, an iterative sub-channel and power allocation algorithm is proposed. The effectiveness of the proposed algorithm in terms of capacity and fairness compared to the existing schemes is verified by simulations.

Index Terms—Cognitive femtocell; fairness; imperfect spectrum sensing; OFDMA; power control; interference.

I. INTRODUCTION

Driven by smart mobile devices and other bandwidth consuming applications, 80% of mobile data traffic occurs indoors, which leads to a heavy burden to macrocells because the coverage for indoor users is not good enough. To offload the overloaded traffics in macrocells and enhance the capacity of wireless networks, one of the most effective methods is to shorten the distance between base stations and user equipment. Femtocell, which is considered as a promising technique, can provide an effective solution to tackle the challenges in this respect [1]. Hence, it is not surprising that femtocell has attracted a lot of interests in last a few years in both industry and academia since it improves the coverage and spatial reuse of spectrum with low power transmission and low infrastructure deployment cost. However, many critical issues on femtocells should be addressed to fully reap their potential gains, such as interference mitigation, spectrum access, resource allocation, and QoS provisioning [2]–[6].

On the other hand, cognitive radio (CR) is an emerging technology that plays an important role in improving efficiency of spectrum access in femtocell networks [7]. The cognitive capabilities can further improve the spectrum efficiency, wireless resource utilization, and interference mitigation by efficient spectrum sensing, interference sensing, and adaptive transmission [8]. Therefore, femtocell combined with cognitive radio can further improve the system performance [9] [10]. In this paper, a cognitive radio enabled femtocell architecture is designed to opportunistically access the spectrum via cognitive femtocell base stations (FBS).

It is also noted that cognitive femtocells working jointly with orthogonal frequency division multiple access (OFDMA) can improve cellular coverage and offload traffics from existing primary macrocells via resource allocation and interference mitigation. In [11], the authors investigated the resource allocation problems based on multistage stochastic programming for stringent QoS requirements of real-time streaming scalable videos in femtocell cognitive radio networks. In [12], the issues on CR-enabled spectrum sensing and interference mitigation were investigated, where interference coordination approach was applied. Opportunistic cooperation between secondary (femtocell) users and primary (macrocell) users was proposed for cognitive femtocell networks based on a generalized Lyapunov optimization technique [13]. In [14], a spectrum-sharing scheme between primary macrocell and secondary femtocell was proposed, and the bounds on maximum intensity of simultaneously transmitting cognitive femtocells that satisfy a given per-tier outage constraint in these schemes were theoretically derived using a stochastic geometry model. In [15], interferences due to different interfering sources were analyzed within cognitive-empowered femtocell networks, and a stochastic dual control approach was introduced for dynamic sensing coordination and interference mitigation without involving global and centralized control efforts. In [16], energy efficiency of spectrum sharing and power allocation was studied using Stackelberg game theory in heterogeneous cognitive radio networks with cognitive femtocells. Recently, interference temperature limits, originated

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Haijun Zhang (email: dr.haijun.zhang@ieee.org) is with the College of Information Science and Technology, Beijing University of Chemical Technology, China, and is also with the Department of Electrical and Computer Engineering, the University of British Columbia, Canada. Chunxiao Jiang (email: chx.jiang@gmail.com) is with the Department of Electronic Engineering, Tsinghua University, China. Xiaotao Mao (email: 2012200789@grad.buct.edu.cn) is with the College of Information Science and Technology, Beijing University of Chemical Technology, China. Hsiao-Hwa Chen (e-mail: hshwchen@mail.ncku.edu.tw) is with the Department of Engineering Science, National Cheng Kung University, Taiwan.

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from the cognitive radio literature [17], [18], have been used to mitigate macrocell-femtocell cross-tier interferences [19].

Among those existing works, the fairness issues among cognitive femtocell users (FUs) have not been well investigated. Although some works [20] have been done for fair resource allocation in cognitive radio and femtocell networks, they focused mainly on the resource allocation with an assumption of perfect spectrum sensing. However, perfect spectrum sensing is extremely difficult to achieve in practical cognitive wireless communications. To the best of our knowledge, joint sub-channel and power allocation in cognitive opportunistic spectrum access femtocells considering co-tier/cross-tier interference mitigation, fairness, and imperfect spectrum sensing (spectrum sensing errors) has not been well investigated. In this paper, we study joint sub-channel and power allocation in OFDMA based cognitive femtocells under FU fairness constraints, QoS requirement, and co-/cross-tier interference temperature limits with imperfect spectrum sensing. The key contributions of our work can be summarized as follows.

- Design a novel spectrum efficient OFDMA cognitive femtocell optimization framework: This is a new trial by jointly considering capacity-maximization, sub-channel occupation fairness, co-tier and cross-tier interference mitigation, spectrum sensing errors, and user QoS requirements in the design of OFDMA cognitive femtocell optimization framework.
- Formulate a joint sub-channel and power allocation algorithm with multiple constraints: We formulate an uplink sub-channel and power allocation problem in cognitive femtocells as a mixed integer programming problem. Both co-tier/cross-tier interference temperature limits are used to protect primary macrocell and neighboring femtocells. A minimum QoS requirement is employed to provide reliable transmission for cognitive FUs. The fairness in terms of minimum and maximum sub-channel occupation is considered for each FU, where spectrum sensing error is taken into account. The non-convex optimization problem is then transformed into a convex one, to propose a joint sub-channel and power allocation algorithm using a close form power and sub-channel solution, where the convergence of the proposed algorithm is also proved by simulations.
- Support fairness and imperfect spectrum sensing: The spectrum sensing errors, due to mis-detection and false alarm, are considered in the optimization problem. Moreover, the fairness in terms of sub-channel occupying upper and lower bounds is also taken into consideration in the design of the joint resource allocation algorithm.
- Make use of co-tier/cross-tier interference temperature limit: The femtocell is enabled with cognitive capabilities, and thus a cognitive femtocell should not affect the operation of primary macrocells. Therefore, both co-tier and cross-tier interference temperatures are considered, in order to mitigate the interference from neighboring femtocells and transform the non-convex problem into a convex one.

The rest of this paper is outlined as follows. Section II introduces a system model. In Section III, a general optimization framework with imperfect spectrum sensing is presented. In Section IV, joint resource optimization with fairness and imperfect spectrum sensing is proposed. Performance of the proposed algorithm is evaluated by simulations in Section V, followed by the conclusions in Section VI.

II. SYSTEM MODEL

We consider an OFDMA uplink of a network comprising one primary macrocell and $K$ co-channel cognitive femtocells, which are deployed randomly in the coverage area of a macrocell. Let $M$ and $F$ denote the numbers of active macro users (MUs) inside the primary macrocell and FUs in each cognitive femtocell, respectively. The OFDMA system has a bandwidth of $B_w$, which is divided into $N_{total}$ sub-channels. The channel model for each sub-channel includes path loss and frequency-flat Rayleigh fading. Note that we focus on resource allocation in the uplink of cognitive femtocells. The FUs opportunistically access the spectrum licensed to the primary macrocells via cognitive FBS, as illustrated in Fig. 1. In each time slot, the secondary network can sense $N_{total}$ sub-channels and opportunistically access idle channels by energy detection based spectrum sensing. In a spectrum sensing period, the cognitive femtocell network senses $N_{total}$ sub-channels licensed to the primary macrocell network and determines available vacant/idle sub-channels, which are denoted as $\mathcal{N} = \{1, 2, ..., N\}$. Throughout this paper, we assume that a cognitive femto base station (FBS) has perfect channel state information (CSI) between FBS and cognitive FUs/primary MUs. Therefore, the total capacity of the cognitive femtocell networks using our proposed resource scheduling schemes will serve as an upper bound of the achievable capacity with channel estimation errors in practical scenarios.

The received signal to interference-plus-noise ratio (SINR) $\ell_{k,i,n}^F$ at the $k$th ($k \in \{1, \ldots, K\}$) cognitive femto base station (FBS) from its FU $i$ ($i \in \{1, \ldots, F\}$) in the $n$th ($n \in \{1, \ldots, N\}$) sub-channel is given as

$$\ell_{k,i,n}^F = \sum_{j=1}^{K} \sum_{v=1}^{F} p_{j,v,n}^F h_{k,j,v,n}^F + p_{w,n}^M h_{k,w,n}^M + \sigma^2.$$

Fig. 1. A cognitive heterogeneous macro/femto network model.
where $p_{FS}^{F,n}$ is FU $i$’s transmit power on sub-channel $n$ in cognitive femtocell $k$; $h_{FS}^{FF,k,n}$ and $h_{FS}^{FM,k,n}$ are the channel gains on sub-channel $n$ from FU $i$ in cognitive femtocell $j$ and from MU $w$ to FBS $k$, respectively; $w$ is a specific MU using sub-channel $n$; $p_{M,w}^{M}$ is MU $w$’s transmit power in sub-channel $n$; and $\sigma^2$ is the additive white Gaussian noise (AWGN) power. In such a case, based on Shannon’s capacity formula, the uplink capacity on sub-channel $n$ of FU $i$ in cognitive femtocell $k$ can be calculated by

$$ R_{k,i,n}^F = \log_2(1 + \frac{E_{s}}{N_0}), $$

(2)

III. OPTIMIZATION FRAMEWORK WITH IMPERFECT SPECTRUM SENSING

A. Imperfect Spectrum Sensing

Spectrum sensing has been extensively investigated in the previous works [21], [22]. Here, we employ a cooperative spectrum sensing scheme [23], in which each cognitive FU senses sub-channels and sends the sensing results to a cognitive FBS. Then, the cognitive FBS makes decision to determine whether or not the sub-channels are vacant.

The interference from cognitive femtocell networks to primary macrocell networks occurs due to the following two reasons. One is the out-of-band emissions, and the other is the spectrum sensing errors. The out-of-band emissions are due to power leakage in the sidelobes of OFDM signals [24]. The amount of out-of-band interference power of sub-channel $n$ introduced to sub-channel $j$ occupied by a primary macrocell (with unit transmit power) can be expressed as

$$ \mathcal{I}_{k,i,n}^s = \int_{f_s}^{f_s + \frac{\Delta f_{rout}}{2}} \frac{1}{T} \sin \left( \frac{\pi (f - f_s^N) T}{\pi - f_s^N} \right)^2 df, $$

(3)

where $f_s^N$ and $f_s$ are the center frequencies of sub-channel $n$ and $s$, respectively, and $h_{k,i,n,s}^{FM}$ is the channel gain from cognitive FU to primary MBS in sub-channel $s$. In (1), power spectrum density (PSD) of OFDM symbol $\varphi(f)$ is given as

$$ \varphi(f) = \frac{1}{T} \left( \sin \left( \frac{\pi (f - f_s^N) T}{\pi - f_s^N} \right) \right)^2, $$

(4)

where $T$ is the duration of an OFDM symbol.

In cognitive heterogeneous networks, which consist of cognitive femtocell networks and primary macrocell networks, imperfect spectrum sensing of cognitive FBS usually causes severe co-channel interference to primary MBS, and thus degrades the performance of the heterogeneous cognitive femto-macro networks. Since it is the cognitive FBS that determines if a sub-channel is occupied by primary MBS or not, thus there are four different cases which are listed as follows.

- Case 1: sub-channel $s$ is vacant in a primary macrocell network, and the determination made by cognitive FBS is vacant;
- Case 2: sub-channel $s$ is vacant in primary macrocell network, but the determination made by cognitive FBS is occupied;
- Case 3: sub-channel $s$ is occupied in primary macrocell network, and the determination made by cognitive FBS is vacant;
- Case 4: sub-channel $s$ is occupied in primary macrocell network, and the determination made by cognitive FBS is occupied.

For the first and forth cases, the cognitive FBS made the correctly decisions. On the other hand, the second case is a mis-detection, and the third case is false alarm. Denote $\mathcal{H}_s^{o}$ and $\mathcal{H}_s^{c}$ as the hypotheses and the sensing result of sub-channel $s$’s occupation by primary MBS, respectively. And denote $\mathcal{H}_s^{o}$ and $\mathcal{H}_s^{c}$ as the hypotheses and the sensing result of sub-channel $s$’s occupation by primary MBS, respectively. The probabilities of the false alarm and mis-detection on sub-channel $s$ is $q_f^s$ and $q_m^s$, respectively. Therefore, we can get the following probabilities for the four different cases.

The probabilities for Cases 1, 2, 3 and 4 for sub-channel $s$ are

$$ \rho_{1,s} = \Pr(\mathcal{H}_s^{o}|\mathcal{H}_s^{c}) $$

$$ = \frac{\Pr(\mathcal{H}_s^{o}|\mathcal{H}_s^{c}) \Pr(\mathcal{H}_s^{c})}{\Pr(\mathcal{H}_s^{o}|\mathcal{H}_s^{c}) \Pr(\mathcal{H}_s^{c}) + \Pr(\mathcal{H}_s^{o}|\mathcal{H}_s^{c}) \Pr(\mathcal{H}_s^{c})} $$

$$ = \frac{1 - q_f^s}{1 - q_f^s + q_m^s q_f^s'}, $$

(5)

$$ \rho_{2,s} = \Pr(\mathcal{H}_s^{o}|\mathcal{H}_s^{c}) $$

$$ = \frac{\Pr(\mathcal{H}_s^{o}|\mathcal{H}_s^{c}) \Pr(\mathcal{H}_s^{c})}{\Pr(\mathcal{H}_s^{o}|\mathcal{H}_s^{c}) \Pr(\mathcal{H}_s^{c}) + \Pr(\mathcal{H}_s^{o}|\mathcal{H}_s^{c}) \Pr(\mathcal{H}_s^{c})} $$

$$ = \frac{1 - q_m^s q_f^s'}{1 - q_m^s q_f^s'} $$

(6)

$$ \rho_{3,s} = \Pr(\mathcal{H}_s^{o}|\mathcal{H}_s^{c}) $$

$$ = \frac{\Pr(\mathcal{H}_s^{o}|\mathcal{H}_s^{c}) \Pr(\mathcal{H}_s^{c})}{\Pr(\mathcal{H}_s^{o}|\mathcal{H}_s^{c}) \Pr(\mathcal{H}_s^{c}) + \Pr(\mathcal{H}_s^{o}|\mathcal{H}_s^{c}) \Pr(\mathcal{H}_s^{c})} $$

$$ = \frac{q_m^s q_f^s'}{1 - q_m^s q_f^s'} $$

(7)

$$ \rho_{4,s} = \Pr(\mathcal{H}_s^{o}|\mathcal{H}_s^{c}) $$

$$ = \frac{\Pr(\mathcal{H}_s^{o}|\mathcal{H}_s^{c}) \Pr(\mathcal{H}_s^{c})}{\Pr(\mathcal{H}_s^{o}|\mathcal{H}_s^{c}) \Pr(\mathcal{H}_s^{c}) + \Pr(\mathcal{H}_s^{o}|\mathcal{H}_s^{c}) \Pr(\mathcal{H}_s^{c})} $$

$$ = \frac{1 - q_f^s}{1 - q_f^s + q_m^s q_f^s'} $$

(8)

respectively, where $q_m^s$ is the probability of primary MBS’s occupation of sub-channel $s$.

Based on the above analysis, the uplink cross-tier interference from cognitive femtocell to primary MBS, caused by out-of-band emission and co-channel interference, can be formulated as

$$ \mathcal{I}_{k,i,n}^F = \rho_{k,i,n}^F \sum_{s \in N_v} \rho_{3,s} \mathcal{I}_{k,i,n}^s + \sum_{s \in N_v} \rho_{4,s} \mathcal{I}_{k,i,n}^s, $$

(9)

where $N_v$ and $N_o$ are the sets of vacant and occupied sub-channels, respectively, determined by cognitive FBS. The amount of out-of-band interference power of sub-channel $n$ introduced to a primary macrocell-occupied sub-channel of unit transmit power, can be expressed as $\mathcal{I}_{k,i,n}^s$. In other words, since $\mathcal{I}_{k,i,n}^s$ is calculated by unit transmit power, $\mathcal{I}_{k,i,n}^s$ is the
unit interference power here, which can be seen as channel gain. Moreover, both $\rho_{k,s}$ and $\rho_{k,t}$ are the probabilities, and therefore $G^{MF}_{k,i,n}$ can be interpreted as the channel gain on sub-channel $n$ from user $i$ in cognitive femtocell $k$ to the primary macro base station (MBS).

**B. General Optimization Framework**

Let us first investigate the constraints in the proposed optimization framework. First, for resource allocation in cognitive femtocell networks, the total transmit power of cognitive FU is constrained by

$$\sum_{n=1}^{N} \tau_{k,i,n}p^F_{k,i,n} \leq P_{max}, \quad \forall k, i,$$

(10)

where $\tau_{k,i,n} \in \{0, 1\}$ is the sub-channel allocation indicator, and $\tau_{k,i,n} = 1$ indicates that sub-channel $n$ is occupied by user $i$ in cognitive femtocell $k$; otherwise $\tau_{k,i,n} = 0$. $P_{max}$ is the maximum transmit power of each cognitive FU.

Second, to maintain communication quality of cognitive FU’s, a QoS requirement in terms of signal to interference-plus-noise ratio (SINR) is introduced for each FU. Thus, we can write the QoS requirement as

$$\sum_{n=1}^{N} \tau_{k,i,n}R^F_{k,i,n} \geq R^0_{k,i}, \quad \forall k, i,$$

(11)

where $R^0_{k,i}$ is the QoS requirement for user $i$ in cognitive femtocell $k$.

Third, a sub-channel should be assigned to no more than one user at a time in a cognitive femtocell. Therefore, the sub-channel assignment can be performed based on

$$\sum_{i=1}^{F} \tau_{k,i,n} \leq 1, \quad \forall k, n.$$

(12)

Fourth, to obtain the fairness on FU’s level, we set the upper and lower bounds of the number of sub-channels assigned to user $i$ in cognitive femtocell $k$ as

$$\Gamma_{U,k,i} \leq \sum_{n=1}^{N} \tau_{k,i,n} \leq \Gamma_{L,k,i}, \quad \forall k, i,$$

(13)

where $\Gamma_{U,k,i}$ and $\Gamma_{L,k,i}$ are the upper and lower bounds on the number of sub-channels assigned to user $i$ in cognitive femtocell $k$, respectively. Finally, to protect the primary macrocell’s transmission, an interference temperature limit is introduced to constrain cross-tier interference from cognitive femtocell to primary macrocell, i.e.,

$$\sum_{k=1}^{K} \sum_{i=1}^{F} \tau_{k,i,n}p^F_{k,i,n} G^{MF}_{k,i,n} \leq I^{MF}_{th,n}, \quad \forall n,$$

(14)

where $I^{MF}_{th,n}$ is the maximum tolerable cross-tier interference temperature in sub-channel $n$ in the primary macrocell.

Resource allocation aims to maximize the total uplink capacity of $K$ cognitive femtocells under a cross-tier interference constraint and FUs’ QoS constraints, i.e.,

$$\max \left\{ \sum_{k=1}^{K} \sum_{i=1}^{F} \sum_{n=1}^{N} \tau_{k,i,n}R^F_{k,i,n} \right\},$$

(15)

s.t. $C_1: \sum_{n=1}^{N} \tau_{k,i,n}p^F_{k,i,n} \leq P_{max}, \forall k, i,$

$C_2: p^F_{k,i,n} \geq 0, \forall k, i, n,$

$C_3: \sum_{n=1}^{N} \tau_{k,i,n}R^F_{k,i,n} \geq R^0_{k,i}, \forall k, i,$

$C_4: \tau_{k,i,n} \in \{0, 1\}, \forall k, i, n,$

$C_5: \sum_{i=1}^{F} \tau_{k,i,n} \leq 1, \forall k, n,$

$C_6: \Gamma_{U,k,i} \leq \sum_{n=1}^{N} \tau_{k,i,n} \leq \Gamma_{L,k,i}, \forall k, i,$

$C_7: \sum_{k=1}^{K} \sum_{i=1}^{F} \tau_{k,i,n}p^F_{k,i,n} G^{MF}_{k,i,n} \leq I^{MF}_{th,n}, \forall n,$

where constraint $C_1$ limits the transmit power of each FU below the maximum power $P_{max}$. $C_3$ sets the QoS requirement $R^0_{k,i}$ for user $i$ in cognitive femtocell $k$, $C_4$ and $C_5$ guarantee that each sub-channel can be assigned to no more than one user in each femtocell, and $C_6$ ensures a fairness among users by setting $\Gamma_{U,k,i}$ and $\Gamma_{L,k,i}$ as the upper and lower bounds of the number of sub-channels assigned to user $i$ in cognitive femtocell $k$, respectively. The priority of the user can be adjusted by setting appropriate values of $\Gamma_{U,k,i}$ and $\Gamma_{L,k,i}$, and $C_7$ imposes the maximum tolerable cross-tier interference temperature $I^{MF}_{th,n}$ in sub-channel $n$ for the primary macrocell.

**IV. JOINT RESOURCE OPTIMIZATION WITH FAIRNESS & IMPERFECT SENSING**

**A. Transformation of the Optimization Problem**

The problem in (15) is a non-convex mixed integer programming problem. It can be solved using a brute-force method, which however suffers a high computational complexity. To make the problem tractable, we introduce an additional co-tier interference temperature constraint $C_8$ as

$$C_8: \sum_{j=1}^{K} \sum_{k \neq k}^{F} \sum_{v,n}^{F} \tau_{j,v,n}p^F_{j,v,n} I^{FF}_{j,v,n} \leq I^{FF}_{th,n}, \forall k, n,$$

(16)

where $I^{FF}_{th,n}$ is the co-tier interference limit in sub-channel $n$ for a cognitive femtocell. Each femtocell will potentially interfere each other when two neighbor femtocells use the same sub-channel. Therefore, from physical/engineering point view in the real world applications, $I^{FF}_{th,n}$ is a co-tier interference limit for neighbor femtocell to mitigate co-tier interference. The value of $I^{FF}_{th,n}$ can be broadcasted by each femtocell or set by each femtocell.

Moreover, inspired by [25], we relax $\tau_{k,i,n}$ in $C_4$ to be a real variable in the range of $[0,1]$, in which case $\tau_{k,i,n}$ can be interpreted as the fraction of time that sub-channel $n$ is assigned to user $i$ in cognitive femtocell $k$ during one transmission frame. Denote $\bar{p}_{k,i,n} = \tau_{k,i,n}p^F_{k,i,n}$ as the actual power allocated to user $i$ in cognitive femtocell $k$ on sub-channel $n$. Denote \( \tilde{I}^{FF}_{th,n} = I^{FF}_{th,n} + \sigma^2 \) and $\tilde{R}_{k,i,n} = \log_2(1 + \bar{p}_{k,i,n} + \tilde{I}^{FF}_{th,n} + \tilde{I}^{FF}_{th,n} + \sigma^2)$ as the upper bound of the
total received interference power and lower bound of the capacity of user \( i \) on sub-channel \( n \) in cognitive femtocell \( k \), respectively. In such a case, the optimization problem in (15) can be rewritten as

\[
\max_{\{\tau_{k,i,n}\}} \sum_{k=1}^{K} \sum_{i=1}^{F} \sum_{n=1}^{N} \tau_{k,i,n} \hat{R}_{k,i,n}^{F},
\]

(17)

s.t. \( C_1 : \sum_{n=1}^{N} \hat{p}_{k,i,n} \leq P_{\text{max}}, \forall k, i, \)

\( C_2 : \hat{p}_{k,i,n} \geq 0, \forall k, i, n, \)

\( C_3 : \sum_{n=1}^{N} \tau_{k,i,n} \hat{R}_{k,i,n}^{F} \geq R_{k,i}^{0}, \forall k, i, \)

\( C_4 : \tau_{k,i,n} \leq 1, \forall k, i, n, \)

\( C_5 : \sum_{k=1}^{K} \tau_{k,i,n} \leq 1, \forall k, n, \)

\( C_6 : \Gamma_{L,k,i} \leq \sum_{n=1}^{N} \tau_{k,i,n} \leq \Gamma_{U,k,i}, \forall k, i, \)

\( C_7 : \sum_{k=1}^{K} \sum_{n=1}^{N} \tau_{k,i,n} \hat{R}_{k,i,n}^{F} \leq R_{k,i}^{\text{MF}}, \forall n, \)

\( C_8 : \sum_{j=1}^{F} \sum_{n=1}^{N} \tau_{j,v,n} \hat{H}_{j,v,n}^{F} \leq \hat{I}_{t,h,n}, \forall k, n. \)

**Theorem 1:** The objective function in (17) is concave, and the corresponding optimization problem under the constraints \( C_1 \) to \( C_8 \) is a convex problem.

**Proof:** The proof is provided in Appendix A. \( \square \)

**B. Joint Sub-channel and Power Allocation with Imperfect Spectrum Sensing**

The joint sub-channel and power allocation problem in (17) can be solved using the Lagrangian dual decomposition method, which has been widely used in solving resource allocation problems. The Lagrangian function is given by

\[
\mathcal{L}(\{\tau_{k,i,n}\}, \{\hat{p}_{k,i,n}\}, \lambda, \nu, \delta, \mu, \eta) = \sum_{k=1}^{K} \sum_{i=1}^{F} \sum_{n=1}^{N} \lambda_{k,i} \left( P_{\text{max}} - \sum_{n=1}^{N} \hat{p}_{k,i,n} \right) + \sum_{k=1}^{K} \sum_{i=1}^{F} \sum_{n=1}^{N} \delta_{k,i} \left( \hat{R}_{k,i,n}^{F} - \sum_{n=1}^{N} \tau_{k,i,n} \hat{R}_{k,i,n}^{F} \right) + \sum_{k=1}^{K} \sum_{n=1}^{N} \eta_{k,n} \left( 1 - \sum_{i=1}^{F} \tau_{k,i,n} \right) + \sum_{k=1}^{K} \sum_{i=1}^{F} \sum_{n=1}^{N} \nu_{k,i} \left( \sum_{n=1}^{N} \tau_{k,i,n} \hat{R}_{k,i,n}^{F} - R_{k,i}^{0} \right) + \sum_{k=1}^{K} \sum_{n=1}^{N} \mu_{k,n} \left( \hat{I}_{t,h,n} - \sum_{j=1}^{F} \sum_{n=1}^{N} \hat{p}_{j,v,n} \hat{H}_{j,v,n}^{F} \right),
\]

(18)

where \( \lambda, \nu, \delta, \mu, \) and \( \eta \) are the Lagrange multiplier vectors for \( C_1, C_3, C_7, C_8, \) and \( C_5 \) in (17), respectively. The boundary constraints \( C_2, C_4, \) and \( C_6 \) in (17) are absorbed in the Karush-Kuhn-Tucker (KKT) conditions [27], which will be shown later. The dual function is defined as

\[
g(\lambda, \nu, \delta, \mu, \eta) = \max_{\{\tau_{k,i,n}\}, \{\hat{p}_{k,i,n}\}} \mathcal{L}(\{\tau_{k,i,n}\}, \{\hat{p}_{k,i,n}\}, \lambda, \nu, \delta, \mu, \eta),
\]

(19)

and the dual problem can be expressed by

\[
\min_{\lambda, \nu, \delta, \mu, \eta} g(\lambda, \nu, \delta, \mu, \eta).
\]

(20)

Decomposing the Lagrangian dual problem into a master problem and \( K \times N \) subproblems, we can solve them iteratively, where MBS solves the master problem and each FBS solves \( N \) subproblems based on local information in each iteration. Accordingly, (18) is rewritten as

\[
\mathcal{L}(\{\tau_{k,i,n}\}, \{\hat{p}_{k,i,n}\}, \lambda, \nu, \delta, \mu, \eta) = \sum_{k=1}^{K} \sum_{n=1}^{N} \mathcal{L}_{k,n}(\{\tau_{k,i,n}\}, \{\hat{p}_{k,i,n}\}, \lambda, \nu, \delta, \mu, \eta)
\]

\[
= \sum_{k=1}^{K} \sum_{n=1}^{N} \left[ \nu_{k,i} \tau_{k,i,n}^{0} - \nu_{k,i} \sum_{n=1}^{N} \tau_{k,i,n} \hat{R}_{k,i,n}^{F} - \sum_{n=1}^{N} \eta_{k,n} \right] + \sum_{k=1}^{K} \sum_{n=1}^{N} \mu_{k,n} \tau_{k,i,n}^{FF} + \sum_{n=1}^{N} \sum_{k=1}^{K} \eta_{k,n},
\]

(21)

where

\[
\mathcal{L}_{k,n}(\{\tau_{k,i,n}\}, \{\hat{p}_{k,i,n}\}, \lambda, \nu, \delta, \mu, \eta) = \sum_{i=1}^{F} \sum_{n=1}^{N} \lambda_{k,i} \hat{p}_{k,i,n} - \sum_{i=1}^{F} \sum_{n=1}^{N} \eta_{k,n} \tau_{k,i,n} + \sum_{k=1}^{K} \sum_{i=1}^{F} \sum_{n=1}^{N} \nu_{k,i} \tau_{k,i,n} \hat{R}_{k,i,n}^{F} + \sum_{g=1}^{K} \sum_{n=1}^{N} \mu_{g,n} \hat{R}_{k,i,n}^{MF} - \sum_{g=1}^{K} \sum_{n=1}^{N} \delta_{g,k,n} \hat{G}_{k,i,n}.
\]

(22)

The calculation of the derivatives with respect to \( \hat{p}_{k,i,n} \) and \( \tau_{k,i,n} \), respectively, gives the KKT condition as

\[
\frac{\partial \mathcal{L}_{k,n}(\cdots)}{\partial \hat{p}_{k,i,n}} = \lambda_{k,i} - \nu_{k,i} \leq 0,
\]

(23)

where

\[
\lambda_{k,i} = \frac{(1+\nu_{k,i}) \tau_{k,i,n}^{MF}}{\ln(2(\tau_{k,i,n}^{FF}+\sum_{g=1}^{K} \mu_{g,n} \hat{R}_{k,i,n}^{MF}+\delta_{g,k,n} \hat{G}_{k,i,n}))} - \frac{\sum_{g=1}^{K} \mu_{g,n} \hat{R}_{k,i,n}^{MF} - \delta_{g,k,n} \hat{G}_{k,i,n}}{\tau_{k,i,n}^{FF}}, \forall k, i,
\]

(24)

According to (23)-(26), we get the optimal power allocated to user \( i \) in cognitive femtocell \( k \) in sub-channel \( n \) for (17) as

\[
p_{k,i,n}^{*} = \frac{\hat{p}_{k,i,n}^{*}}{\tau_{k,i,n}^{*}} = \frac{\lambda_{k,i} - \nu_{k,i}}{\ln(2(\tau_{k,i,n}^{FF}+\sum_{g=1}^{K} \mu_{g,n} \hat{R}_{k,i,n}^{MF}+\delta_{g,k,n} \hat{G}_{k,i,n}))} - \frac{\sum_{g=1}^{K} \mu_{g,n} \hat{R}_{k,i,n}^{MF} - \delta_{g,k,n} \hat{G}_{k,i,n}}{\tau_{k,i,n}^{FF}}, \forall k, i,
\]

(27)

where \((x)^{+} = \max(0, x)\). Moreover, we have

\[
\frac{\partial \mathcal{L}_{k,n}(\cdots)}{\partial \tau_{k,i,n}} = \Xi_{k,i,n} - \eta_{k,n} \leq 0,
\]

(28)

where

\[
\Xi_{k,i,n} = (1+\nu_{k,i}) \ln(2(\tau_{k,i,n}^{FF}+\sum_{g=1}^{K} \mu_{g,n} \hat{R}_{k,i,n}^{MF}+\delta_{g,k,n} \hat{G}_{k,i,n})) - \frac{\tau_{k,i,n}^{FF}}{\sum_{g=1}^{K} \mu_{g,n} \hat{R}_{k,i,n}^{MF}+\delta_{g,k,n} \hat{G}_{k,i,n}}, \forall k, i,
\]

(29)

\[
\tau_{k,i,n}^{*} = \Xi_{k,i,n} - \eta_{k,n} = 0,
\]

(30)
Algorithm 1, sub-channels will be assigned to cognitive femto users, and the used sub-channels will be removed from sub-channel set $N$ based on line 12 of Algorithm 1. Algorithm 1 will check that whether any unused sub-channels are left in line 18; if true, lines 19-20 will be executed until the sub-channel set is empty. Therefore, lines 9-24 can ensure a full utilization of all vacant sub-channels.

In practical scenarios, users’ sub-channel requirements are different, and traditional capacity-maximum sub-channel algorithms tend to allocate the sub-channels to the users with better channel conditions according to users’ sub-channel requirements. Therefore, the sub-channel requirements of the other users with relatively poor channel conditions may not be satisfied. This is unfair for the users with poor sub-channel conditions. In Algorithm 1, Procedure 1 can guarantee the lowest requirements of sub-channels for users with poor channel conditions, and Procedure 2 can maximize the users’ capacity while keeping the number of users’ sub-channel occupation below the upper bound.

Note that $\bar{h}_{g,k,i,n}$ required in (27), (29), and (35) can be known by a cognitive FBS from a FBS gateway or through available interfaces between FBS’s, and $G_{k,i,n}^{MF}$ required in (27) and (36) can be estimated by user $i$ in femtocell $k$ by measuring downlink channel gain of sub-channel $n$ from the MBS, assuming a symmetry between uplink and downlink interfaces between FBS’s, and $\hat{G}_{k,i,n}$ is known by a cognitive FBS from a FBS gateway or through available interfaces between FBS’s. Furthermore, it is assumed that there are wired connections between FBS’s and MBS [19] [29], so that $\hat{G}_{k,i,n}$ can be exchanged between the MBS and FBS’s.

Let us discuss the complexity of the proposed algorithm. Suppose that the subgradient method used in Algorithm 1 needs $\Delta$ iterations to converge. Since the updates of each $\lambda$ and $\nu$ need $O(F)$ operations [27], the computation of $\mu$ and $\delta$ requires $O(N)$ operations each, and $\Delta$ is a polynomial function of $FN$. Therefore, the asymptotic complexity of

\[
\eta_k,n (1 - \sum_{i=1}^{F} T_{k,i,n}) = 0. \quad (31)
\]

Based on (28)-(31), sub-channel $n$ is assigned to the user with the largest $\Xi_{k,i,n}$ in femtocell $k$, that is

\[
\hat{\gamma}_{k,i,n} = 1_{i_{\text{max}} = \max_{i} \Xi_{k,i,n}}. \quad \forall k,n. \quad (32)
\]

Since the dual function is differentiable, the subgradient method can be used to solve the master dual minimization problem in (20). Based on the subgradient method, the master dual problem in (20) can be solved as

\[
\lambda_{k,i}^{(l+1)} = \left[ \lambda_{k,i}^{(l)} - \varepsilon_{1}^{(l)} \left( P_{\text{max}} - \sum_{n=1}^{N} \hat{p}_{k,i,n} \right) \right]^+, \quad \forall k,i. \quad (33)
\]

\[
\psi_{k,i}^{(l+1)} = \left[ \psi_{k,i}^{(l)} - \varepsilon_{2}^{(l)} \left( \sum_{n=1}^{N} \hat{g}_{k,i,n} - p_{k,i} \right) \right]^+, \quad \forall k,i. \quad (34)
\]

\[
\mu_{k,n}^{(l+1)} = \left[ \mu_{k,n}^{(l)} - \varepsilon_{3}^{(l)} \left( h_{\text{MF}}^{(l)} - \sum_{k' \neq k \in U} \sum_{j,l,n} \hat{g}_{k',j,l,n}^{\text{MF}} \right) \right]^+, \quad \forall k,n. \quad (35)
\]

\[
\delta_{n}^{(l+1)} = \left[ \delta_{n}^{(l)} - \varepsilon_{4}^{(l)} \left( I_{\text{MF}}^{(l)} - \sum_{k=1}^{K} \sum_{n=1}^{N} \hat{p}_{k,i,n} \hat{g}_{k,i,n}^{\text{MF}} \right) \right]^+, \quad \forall n. \quad (36)
\]

where $\varepsilon_{1}^{(l)}$, $\varepsilon_{2}^{(l)}$, $\varepsilon_{3}^{(l)}$, and $\varepsilon_{4}^{(l)}$ are step sizes of iterations $i$, $l \in \{1,2,\ldots,L_{\text{max}}\}$, $L_{\text{max}}$ is the maximal number of iterations. The step sizes should satisfy $\sum_{l=1}^{\infty} \varepsilon_{i}^{(l)} = \infty$, and $\lim_{l \to \infty} \varepsilon_{i}^{(l)} = 0$, $\forall t \in 1,\ldots,4$. $\lambda$, $\nu$, and $\mu$ are updated by the cognitive femtocells in a distributed manner, and $\delta_{n}^{(l+1)}$ is updated by the primary MBS. Fig. 2 shows the three-layer architecture of the decomposed dual problem.

C. Iterative Resource Optimization Algorithm with Fairness

Although in the solution in (27), (32)-(36) give a complete algorithm for the original problem, the fairness in sub-channel occupation was not considered. We still need to design an algorithm to indicate the execution structure for the equations. Therefore, we propose Algorithm 1 as an implementation of our joint sub-channel and power allocation scheme, as shown in the pseudocode below.

In this paper, the fairness is taken into consideration in terms of sub-channel allocation. Specifically, to ensure the fairness on FUs’ level, we set up the upper and lower bounds of the number of sub-channels assigned to the users in a cognitive femtocell as shown in (13). In the problem formulation, $C_0$ ensures fairness among users by setting $\Gamma_{U,k,i}$ and $\Gamma_{L,k,i}$ as the upper and lower bounds of the number of sub-channels assigned to user $i$ in cognitive femtocell $k$, respectively, and the priority of the user can be adjusted by setting values of $\Gamma_{U,k,i}$ and $\Gamma_{L,k,i}$ appropriately. After the transformation, we get the solution of the formulated optimization problem by proposed Algorithm 1.

In Algorithm 1, we use the following two procedures to ensure users’ fairness in sub-channel allocation. First, we allocate sub-channels for the users whose sub-channel occupation is below $\Gamma_{L,k,i}$, and this procedure is named as “sub-channel allocation for user fairness” in Algorithm 1, to guarantee users’ lowest requirement. In the second procedure, which is called “sub-channel allocation for capacity enhancement”, the algorithm tries to enhance the user’s capacity while keeping users’ sub-channel occupation below the upper bound of $\Gamma_{U,k,i}$. With the help of the two procedures, Algorithm 1 can ensure that the sub-channels assigned to user $i$ in femtocell $k$ is between $\Gamma_{U,k,i}$ and $\Gamma_{L,k,i}$. Moreover, from lines 9-16 of Algorithm 1, sub-channels will be assigned to cognitive femto users, and the used sub-channels will be removed from sub-channel set $N$ based on line 12 of Algorithm 1. Algorithm 1 will check that whether any unused sub-channels are left in line 18; if true, lines 19-20 will be executed until the sub-channel set is empty. Therefore, lines 9-24 can ensure a full utilization of all vacant sub-channels.

In practical scenarios, users’ sub-channel requirements are different, and traditional capacity-maximum sub-channel algorithms tend to allocate the sub-channels to the users with better channel conditions according to users’ sub-channel requirements. Therefore, the sub-channel requirements of the other users with relatively poor channel conditions may not be satisfied. This is unfair for the users with poor sub-channel conditions. In Algorithm 1, Procedure 1 can guarantee the lowest requirements of sub-channels for users with poor channel conditions, and Procedure 2 can maximize the users’ capacity while keeping the number of user’s sub-channel occupation below the upper bound.

Note that $\bar{h}_{g,k,i,n}$ required in (27), (29), and (35) can be known by a cognitive FBS from a FBS gateway or through available interfaces between FBS’s, and $G_{k,i,n}^{MF}$ required in (27) and (36) can be estimated by user $i$ in femtocell $k$ by measuring downlink channel gain of sub-channel $n$ from the MBS, assuming a symmetry between uplink and downlink channels. Furthermore, it is assumed that there are wired connections between FBS’s and MBS [19] [29], so that $G_{k,i,n}^{MF}$ can be exchanged between the MBS and FBS’s.
Algorithm 1 Iterative Resource Allocation Algorithm.

1. Cognitive FBS set: $K = \{1, 2, ..., K\}$; Cognitive FU set per femtocell: $U = \{1, 2, ..., F\}$.
2. Initialize $\lambda_{max}$ and Lagrangian variables vectors $\lambda, \nu, \mu$, and set $i = 0$.
3. Allocate the same power to each sub-channel, set $\tau_{k,i,n} = 0$, $\forall k, i, n$.
4. repeat
5.  Cognitive FBS $k$ measures $h_{k,i,n}$ and $I_{k,i,n}$, $\forall k, i, n$;
6.  for each FBS do
7.    sub-channel set: $N = \{1, 2, ..., N\}$;
8.    Set $N_i = 0$, $\forall i \in U$
9.    sub-channel allocation for user fairness
10.   while $N_i < \Gamma_{L,k,i}, \forall i \in U$ do
11.     a) find $n^* = \arg \max_{n \in N} \sum_{k,i,n}$ according to (29);
12.     b) $\tau_{k,i,n^*} = 1, N = N - \{n^*\}$, $N_i = N_i + 1$;
13.     if $N_i = N_{U,k,i}$ then
14.        $U = U - \{i\}$;
15.    end if
16.    end while
17.    sub-channel allocation for capacity enhancement
18.    while $N \neq \emptyset$ do
19.      a) find $(i^*, n^*) = \arg \max_{i \in U, n \in N} \sum_{k,i,n}$;
20.      b) $\tau_{i^*,n^*} = 1, N = N - \{n^*\}$;
21.      if $N_i = N_{U,k,i^*}$ then
22.         $U = U - \{i^*\}$;
23.    end if
24.    end while
25.   Every FBS $j (j \neq k)$ measures $\mu_{j,n}h_{j,k,i,n}$, and feeds it back to FBS $k$
26.   Power Allocation
27.   for $n = 1$ to $N$ do
28.     a) FUs update $p_{k,i,n}$ according to (27);
29.     b) Cognitive FBS $k$ updates $\lambda, \nu, \mu$ according to (33), (34) and (35), respectively;
30.     c) Cognitive FBS $k$ updates $\sum_{k,i,n}$ according to (29).
31.   end for
32. end for
33. Primary MBS updates $\delta$ according to (36), and broadcasts the updated value to all FBSs via backhaul, $l = l + 1$.
34. until convergence or $l = L_{max}$

Algorithm 1 is $O(KFN(\log_2 N + \log_2 F)\Delta)$. Compared to the brute-force method, which has a complexity of $O(KFN)$, the proposed Algorithm 1 has a lower complexity, especially for a large $N$.

V. SIMULATION RESULTS AND DISCUSSIONS

In the simulations, the primary macrocell’s radius was set to 500 m, and the radius of each cognitive femtocell is set to 10 m. Cognitive femtocells and MUs are distributed randomly in the macrocell coverage area. The carrier frequency is 2 GHz. $B_w = 10$ MHz, $N_0 = -174$ dBm/Hz, $N = 50$, and $M = 20$ were used in the simulations, respectively. The block-fading channel gains are modeled as i.i.d. exponential random variables with unit mean. MUs’ maximum transmit power is 23 dBm. The standard deviation of lognormal shadowing between MBS and users is 8 dB, while that between an FBS and users is 10 dB. The probability of false alarm $q_i^f$, mis-detection $q_i^n$, and primary MU’s occupation $p_i^k$ are uniformly distributed over $[0.05, 0.1]$, $[0.01, 0.05]$, and $[0.1]$, respectively. Without loss of generality, we assume $N_o = \{3, 5, 7, ..., 50\}$, while the upper bounds $[7, 14, 14]$ and the lower bounds $[3, 3, 7]$ of sub-channel assignment for FUs $i \in \{1, 2, 3, 4\}$ per femtocell were assumed. For comparison purpose, we included the scheduling scheme in [30] in conjunction with the power allocation scheme in Algorithm 1, and refer to it as the “existing scheme” hereafter. The indoor and outdoor pathloss models are based on [31].

Fig. 3 shows the convergence of the proposed algorithm in terms of the average capacity per femtocell versus the number of iterations. $K = 10$, $R_o^0 - 90$ dBm, and $R_{th,n} = 100$ dBm.

Fig. 4 shows the total capacity of $K$ cognitive femtocells versus those for different co-/cross-tier interference limits. The proposed algorithm with higher co-/cross-tier interference
limits, $I_{th,n}^{FF}$ and $I_{th,n}^{MF}$, provides a higher total capacity of $K$ cognitive femtocells, because of the higher transmit power used by users under the slacker constraint of co-/cross-tier interference. The effect on how additional constraint of $C_S$ affects the overall performance of the proposed algorithm is investigated in the simulations, as showed in Fig. 4. The brute force method without constraint of co-tier interference limit $C_S$ has a better performance in terms of total capacity of $K$ cognitive femtocells than the proposed algorithm with $C_S$, because of the slacker constraint of co-/cross-tier interference in the optimization problem.

Fig. 5 shows the average number of sub-channels allocated by the proposed algorithm to each FU as compared with the “existing algorithm”. It can be seen that sub-channel assignments of the proposed algorithm meet the requirements of different users given in $C_0$, while the “existing algorithm” does not always satisfy $C_0$, e.g., the number of assigned sub-channels may fall below the lower bound. The proposed algorithm tends to allocate a number of sub-channels, which is only slightly larger than the lower bound to each FU, leading to an efficient reuse of sub-channels. The procedure of “sub-channel allocation for user fairness” guarantees the lower bound for users’ sub-channel requirement, while the procedure of “sub-channel allocation for capacity enhancement” guarantees that it does not exceed the upper bound.

Fig. 6a shows the average cross-tier interference suffered in each sub-channel of primary macrocell when maximum transmit power $P_{max}$ increases from 20 dBm to 30 dBm, for the number of users per femtocell $F = 4$ and the number of femtocells $K = 10$. The other simulation parameters were set as $R_{th,k}^{FF} = 9$ bps/Hz for all $k$, and $I_{th,n}^{MF} = I_{th,n}^{FF} = -100$ dBm for all $n$. The total cross-tier interference increases as the increase of $P_{max}$. This is because that the cross-tier interference is caused by transmit power per sub-channel and the cross-tier channel gain, and a large value of $P_{max}$ enlarges the feasible domain of the optimizing variable. It also can be seen from the figure that the perfect spectrum sensing scheme has a higher cross-tier interference than the imperfect spectrum sensing scheme. The reason of this phenomenon is that mis-detection and false alarm in imperfect spectrum sensing overestimate the cross-tier interference. Moreover, the average interference from cognitive femtocell to primary macrocell in each sub-channel in imperfect spectrum sensing is below the cross-tier interference threshold. Fig. 6b shows the average co-tier interference suffered in each sub-channel of neighboring femtocells when maximum transmit power $P_{max}$ increases from 20 dBm to 30 dBm. Note that perfect spectrum sensing of cross-tier channel gain at cognitive FBS side results in a higher co-tier interference than the imperfect spectrum sensing scheme, because mis-detection and false alarm in imperfect spectrum sensing overestimate the cross-tier interference.

Fig. 7 shows the total capacity of all cognitive femtocells when maximum transmit power $P_{max}$ increases from 20 dBm to 30 dBm, for the number of users per femtocell $F = 4$ and the number of femtocells $K = 10$. The other simulation parameters are set as $R_{th,k}^{FF} = 9$ bps/Hz for all $k$, and $I_{th,n}^{MF} = I_{th,n}^{FF} = -100$ dBm for all $n$. The total capacity of all femtocells increases as the increase of $P_{max}$.

![Fig. 5. Number of sub-channels occupied by each FU.](image1.png)

![Fig. 6a. Average cross-tier interference to primary macrocell in each sub-channel.](image2.png)

![Fig. 6b. Average co-tier interference to neighboring femtocells in each sub-channel.](image3.png)
This is because a large value of $P_{\text{max}}$ enforces the feasible domain of the optimizing variable. It also can be seen from the figure that perfect spectrum sensing scheme has a higher capacity of all cognitive femtocells than the imperfect spectrum sensing scheme, because mis-detection and false alarm in imperfect spectrum sensing overestimate the cross-tier interference, which shrinks the feasible domain of the optimizing variable.

Fig. 8 shows the total capacity of all cognitive femtocells when minimum transmit power $R_{k,i}^0$ increases from 2 bps/Hz to 10 bps/Hz for the number of users per femtocell $F = 2, 3, 4$ and the number of femtocells $K = 10$. Other simulation parameters are set as $I_{th,n}^M = I_{th,n}^F = -100$ dBm for all $n$. The total capacity of all femtocells decreases as the decrease of $R_{k,i}^0$. This is because a large value of $R_{k,i}^0$ narrows the feasible domain of the optimizing variable. It can also be seen from the figure that a larger number of FUs per femtocell results in a higher capacity, because of the mult-user diversity in the resource allocation.

Fig. 9 shows the running time to compare how much improvement can be gained in speed by introducing the constraint used in Eqn. (16) to better reflect a real-world scenario, where $R_{k,i}^0 = 9$ bps/Hz for all FUs, $P_{\text{max}} = 23$ dBm, and $f_{\text{FU}}^p = f_{\text{FU}}^c = -100$ dBm. It can be seen from the figure that Algorithm 1 reduces the running time significantly, if compared to the brute force method.

VI. CONCLUSIONS

In this paper, we have proposed a joint sub-channel and power allocation algorithm for cognitive femtocells, considering minimum user data rate requirement, sub-channel assignment fairness, co-/cross-tier interference limits, and imperfect spectrum sensing. Simulation results have shown that the proposed algorithm converges quickly and outperforms the existing algorithms in terms of cognitive femtocell capacity and sub-channel reuse efficiency. In the future works, a joint optimization of both spectrum sensing process and resource allocation will be considered.

APPENDIX A

PROOF OF THEOREM 1

First, we define the element $\tau_{k,i,n,\hat{R}_{k,i,n}}^{FF}$ in (17) as $f(\tau_{k,i,n,\hat{R}_{k,i,n}}^{FF}) = \tau_{k,i,n,\hat{R}_{k,i,n}}^{FF}$. The objective function in (17) is the sum of $f(\tau_{k,i,n,\hat{R}_{k,i,n}}^{FF})$ over all possible values of $k$, $i$, and $n$. Substituting $\hat{R}_{k,i,n}^{FF} = \log_2(1 + \frac{\hat{p}_{k,i,n}h_{k,i,n}^{FF}}{\tau_{k,i,n}I_{k,i,n}})$ into $f(\tau_{k,i,n,\hat{R}_{k,i,n}}^{FF})$, we get

$$f(\tau_{k,i,n,\hat{R}_{k,i,n}}^{FF}) = \tau_{k,i,n,\hat{R}_{k,i,n}}^{FF} \log_2(1 + \frac{\hat{p}_{k,i,n}h_{k,i,n}^{FF}}{\tau_{k,i,n}I_{k,i,n}}).$$

(37)

Based on (37), we can obtain

$$\frac{\partial^2 f}{\partial \tau_{k,i,n}^2} = -\frac{1}{\ln 2} \frac{(\hat{p}_{k,i,n}h_{k,i,n}^{FF})^2}{\tau_{k,i,n}(\tau_{k,i,n}I_{k,i,n} + \hat{p}_{k,i,n}h_{k,i,n}^{FF})^2},$$

(38)

$$\frac{\partial f}{\partial \tau_{k,i,n} \partial p_{k,i,n}} = \frac{\partial^2 f}{\partial \tau_{k,i,n}^2} \frac{\partial^2 f}{\partial p_{k,i,n}^2} = \frac{1}{\ln 2} \frac{(\hat{p}_{k,i,n}h_{k,i,n}^{FF})^2}{(\tau_{k,i,n}I_{k,i,n} + \hat{p}_{k,i,n}h_{k,i,n}^{FF})^2}.$$

(39)
$$\frac{\partial^2 f}{\partial p_{k,i,n}^2} = -\frac{1}{\lambda_k} (\tau_{k,i,n}(H_{FF})_{k,i,n})^2 \frac{\partial^2 f}{\partial p_{k,i,n}^2}.$$  
\[ (40) \]

Consequently, the Hessian matrix of \( f(\tau_{k,i,n}, \hat{p}_{k,i,n}) \) can be written as

\[ H = \begin{bmatrix}
\frac{\partial^2 f}{\partial \tau_{k,i,n}^2} & \frac{\partial^2 f}{\partial \tau_{k,i,n} \partial \hat{p}_{k,i,n}} \\
\frac{\partial^2 f}{\partial \tau_{k,i,n} \partial \hat{p}_{k,i,n}} & \frac{\partial^2 f}{\partial \hat{p}_{k,i,n}^2}
\end{bmatrix}. \]
\[ (41) \]

To prove negative semi-definiteness of \( H \), let us first introduce Lemma 1 as [26].

**Lemma 1:** Let \( A \) be an \( N \times N \) symmetric matrix, \( A \) is negative semidefinite if and only if all the \( k \)th order principal minors of \( A \) are no larger than zero if \( k \) is odd, and not less than zero if \( k \) is even, where \( 1 \leq k \leq N \).

Substituting (38)-(40) to (41), we can easily verify that the first order principal minors of \( H \) are negative, and the second order principal minor of \( H \) is zero. Therefore, \( H \) is negative semidefinite according to Lemma 1, and \( f(\tau_{k,i,n}, \hat{p}_{k,i,n}) \) is concave. The objective function of (17) is concave because any positive linear combination of concave functions is concave [27] [28]. As the inequality constraints in (17) are convex, the feasible set of the objective function in (17) is convex, and the corresponding optimization problem is a convex problem. This completes the proof.

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Chunxiao Jiang (S’09–M’13) received his B.S. degree in information engineering from Beijing University of Aeronautics and Astronautics (Beihang University) in 2008 and the Ph.D. degree from Tsinghua University (THU), Beijing in 2013, both with the highest honors. During 2011–2013, he visited the Signals and Information Group at the Department of Electrical & Computer Engineering of the University of Maryland with Prof. K. J. Ray Liu. Dr. Jiang is currently a post-doctor in EE department of THU with Prof. Yong Ren. His research interests include the applications of game theory and queuing theory in wireless communication and networking and social networks. Dr. Jiang received the Best Paper Award from IEEE GLOBECOM in 2013, the Beijing Distinguished Graduate Student Award, Chinese National Fellowship and Tsinghua Outstanding Distinguished Doctoral Dissertation in 2013.

Xiaotao Mao received the BS degree in communication engineering from Beijing University of Chemical Technology, Beijing, China, in 2012. She is currently pursuing the M.S. degree at the Department of Computer Science and Technology, Beijing University of Chemical Technology, Beijing, China. She won the Outstanding Graduates Award from Beijing University of Chemical Technology in 2012. Her research interests include wireless communications and convex optimization. Currently, her research focuses on resource allocation in wireless networks.

Hsiao-Hwa Chen (S’89–M’91–SM’00–F’10) is currently a Distinguished Professor in the Department of Engineering Science, National Cheng Kung University, Taiwan. He obtained his BSc and MSc degrees from Zhejiang University, China, and a PhD degree from the University of Oulu, Finland, in 1982, 1985 and 1991, respectively. He has authored or co-authored over 400 technical papers in major international journals and conferences, six books and more than ten book chapters in the areas of communications. He served as the general chair, TPC chair and symposium chair for many international conferences. He served or is serving as an Editor or Guest Editor for numerous technical journals. He is the founding Editor-in-Chief of Wiley’s Security and Communication Networks Journal (www.interscience.wiley.com/journal/security). He is the recipient of the best paper award in IEEE WCNC 2008 and a recipient of IEEE Radio Communications Committee Outstanding Service Award in 2008. Currently, he is also serving as the Editor-in-Chief for IEEE Wireless Communications. He is a Fellow of IEEE, a Fellow of IET, and an elected Member at Large of IEEE ComSoc.