Robust fuzzy control of nonlinear fuzzy impulsive systems with time-varying delay

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Abstract: The problem of robust fuzzy control for a class of nonlinear fuzzy impulsive systems with time-varying delay is investigated by employing Lyapunov functions. The nonlinear delay system is represented by the well-known T–S fuzzy model. The so-called parallel distributed compensation idea is employed to design the state feedback controller. Sufficient conditions for global exponential stability of the closed-loop system are derived in terms of linear matrix inequalities (LMIs), which can be easily solved by LMI technique. An example is given to demonstrate the effectiveness of the proposed method.

1 Introduction

Over the past few decades, fuzzy logic control of nonlinear systems has received considerable attentions because this approach is effective to obtain nonlinear control systems, especially in the incomplete knowledge of the plant or even of the precise control action appropriate to a given situation. Among various kinds of fuzzy methods, fuzzy-model-based control is widely used because the design and analysis of the overall fuzzy system can be systematically performed using the well-established classical linear systems theory [1–4]. The stability analysis and controller design for nonlinear systems based on T–S fuzzy model are discussed in [1–3]. On the other hand, time delay is commonly encountered and is often the sources of instability. Recently, the robust stability analysis problems for fuzzy time-delay systems have received considerable attentions [5–8].

In modern science and technology, there are natural phenomena in real world which are characterised by the fact that at certain moment of time they experience a change of state abruptly. Consequently, it is natural to assume that these perturbations act instantaneously, that is, in the form of impulses. It is known, for example, that many biological phenomena involving thresholds, bursting rhythm models in medicine and biology, optimal control models in economics, pharmacokinetics and frequency-modulated systems do exhibit impulsive effects [9–11]. The impulsive disturbances can severely degrade the closed-loop system performance and even make a stable system unstable. During the past few years, the qualitative properties of impulsive differential equations have been intensively developed; see [9–13] and the references cited therein. Recently, the development of impulsive fuzzy differential equations has been initiated in [12] and the impulsive functional differential inclusions are studied in [13]. On the other hand, the stability of systems with impulsive effect has sparked the interest of many researchers; see [14–17] and the references cited therein. The problem of robust decentralised stabilisation for a class of large-scale, time delay and uncertain impulsive dynamical systems has been investigated in [15]. Several criteria on robust stability, robust asymptotic stability and robust exponential stability for uncertain impulsive dynamical systems have been established in [16].

Very recently, there have been growing attentions on the study of T–S fuzzy systems with impulse [18, 19]. In [18], a class of nonlinear fuzzy impulsive systems is defined by extending the ordinary T–S fuzzy model and sufficient conditions for global exponential stability of the closed-loop systems are derived. In [19], some criteria of uniform stability and uniform asymptotic stability for T–S fuzzy delay systems with impulse have been presented. On the other hand, there are some interesting applications on
impulsive control or synchronisation of chaotic systems based on T–S fuzzy model [20, 21].

In the design of controller systems, one is not only interested in global stability, but also in some other performances. Particularly, it is often desirable to have systems that converge fast enough in order to achieve fast response. Considering this, many researchers have studied the exponential stability analysis problem for neural networks [22], stochastic systems [23] and so on. To the best of our knowledge, so far, the problem of global exponential stabilisation for fuzzy impulsive systems with time-varying delay has not been addressed in the literature, which is still open and remains unsolved.

Motivated by the aforementioned discussions, we investigate the problem of robust fuzzy control for a class of nonlinear systems with time-varying delay. The nonlinear delay system is represented by the well-known T–S fuzzy model. The so-called parallel distributed compensation (PDC) idea is employed to design the state feedback controller. Sufficient assumptions are made about the premise variable $x(t)$, which can be easily solved by LMI technique.

The remainder of this paper is organised as follows. In Section 2, the problem to be investigated is given and some necessary definitions and useful lemmas are also presented. In Section 3, some criteria are derived to ensure the global exponential stability of the closed-loop system. In Section 4, an example is given to demonstrate the effectiveness of the proposed method. Finally, conclusions are drawn in Section 5.

**Notation:** Throughout this paper, the superscripts ‘−1’ and ‘T’ stand for the inverse and transpose of a matrix, respectively; $\mathbb{R}^n$ denotes the $n$-dimensional Euclidean space; let $\mathbb{R}_{+} = [0, \infty)$, $\mathbb{N} = \{0, 1, 2, \ldots\}$; $\mathbb{R}^{n \times m}$ is the set of all $n \times m$ real matrices; for real symmetric matrices $X$ and $Y$, the notation $X \geq Y$ (respectively, $X > Y$) means that the matrix $X - Y$ is positive semi-definite (respectively, positive definite); $I$ is an appropriately dimensioned identity matrix and $\lambda_{\min}(P)$ ($\lambda_{\max}(P)$) denotes the smallest (largest) eigenvalues of $P$. The vector norm of $x \in \mathbb{R}^n$ is Euclidean, that is, $\|x\| = \sqrt{x^T x}$.

## 2 Problem statement and basic assumptions

Consider the following nonlinear system with time-varying delay represented by T–S fuzzy model.

**Plant Rule i**

| IF $z_i(t)$ is $M^i_1$ and, \ldots, and $z_q(t)$ is $M^i_g$ | THEN $\dot{x}(t) = A_ix(t) + B_ix(t - \tau(t)) + C_iu(t), t \neq t_k$ |

**where** $M^i_j$ **is** the **fuzzy** set, $g$ **the** number of **rules**, $x(t) = [z_1(t), z_2(t), \ldots, z_q(t)]^T$ **the** premise variable, $x(t) \in \mathbb{R}^q$ **the** state vector, $u(t)$ **the** control input, $A_i, B_i, C_i, D_i \in \mathbb{R}^{q \times q}$ **and** $C_i \in \mathbb{R}^{q \times m}$ **the** constant matrices, $0 \leq \tau(t) \leq \tau_0$ **the** unknown bounded time-varying delay in the state and $\Delta x(t_k) = x(t_k^+) - x(t_k^-)$ **the** difference of the state at the $k$th impulsive point, $x(t_k^-)$ **the** limit-point of $x(t)$ **as** $t \searrow t_k$. Without loss of generality, we assume that $\lim_{t \to t_k^-} x(t) = x(t_k)$, which means that the solution $x(t)$ is right continuous at time $t_k$. $\tilde{G}_i \in \mathbb{R}^{n \times q}$ **is** constant coefficients, the impulsive time instants $\{t_k\}$ satisfy $0 < t_0 < t_1 < t_2 < \cdots < t_{k-1} < t_k < \cdots$, $\lim_{k \to \infty} t_k = +\infty$. We assume that there exists a constant $L > 1$, such that $t_k - t_{k-1} \geq L\tau_0$.

**Remark 1:** In (1), if $u(t) = 0$, then the nonlinear system reduces to

**IF** $z_i(t)$ **is** $M^i_1$ **and** \ldots, and $z_q(t)$ **is** $M^i_g$ **THEN** $\dot{x}(t) = A_ix(t) + B_ix(t - \tau(t)), t \neq t_k$

$\Delta x(t_k) = \tilde{G}_ki x(t_k^-), k = 1, 2, \ldots$

$x(t) = \phi(t), t \in [t_0 - \tau_0, t_0]$

$i = 1, 2, \ldots, g$  

which is called the unforced fuzzy impulsive system with time-varying delay.

In (1), if $\tau(t) = 0$, $\tilde{G}_i = 0$, $i = 1, 2, \ldots, g$, **then** the nonlinear system reduces to

**IF** $z_i(t)$ **is** $M^i_1$ **and** \ldots, and $z_q(t)$ **is** $M^i_g$ **THEN** $\dot{x}(t) = A_ix(t) + C_iu(t)$

$i = 1, 2, \ldots, g$

which is a typical continuous T–S fuzzy model. Stability of this T–S fuzzy model has been extensively investigated [1–3].

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which is a typical continuous T–S fuzzy time-delay model. Stability of this T–S fuzzy model has been extensively investigated [5–7].

**Remark 2:** As is shown in [15], the condition $t_k - t_{k-1} \geq L\tau_0$, $L > 1$ describes the relationship between the discontinuity point $t_k$ and the maximum time delay $\tau_0$. 
where the parameter $L$ can be regarded as a proportional coefficient of $t_k - t_{k-1}$ to the maximum time delay $\tau_0$. This inequality also implies that the discontinuity point requires no denseness condition in order to ensure the exponential stability of the system.

By using the fuzzy inference method with a singleton fuzzification, product inference and centre average defuzzification, the overall fuzzy model is of the following form

\[
\dot{x}(t) = \sum_{i=1}^{q} h_i(x(t))[Ax(t) + Bu(t) + C_i \dot{x}(t)], \quad t \neq t_k
\]

\[
\Delta x(t_k) = \sum_{i=1}^{q} h_i(x(t))G_i(x(t_k)), \quad k = 1, 2, \ldots \tag{3}
\]

where

\[
h_i(x(t)) = \frac{w_i(x(t))}{\sum_{i=1}^{q} w_i(x(t))}, \quad w_i(x(t)) = \prod_{j=1}^{q} M_j^i(x_j(t))
\]

We assume that $w_i(x(t)) \geq 0$ and $\sum_{i=1}^{q} w_i(x(t)) > 0$. It is clear that

\[
h_i(x(t)) \geq 0, \quad \sum_{i=1}^{q} h_i(x(t)) = 1
\]

The control objective is to design a state feedback fuzzy controller such that the closed-loop system is exponential stable, that is to say, there exist $M, \gamma > 0$ such that

\[
\|x(t)\| \leq M\|\hat{\phi}\|e^{-\gamma(t-t_0)} \to 0, \quad t \to \infty \tag{4}
\]

where $\|\hat{\phi}\| = \sup_{t_0 \leq t \leq t_0 + h} \|\phi(t)\|$.

Based on the so-called PDC idea, the state feedback fuzzy controller is designed as follows

**Plant Rule i**

IF $z_i(t)$ is $M_i^l$ and, \ldots, and $z_q(t)$ is $M_q^l$

THEN $u(t) = F_i x(t)$

\[
u(t) = \sum_{i=1}^{q} h_i(x(t))F_i x(t) \tag{5}
\]

where $F_i \in \mathbb{R}^{m \times p_i}, i = 1, 2, \ldots, q$ are constant control gains to be determined later.

By using the fuzzy inference method with a singleton fuzzification, product inference and centre average defuzzification, the overall fuzzy regulator is represented by

\[
\dot{x}(t) = \sum_{i=1}^{q} h_i(x(t))h_i(x(t))[(A_i + C_i F_i)\dot{x}(t) + B_i u(t - \tau(t))], \quad t \neq t_k
\]

\[
\Delta x(t_k) = \sum_{i=1}^{q} h_i(x(t))G_i(x(t_k)), \quad k = 1, 2, \ldots \tag{7}
\]

Before proceeding, we recall some preliminaries which will be used throughout the proofs of our main results.

**Definition 1 [11, 19]:** Let $V : \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}^+$, then $V$ is said to belong to class $\mathcal{V}_0$ if

(i) $V$ is continuous in each of the sets $[t_k - 1, t_k) \times \mathbb{R}^n$, and for each $t \in \mathbb{R}^n$, $k = 1, 2, \ldots, \lim_{t \to t_k} V(t, p(t)) = V(t_k^+, x)$ exists;

(ii) $V$ is locally Lipschitzian in $x \in \mathbb{R}^n$.

Let $\dot{x} = f(t, x)$, then we have the following generalized derivative of a Lyapunov function $V(t, x)$.

**Definition 2 [11, 19]:** For $(t, x) \in [t_{k-1}, t_k) \times \mathbb{R}^n$, we define

\[
D^+ V(t, x) \doteq \lim_{h \to 0^+} \frac{1}{h} [V(t + h, x + h f(t, x)) - V(t, x)]
\]

**Lemma 1 (Halanay Lemma, [24]):** Let $m(t)$ be a scalar positive function and assume that the following condition holds

\[
D^+ m(t) \leq -a m(t) + b \bar{m}(t), \quad t \geq t_0
\]

where constants $a > b > 0$. Then, there exist $\alpha > 0$, such that for all $t \geq t_0$

\[
m(t) \leq \tilde{m}(t) e^{-\alpha(t-t_0)}
\]

here, \(\tilde{m}(t) = \sup_{t_0 \leq \tau \leq t} \{m(\tau)\}\) and $\alpha > 0$ satisfies $\alpha = a + be^{\alpha t_0} = 0$.

**Lemma 2 [25]:** Suppose that matrices $M_i \in \mathbb{R}^{m \times r}, i = 1, 2, \ldots, r$, and a positive semi-definite matrix $P \in \mathbb{R}^{m \times m}$ are given. If $\sum_{i=1}^{r} p_i = 1$ and $0 \leq p_i \leq 1$, then

\[
\left( \sum_{i=1}^{r} p_i M_i \right)^T P \left( \sum_{i=1}^{r} p_i M_i \right) \leq \sum_{i=1}^{r} p_i M_i^T P M_i
\]

### 3 Design of controller and stability analysis

Now, we present the design of controller and stability analysis for the nonlinear fuzzy impulsive system (1) with time-varying delay.
Theorem 1: If there exist symmetric and positive definite matrix $X_i$ and some matrices $H_{ki}$ such that the following LMIs hold

$$
\begin{bmatrix}
XA_i^T + (C_i X_i H_j + aX) & X^T B_i \\
XB_i^T & -bX
\end{bmatrix} < 0,
$$

where $i = 1, 2, \ldots, g$, $j = 1, 2, \ldots, q$.

From (11), we have

$$
D^+ V(x(t)) \leq -aV(x(t)) + bV(x(t))
$$

where $V(x(t)) = \sup_{t \geq 0} [V(x(t))]$.

Since $a > b > 0$, by Lemma 1 and (12), there exists a constant $\alpha > 0$ such that for all $t \in [t_k-1, t_k)$, $k = 1, 2, \ldots$

$$
V(x(t)) \leq \tilde{V}(x(t_{k-1})) e^{-\alpha (t-t_{k-1})}
$$

where $\alpha$ satisfies $\alpha - a + b e^{\alpha \tau_0} = 0$.

On the other hand, when $t = t_k$, by Lemma 2, we have

$$
\begin{align*}
V(x(t_k)) &= x^T(t_k)X^{-1}x(t_k) \\
&= x^T(t_k) \left( \sum_{i=1}^g b_i(z(t)) \tilde{G}_{ki} \right)^T \\
&\quad \times X^{-1} \left( \sum_{i=1}^g b_i(z(t)) (I + \tilde{G}_{ki}) \right)^T \\
&\quad \times X^{-1} (I + \tilde{G}_{ki}) x(t_k) \\
&\leq \sum_{i=1}^g b_i(z(t)) x^T(t_k) (I + \tilde{G}_{ki}) X^{-1} (I + \tilde{G}_{ki}) x(t_k)
\end{align*}
$$

Taking $e^{\alpha \tau_0} \leq \lambda_k \leq e^{\alpha (t_k-t_{k-1})}$, we have

$$
\begin{align*}
V(x(t_k)) &\leq \sum_{i=1}^g b_i(z(t)) x^T(t_k) (I + \tilde{G}_{ki}) X^{-1} (I + \tilde{G}_{ki}) x(t_k) \\
&\quad + x(t_k) X^{-1} x(t_k)
\end{align*}
$$

Pre- and post-multiplying both sides of (9) by

$$
\begin{bmatrix}
X^{-1} & 0 \\
0 & X^{-1}
\end{bmatrix}
$$

we have

$$
\begin{bmatrix}
-x(t_k) X^{-1} (I + \tilde{G}_{ki}) & -X
\end{bmatrix}
\begin{bmatrix}
-\lambda_k X^{-1} (I + \tilde{G}_{ki}) X^{-1} x(t_k) \\
(I + \tilde{G}_{ki}) X^{-1} x(t_k)
\end{bmatrix} < 0
$$

Applying Schur complement to (15) yields

$$
(I + \tilde{G}_{ki}) X^{-1} (I + \tilde{G}_{ki}) - \lambda_k X^{-1} < 0
$$

Therefore we obtain

$$
V(x(t_k)) \leq \lambda_k V(x(t_k))
$$
By mathematical induction, one can show that

\[
V(x(t)) \leq \lambda_1 \lambda_2 \cdots \lambda_{k-1} \lambda_k \max(X^{-1}) \|\dot{x}\|^2 e^{-\alpha(t-t_0)},
\]

\[
t \in [t_{k-1}, t_k), \quad k = 1, 2, \ldots
\]

(18)

Indeed, when \( k = 1 \), since \( \|x(t)\| = \|\phi(t)\| \leq \|\phi\| \),
\( t \in [t_0 - \tau_0, t_0] \), we have

\[
V(x(t)) \leq \max(X^{-1}) \|x(t)\|^2 \leq \lambda_1 \max(X^{-1}) \|\phi\|^2,
\]

\[
t \in [t_0 - \tau_0, t_0]
\]

Hence

\[
\dot{V}(x(t_0)) \leq \lambda_1 \max(X^{-1}) \|\phi\|^2
\]

\[
V(x(t)) \leq \dot{V}(x(t_0))e^{-\alpha(t-t_0)}
\]

\[
\leq \lambda_1 \max(X^{-1}) \|\phi\|^2 e^{-\alpha(t-t_0)}, \quad t \in [t_0, t_1)
\]

Thus, (18) holds for \( k = 1 \).

Next, assume that (18) holds for \( k \leq m, m \geq 1 \). Then, we need to show (18) still holds when \( k = m + 1 \).

By (13) and (17) and the above induction assumption, one has

\[
V(x(t_k)) \leq \lambda_1 \lambda_2 \cdots \lambda_{k-1} \lambda_k \lambda_k \max(X^{-1}) \|\phi\|^2 e^{-\alpha(t-t_0)}
\]

\[
(t \in [t_k, t_{k+1}])
\]

\[
V(x(t)) \leq \max \{ V(x(t_k))e^{-\alpha(t-t_k)}
\]

\[
\leq \sup_{t_k - \tau_0 \leq t \leq t_k} \max \{ V(x(t_k))e^{-\alpha(t-t_k)}
\]

\[
\leq \max \{ \lambda_1 \lambda_2 \cdots \lambda_{k-1} \lambda_k \max(X^{-1}) \|\phi\|^2 e^{-\alpha(t_0 - \tau_0)}
\]

\[
\lambda_1 \lambda_2 \cdots \lambda_{k-1} \lambda_k \lambda_k \max(X^{-1}) \|\phi\|^2 e^{-\alpha(t-t_0)}
\]

\[
\leq \max \{ \lambda_1 \lambda_2 \cdots \lambda_{k-1} \lambda_k \max(X^{-1}) \|\phi\|^2 e^{-\alpha(t-t_0)}
\]

\[
\lambda_1 \lambda_2 \cdots \lambda_{k-1} \lambda_k \lambda_k \max(X^{-1}) \|\phi\|^2 e^{-\alpha(t-t_0)}
\]

\[
\lambda_1 \lambda_2 \cdots \lambda_{k-1} \lambda_k \lambda_k \max(X^{-1}) \|\phi\|^2 e^{-\alpha(t-t_0)}
\]

\[
\lambda_1 \lambda_2 \cdots \lambda_{k-1} \lambda_k \lambda_k \max(X^{-1}) \|\phi\|^2 e^{-\alpha(t-t_0)}
\]

Therefore

\[
V(x(t)) \leq \lambda_1 \lambda_2 \cdots \lambda_{k-1} \lambda_k \lambda_k \max(X^{-1}) \|\phi\|^2 e^{-\alpha(t-t_0)}
\]

Since \( e^{\alpha t_0} \leq \lambda_1 \leq e^{\alpha(t-t_0)}/L \), thus \( 0 < \lambda_1 e^{\alpha(t-t_0)}/L \leq 1 \).

Therefore for \( t \in [t_k, t_{k+1}) \)

\[
\lambda_{k+1} \max(X^{-1}) \|x(t)\|^2 \leq V(x(t)) \leq \lambda_{k+1} \max(X^{-1}) \|\phi\|^2
\]

\[
eq e^{-\alpha(t-t_0)/L} \|x(t)\|^2 \leq e^{\alpha(t-t_0)/L} \lambda_{k+1} \max(X^{-1}) \|\phi\|^2
\]

\[
\leq e^{\alpha(t-t_0)/L} \lambda_{k+1} \max(X^{-1}) \|\phi\|^2 e^{-\alpha(t-t_0)/L}
\]

Let \( M = \sqrt{\lambda_{k+1} \max(X^{-1})/\lambda_{k+1} \max(X^{-1})} \|\phi\| \) and \( \gamma = \alpha(L - 1)/L \), it is easy to see that

\[
\|x(t)\| \leq M\|\phi\|e^{-\gamma(t-t_0)}, \quad t \geq t_0
\]

Therefore the T–S fuzzy system (1) with time-varying delay is global exponential stable via the state feedback fuzzy controller (5). In this case, the feedback gains \( F_i \) are given by \( F_i = H_i X^{-1}, \ i = 1, 2, \ldots , q \).

Remark 3: Theorem 1 provides sufficient conditions for the global exponential stability of the T–S fuzzy system (1) with time-varying delay. The conditions in Theorem 1 are all in terms of LMIs, which can be efficiently verified via solving the LMIs numerically by interior point. And the feedback gains can also be obtained via solving the LMIs.

Remark 4: Compared with the existing results of fuzzy time-delay system, our results may be conservative. As shown in [18], we can also obtain less conservative conditions by using the same approaches given by Tanaka et al. [2] and Kim and Lee [3]. At the same time, the conditions in Theorem 1 do not include information on the delay. Therefore our results are delay-independent stability criteria. Generally speaking, delay-independent stability criteria are more conservative that delay-dependent criteria when the delay is small, especially there exist uncertainties in systems. So, our future work is to find delay-dependent exponential stability conditions for nonlinear fuzzy impulsive system with time-varying delay.

Remark 5: The controller design procedure is as follows: Choose two constants \( a \) and \( b \) such that \( a > b > 0 \). Then we obtain a constant \( \alpha \), which is the unique positive root of the equation (10). Choose parameters \( \lambda_k \) such that \( e^{\alpha t_0} \leq \lambda_k \leq e^{\alpha(t-t_0)/L}, \ k = 1, 2, \ldots \). If there exist symmetric and positive definite matrix \( X_i \) and some matrices \( H_i \) such that the LMIs (8) and (9) hold, thus the feedback gains \( F_i \) are given by \( F_i = H_i X^{-1}, \ i = 1, 2, \ldots , q \). Then the closed-loop fuzzy system is global exponential stable with decay rate \( \gamma = \alpha(L - 1)/L \).

For the fuzzy impulsive time-delay system (1) with \( \alpha(t) = 0 \), that is, the unforced fuzzy impulsive system (2) with time-varying delay, we can obtain the following result.
Corollary 1: If there exist symmetric and positive definite matrix $X$, such that the following LMIs hold

$$
\begin{bmatrix}
X^T A_i + A_i X + a X & B_i X \\
X B_i^T & -bX
\end{bmatrix} < 0, \quad i = 1, 2, \ldots, q
$$

(19)

$$
\begin{bmatrix}
-\lambda_i X & (X + \bar{G}_i X)^T \\
X + \bar{G}_i X & -X
\end{bmatrix} < 0, \quad i = 1, 2, \ldots, q_i
$$

(20)

where $a > b > 0$ and $\alpha > 0$ is the unique positive root of the following equation

$$
\alpha - a + b e^{\alpha \tau_0} = 0
$$

The parameters $\lambda_i$ are specified by the designer, where $e^{\alpha \tau_0} \leq \lambda_i \leq e^{\alpha \tau_0-i_{i-1})/k}$, $k = 1, 2, \ldots$. Then the fuzzy impulsive time-delay system (1) with $u(t) = 0$ is global exponentially stable.

4 Numerical examples

Example 1: Consider a continuous stirred tank reactor nonlinear system [5, 26]. As in [5, 26], the system model is given by the following equations

$$
\dot{x}_1 = -\frac{1}{\lambda} x_1(t) + D_o (1 - x_1(t))
$$

$$
\times \exp \left( \frac{x_2(t)}{(1 + x_2(t))/\gamma_0} \right) + \left( \frac{1}{\lambda} - 1 \right) x_1(t - \tau(t))
$$

$$
\dot{x}_2 = \left( \frac{1}{\lambda} + \beta \right) x_2(t) + HD_o (1 - x_1(t))
$$

$$
\times \exp \left( \frac{x_2(t)}{(1 + x_2(t))/\gamma_0} \right) + \left( \frac{1}{\lambda} - 1 \right) x_2(t - \tau(t)) + \beta u(t)
$$

$$
x_i = \phi_i(t), \quad t \in [-\tau_0, 0], \quad i = 1, 2
$$

(21)

where $\gamma_0 = 20$, $H = 8$, $D_o = 0.072$, $\lambda = 0.8$ and $\beta = 0.3$. The state $x_1(t)$ corresponds to the conversion rate of the reactor, $0 \leq x_1(t) \leq 1$ and $x_2(t)$ is the dimensionless temperature. Assume that only the temperature can be measured on line, that is, $y(t) = [0 \ 1] x(t)$, $x(t) = [x_1(t), x_2(t)]^T$.

Considering the impulsive effect of the system (21) and using the same modelling approach developed in [5, 26], we can obtain the following T–S fuzzy model with impulse to represent system (21).

Plant Rule $i$

IF $x_2(t)$ is $M_i^1$

THEN

$$
\dot{x}(t) = A_i x(t) + B_i x(t - \tau(t)) + C_i u(t),
$$

$$
\Delta x(t) = \tilde{G}_i x(t), \quad k = 1, 2, \ldots
$$

$$
x(t) = \phi(t), \quad t \in [-\tau_0, 0]
$$

$$
i = 1, 2, 3
$$

(22)

with

$$
A_1 = \begin{bmatrix}
-1.4274 & 0.0757 \\
-1.4189 & -0.9442
\end{bmatrix},
$$

$$
A_2 = \begin{bmatrix}
-2.0508 & 0.3958 \\
-6.4066 & 1.6268
\end{bmatrix},
$$

$$
A_3 = \begin{bmatrix}
-4.5279 & 0.3167 \\
-26.2228 & 0.9387
\end{bmatrix},
$$

$$
B_1 = B_2 = B_3 = \begin{bmatrix}
0.25 & 0 \\
0 & 0.25
\end{bmatrix},
$$

$$
C_1 = C_2 = C_3 = \begin{bmatrix}
0 \\
0.3
\end{bmatrix}, \quad \phi(t) = \begin{bmatrix}
\phi_1(t) \\
\phi_2(t)
\end{bmatrix}
$$

The membership functions are selected as follows

$$
M_1^1(x_2(t)) = \begin{cases}
1, & x_2 \leq 0.8862 \\
1 - \frac{x_2 - 0.8862}{2.7520 - 0.8862}, & 0.8862 < x_2 < 2.7520, \\
0, & x_2 \geq 2.7520
\end{cases}
$$

$$
M_1^2(x_2(t)) = \begin{cases}
1 - M_1^1(x_2(t)), & x_2 < 2.7520 \\
M_1^2(x_2(t)), & x_2 \geq 2.7520
\end{cases}
$$

$$
M_2^2(x_2(t)) = \begin{cases}
0, & x_2 \leq 2.7520 \\
\frac{x_2 - 2.7520}{4.7052 - 2.7520}, & 2.7520 < x_2 < 4.7052, \\
1, & x_2 \geq 4.7052
\end{cases}
$$

The time delay is chosen to be $\tau(t) = 0.7 \sin^2(t + \pi/3)$. The impulsive matrices are as follows: for $m \in \mathbb{N}$, $i = 1, 2, 3$, if $k = 4m$, then $\bar{G}_i = \text{diag}[0.1, 0.1]$; if $k = 4m + 1$, then $\bar{G}_i = \text{diag}[-0.1, -0.1]$; if $k = 4m + 2$, then $\bar{G}_i = \text{diag}[0.1, -0.1]$; if $k = 4m + 3$, then $\bar{G}_i = \text{diag}([-0.1, 0.1])$.

The design parameters are chosen as follows: $\Delta t_k = t_k - t_{k-1} = 1$, $k = 1, 2, \ldots$; $\tau_0 = 0.8$, $L = 1.1$, $a = 1$, $b = 0.5$, $\alpha = 0.34242962$ is the unique positive root of the equation $\alpha - a + b e^{\alpha \tau_0} = 0$; $e^{a \tau_0} \leq \lambda_1 = 1.3610 = e^{0.9a} \leq e^{\alpha \tau_0/2}$, $k = 1, 2, \ldots$. Using MATLAB LMI toolbox, we obtain that

Figure 1 Responses of system state

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\( F_1 = [6.7613 - 1.3350], \ F_2 = [16.5423 - 8.1535] \) and \( F_3 = [34.9576 - 13.1386] \). Simulation results are shown in Figs. 1 and 2 under initial condition \( x(t) = \phi(t) = [0.5 - 1]^T, t \in [-\tau_0, 0] \).

5 Conclusions

We have investigated the problem of robust fuzzy control for a class of nonlinear systems with time-varying delay. Based on Lyapunov method and LMI technique, some criteria have been proposed to guarantee the global exponential stability of the closed-loop system. Numerical simulations have been included to demonstrate the effectiveness of the proposed controller.

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7 References


