Optimal Throughput for 802.11 DCF with Multiple Packet Reception

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Abstract—In this paper, we propose an analytical model for evaluating the MAC throughput in an unsaturated IEEE 802.11 wireless local area network (WLAN) where multiple packets reception (MPR) is possible using multiuser detection techniques. In particular, a recently proposed successive interference cancellation (SIC) scheme for MPR is considered where users can randomly choose the transmission power from a set of discrete power levels. We derive an explicit expression for throughput of the WLAN based on such an SIC scheme and validate the accuracy of the model via ns-2 simulation results. We show that the throughput is significantly improved compared to the conventional 802.11 MAC protocol just by resolving collisions between two packets with different transmission power levels. In addition, we provide the optimal power distribution to maximize the throughput achievable in an SIC-enabled WLAN.

I. INTRODUCTION

Since its inception, IEEE 802.11 wireless local area networks (WLANs) have attracted a lot of interests. Due to its simplicity and scalability characteristics, IEEE 802.11 WLANs have become one of the most deployed worldwide wireless networks which are expected to play an important role in multimedia home networks and next-generation wireless communications [1].

In the 802.11 standard, two mechanisms to access the medium are defined. The fundamental mechanism is the contention-based distributed coordination function (DCF), while the optional mechanism is the contention-free point coordination function. The DCF scheme is focused in this paper. DCF is based on the carrier sense multiple access with collision avoidance (CSMA/CA) multiple access protocol. Collisions happen when more than one packet are transmitted simultaneously and result in no useful throughput. However, with the development of multiuser detection (MUD) techniques [2], it becomes possible to receive multiple packets which are transmitted simultaneously. For instance, by using MUD techniques, in CDMA [3] and multi-antenna systems [4], multiple packet reception (MPR) can be achieved. There have been various proposed MUD techniques which include zero-forcing, minimum-mean-square-error, maximum-likelihood, parallel interference cancellation and successive interference cancellation (SIC) [2].

The development of MUD techniques brings the beneficial MPR capability, but it also raises two important questions [5]: how does the MPR capability affect the MAC protocol performance and how to adjust the protocol to fully utilize the MPR capability? There have been many literatures which focused on modeling the random access networks with MPR capability. In [6] and [7], a model for general MPR channels was proposed and the performance for slotted ALOHA with infinite-user assumption was analyzed. It was extended to finite-user ALOHA system in [8]. In [9] and [5], the protocols to exploit the MPR capability were proposed. A Multi-Queue Service Room (MQSR) protocol was introduced in [9] which could yield the optimal utilization of the MPR channel, but with the drawback of high computational complexity. In [5], a much simpler protocol with comparable performance to MQSR was proposed. In [10], the effect of MPR on the MAC layer collision resolution schemes was studied. It was shown that the widely used binary exponential backoff scheme did not yield the close-to-optimal network throughput for both non-carrier-sensing networks and carrier-sensing networks when operated in basic access mode of DCF. Furthermore, protocol which considered both the MAC layer design and the PHY layer signal processing design to enable MPR capability in distributed random access WLANs was also proposed.

When SIC is used in high rate applications, its performance deteriorates when the interfering signals have equal power. A simple way to address this issue is to use a central coordinator to allocate different power levels to different users. However, such a coordinator does not exist in distributed multi-access systems. Therefore, a practical SIC scheme [11] is proposed recently to enhance the MPR capability. In this scheme, power randomization is imposed at the transmitters of an SIC-based MUD system. Using the slotted ALOha as an example, it
has been shown that, for the same total power, the system throughput is significantly improved when this SIC scheme is used. In this paper, we investigate the throughput improvement of DCF when SIC is used at the physical layer.

Below we consider an unsaturated IEEE 802.11 network in an infrastructure topology based on the physical layer utilizing SIC scheme [11]. To this end, when a node transmits a packet, it randomly chooses a power level from a set of discrete power levels. Different from the traditional DCF, collisions between two packets which are transmitted with different power levels can be resolved. The throughput performance with this MPR capability is modeled using a similar approach as in [12]. The closed-form expression for optimal distribution of discrete power levels to maximize system throughput is also derived.

The remainder of the paper is organized as follows. In Section II, the SIC scheme with power randomization will be briefly introduced. In Section III, the throughput performance at the MAC layer of IEEE 802.11 WLANs enhanced by such an SIC scheme is modeled. The accuracy of the model is validated in Section IV. In Section V, the optimal distribution of discrete power levels to maximize throughput is derived. Section VI concludes the paper.

II. SIC WITH POWER RANDOMIZATION

In this section, the SIC with power randomization (SPR) scheme [11] is briefly explained.

Consider a 2-user Gaussian multiple-access channel. The received signal $y$ can be represented by

$$y = \sum_{i=1}^{2} \sqrt{e_i} x_i + n$$

where $x_i$ is the transmitted signal of user $i$ (here $i = 1, 2$) with power normalized to 1, $e_i$ is power level used to transmit $x_i$, and $n$ is the complex Gaussian noise with mean 0 and variance $N_0$. For convenience, we assume that the rate of each user is equal and denoted as $R$ ($R \geq 1$). Consider that SIC is deployed in the receiver. Assume that user 1 is decoded first where the signal of user 2 is regarded as the noise. For reliable transmission, the following constraint is required,

$$\log_2(1 + \frac{e_1}{e_2 + N_0}) \geq R. \quad (2)$$

Upon successful decoding, signal of user 1 is subtracted from $y$. Then the information for user 2 can be successfully decoded from the residual signal after subraction provided that,

$$\log_2(1 + \frac{e_2}{N_0}) \geq R. \quad (3)$$

Similar discussions can be given to the situation with user 2 being decoded first.

We now design a transmission strategy based on the constraints in (2) and (3). Denote by $\mathcal{E} = \{E_i\}$ a set of positive real values recursively defined below.

$$E_i = \begin{cases} 0 & i = 0, \\ (2^R - 1)(E_{i-1} + N_0) & i > 0. \end{cases} \quad (4)$$

The set $\mathcal{E}$ has the following two properties.

Property (i): For $i > j$, $E_i$ and $E_j$ satisfy (2), i.e.,

$$\log_2(1 + \frac{E_i}{E_j + N_0}) \geq R. \quad (5)$$

Property (ii): For any $j > 0$, $E_j$ satisfies (3), i.e.,

$$\log_2(1 + \frac{E_j}{N_0}) \geq R. \quad (6)$$

Now let the power level of each of users 1 and 2 be randomly drawn from $\mathcal{E}$. According to Properties (i) and (ii), when two users are transmitting simultaneously, as long as their chosen power levels are different, both of their signals can be successfully decoded. Note that the situation with $e_i = 0$ means user $i$ is not transmitting and so there is no collision. We will regard this as a successful case.

Incidentally, it has been proved in [11] that (4) leads to the optimal power profile based on which the achieved throughput is not worse than any other profile while less average power is consumed. In Section V, based on the optimal power profile, we will derive the optimal distribution for those discrete power levels in $\mathcal{E}$ in order to maximize the system throughput.

III. THROUGHPUT OF DCF WITH SPR

There have been many works which focused on modeling the performance of IEEE 802.11 DCF. In [13], a seminal analytical model was proposed to accurately compute the 802.11 DCF throughput for saturated networks. Extending the model in [13], an unsaturated model was proposed in [14] which was proved to have good accuracy. However, the complexity of the model was a concern since it involves a large state space. A much simpler model was proposed in [15], but with the sacrifice of accuracy. In [12], a simple model for unsaturated case was proposed which could obtain comparable accuracy compared with other existing unsaturated analytical models.

In this section, we extend the model in [12] to evaluate the throughput of basic access mode of DCF with the capability of multiple packet reception enabled by SPR. Consider an unsaturated IEEE 802.11 network consisting of $N$ nodes in a infrastructure mode. The minimum contention window and the retransmission limit used in DCF are denoted as $W$ and $K$, respectively.

Let $\tau'$ be the attempt rate per slot given that a node has packet to send and $\tau$ be the unconditional attempt rate per slot for each node. Also, let $\rho$ be the probability that a node has packets to send (i.e. its queue is not empty). Hence, we have

$$\tau = \rho \tau'. \quad (7)$$

Assuming that packets arrive at a node with rate $\lambda$ [pkts/sec], and that each node has an infinite buffer. Each node can be modeled as an $G/G/1/\infty$ queue [12], with the service time being determined by the DCF protocol. Let $\bar{S}$ be the average service time, we have the probability of a nonempty buffer $\rho = \lambda \bar{S}$. \quad (8)
Thus the general expression for $\tau$ is given by
\[
\tau = \min(1, \rho)\tau'.
\] (9)
where the $\min()$ function is used to prevent the probability of a nonempty buffer from exceeding one since $\rho$ might be larger than one when the network becomes saturated.

First, we need to find the expression for $\tau'$. Let $\gamma$ be the collision probability experienced by a tagged node, $R(\gamma)$ and $\overline{W}$ be the average number of attempts and the mean backoff time (in slots) till a packet transmission is finished (either successful or not), respectively. Denote by $b_i$ the mean backoff duration (in slots) at the $i$th attempt, $0 \leq i \leq K$. With the binary exponential backoff, $b_i$ is given by
\[
b_i = \begin{cases} 
\frac{W}{2} & i = 0, \\
2^ib_0 & 1 \leq i \leq m-1, \\
2^mb_0 & m \leq i \leq K,
\end{cases}
\] (10)
where $m$ determines maximum backoff window size (i.e. $CW_{max} = 2^mW$). Then it is straightforward to obtain [16]
\[
R(\gamma) = 1 + \gamma + \ldots + \gamma^K, \\
\overline{W} = b_0 + \gamma b_1 + \ldots + \gamma^K b_K.
\] (11)
As in [16], $\tau'$ can be expressed as a ratio between $R(\gamma)$ and $\overline{W}$,
\[
\tau' = \frac{R(\gamma)}{\overline{W}} = \frac{1 + \gamma + \ldots + \gamma^K}{b_0 + \gamma b_1 + \ldots + \gamma^K b_K}.
\] (12)
When accessing the channel, each node can select a power level from $\{E_1, E_2, \ldots\}$ to transmit its packets (i.e. MAC frame). With SPR, packets can be received successfully if there is no collision, or collision of two packets that have been transmitted on different power levels. Let $p_i$ be the probability that a node chooses to transmit at power level $E_i$. The probability $P[E_i \neq E_j]$ that two nodes do not choose the same power levels is given by
\[
P[E_i \neq E_j] = 1 - \sum_{i=1}^{M} p_i^2.
\] (13)
In the special case that power levels are uniformly distributed, then
\[
P[E_i \neq E_j] = 1 - M(\frac{1}{M})^2
\] (14)
where $M$ is the number of non-zero powers available.
Let $P_b$ and $P_s$ be the probability that a chosen slot is busy and that a packet transmission is successful given that there is an activity in that slot, respectively. We have
\[
P_b = 1 - (1 - \tau)^N
\] (15)
\[
P_s = \frac{P_1 + P_2}{P_b}
\] (16)
where
\[
P_1 = \binom{N}{1} \tau (1 - \tau)^{N-1}
\] (17)
is the probability that there is only one packet transmitted in a slot, and
\[
P_2 = \binom{N}{2} \tau^2 (1 - \tau)^{N-2} P[E_i \neq E_j]
\] (18)
is the probability that two packets are simultaneously transmitted in a single slot but with different power levels.
To calculate the $S$, first we determine the virtual slot time $T_v$ (in seconds) which is the mean time that elapses for one decrement of the backoff counter. Considering that, with probability $1 - P_b$, the slot time is idle; with probability $P_b P_s$, it contains a successful transmission and with probability $P_b(1 - P_s)$ it contains a collision. Thus, the virtual slot time $T_v$ is given by [12]
\[
T_v = (1 - P_b)\sigma + P_b P_s (T_s + \sigma) + P_b(1 - P_s)(T_c + \sigma)
\] (19)
where $\sigma$ is an idle slot time,
\[
T_s = T_{data} + T_{SIFS} + T_{ACK} + T_{DIFS}
\] (20)
and
\[
T_c = T_{data} + T_{timeout} + T_{DIFS}
\] (21)
where $T_{timeout} = T_{ACK} + T_{SIFS}$. $T_{data}$ denotes the transmission time of a data packet, $T_{ACK}$ is the transmission time of an ACK packet, $T_{SIFS}$ and $T_{DIFS}$ represent the duration of $SIFS$ and $DIFS$, respectively. The average service time $\overline{S}$ (in seconds) is then
\[
\overline{S} = \overline{W} T_v.
\] (22)
Finally $\gamma$ can be calculated as
\[
\gamma = 1 - (1 - \tau)^{N-1} - (N - 1) \tau (1 - \tau)^{N-2} P[E_i \neq E_j].
\] (23)
Equations (9), (12) and (23) establish a fixed point from which $\gamma$ can be numerically computed. The channel throughput [bits/second] is
\[
T = \frac{LP_1 + 2LP_2}{T_v}
\] (24)
where $L$ is the packet size in bits.
For easier referencing, the notations of variables used in this paper are summarized as follows,

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>number of nodes.</td>
</tr>
<tr>
<td>$W$</td>
<td>minimum contention window size.</td>
</tr>
<tr>
<td>$K$</td>
<td>retransmission limit.</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>collision probability.</td>
</tr>
<tr>
<td>$\tau'$</td>
<td>attempt rate per slot given that a node has packet to send.</td>
</tr>
<tr>
<td>$\tau$</td>
<td>unconditional attempt rate per slot.</td>
</tr>
</tbody>
</table>
mean backoff time till a packet transmission is finished.

$R(\gamma)$ mean number of attempts till a packet transmission is finished.

$m$ determines maximum backoff window size (i.e. $2^mW$).

$\lambda$ packet arrival rate at a node (pkts/sec).

$S$ average service time of a packet (which includes the access delay and transmission time of the packet).

$\rho$ probability that a node has packets to send.

$E_i$ $i$th discrete power level.

$p_i$ probability that a node chooses to transmit at power level $E_i$.

$M$ number of discrete power levels.

$P_b$ probability that a chosen slot is busy.

$P_s$ probability that a packet transmission is successful given that there is an activity in that slot.

$P_1$ probability that there is only one packet transmitted in the slot.

$P_2$ probability that two packets are transmitted in the slot but with different power levels.

$\sigma$ an idle slot time.

$T_v$ virtual slot time.

$T_s$ mean time for a successful transmission.

$T_c$ mean time for an unsuccessful transmission.

$T$ channel throughput.

$L$ packet size.

IV. Validation

The model is verified by ns-2 version 2.29. The simulation parameters are listed in Table I. Average throughput is normalized by the data transmission rate $R_{data}$. And the normalized offered load is given by

$$l = \frac{N\lambda L_{pay}}{R_{data}}$$

where $L_{pay}$ is the packet payload in bits.

For simplicity, we first study the performance of DCF with SPR under uniform distribution of discrete power levels, i.e., $P[E_i \neq E_j] = 1 - M(1/M)^2$. The packet payload is set to be 500 bytes. Fig. 1 and Fig. 2 plot the collision probability and throughput against offered load, respectively, for various $M$ and $N$. The accuracy of the proposed performance model is demonstrated by the agreement between the simulation and analytical results.

Next, we evaluate the performance of DCF with SPR under arbitrary distributions of power levels. Fig. 3 plots the throughput against a set of power level distributions in which the number of nodes and available power levels is set to be 5 and 3, respectively. The set contains some combinations of probabilities of the three power levels. In Fig. 3, the x-axis starts with the first combination $\{0,0,1.0\}$, which means that the third power level is always chosen. The subsequent combinations are obtained by changing two of the probabilities by a step size 0.1. For example, the following combinations are $\{0,0.1,0.9\}, \{0,0.2,0.8\}, \{0,0.3,0.7\}$ and so on. The arrival rate $\lambda$ and the packet payload are set to be 250 pkts/sec and 500 bytes, respectively. Again, the accuracy of the proposed model is demonstrated by the agreement between the simulation and analytical results.

V. Optimization

In this section, we aim to find the optimal distribution of the discrete power levels used at a node that maximizes the overall throughput. From Section III, the channel throughput is given in (24). Accordingly, we define the following optimization problem

$$\text{maximize } L \frac{P_1 + 2P_2}{T_v}$$

subject to

$$\sum_{i=1}^{M} p_i = 1$$

$$\sum_{i=1}^{M} p_i E_i = E_{av}$$

$$0 \leq p_i \leq 1.$$
to one and that the total average energy is kept at a fixed value. This constraint optimization problem can be further expressed in the equivalent unconstrained form using the Lagrange multipliers $\alpha$ and $\beta$ as follows.

\[
T(p_1, \ldots, p_M) = \frac{\tau(1 - \tau)^{N-1} + (N - 1)\tau^2(1 - \tau)^{N-2}(1 - \sum_{i=1}^{M} p_i^2)}{\sigma + T_s[1 - (1 - \tau)^N]} + \alpha(\sum_{i=1}^{M} p_i - 1) + \beta(\sum_{i=1}^{M} p_i E_i - E_{av}).
\]

(28)

Note that $\tau$ in (28) is a function of $p_i$ through the expression in (9) which is a part of the fixed point formulation defined in Section III. To maximize the function $T(p_1, \ldots, p_M)$ in (28) for each of the individual probabilities $p_i$, let us consider its partial derivative with respect to the variable $p_i$ at a fixed $\tau$ value. Given a fixed $\tau$,

\[
\frac{\partial T}{\partial p_i} = -2p_i \frac{LN(N - 1)\tau^2(1 - \tau)^{N-2}}{\sigma + T_s[1 - (1 - \tau)^N]} + \alpha + \beta E_i.
\]

(29)

Set $\frac{\partial T}{\partial p_i} = 0$, we obtain

\[
p_i = \frac{\alpha + \beta E_i}{2A}
\]

(30)

where

\[
A = \frac{LN(N - 1)\tau^2(1 - \tau)^{N-2}}{\sigma + T_s[1 - (1 - \tau)^N]}.
\]

Substituting (30) into two linear equality constraints in (26), we obtain

\[
\frac{M\alpha + E_N\beta}{2A} = 1
\]

and

\[
\frac{2A}{E_N\alpha + E_s\beta} = E_{av}
\]

(31)

where

\[
E_N = \sum_{i=1}^{M} E_i, \quad E_s = \sum_{i=1}^{M} E_i^2.
\]

Solving (31) for $\alpha$ and $\beta$ we have

\[
\alpha = 2A \left[ \frac{1 - E_N(E_{av}M - E_N)}{M - ME_s - E_N^2} \right]
\]

\[
\beta = 2A \frac{E_{av}M - E_N}{ME_s - E_N^2}
\]

(32)

which gives the closed-form expression of $p_i$ as

\[
p_i = \frac{E_s - E_{av}E_N + ME_{av}E_i - E_NE_i}{ME_s - E_N^2}.
\]

(33)

Note that the above expression (33) for optimal $p_i$ is independent of $\tau$ and thus it will also maximize (28) with an arbitrary $\tau$ value.
In order to validate the accuracy of the obtained results, we perform a detailed comparison between the derived closed-form expression results for $p_i$ and that obtained by standard optimization toolbox [17] for different number of power levels ($M$). Results are shown in Table II, where the following set of parameters is used: $N = 10$, $\lambda = 200$, and $W_0 = 32$, $m = 5$, $K = 7$, $R = 1$, $N_0 = 1$. It can be seen from the table that the closed-form derivation gives exactly the same results as the one by using optimization toolbox.

VI. CONCLUSION

Recently, a successive interference cancellation (SIC) scheme is proposed to enhance the MPR capability. By adopting this SIC-based MUD technique at the physical layer, users can randomly choose the transmission power from a set of discrete power levels. As a result, collisions between two simultaneously transmitted packets with different transmission power levels can be resolved. We proposed a simple but accurate model to evaluate the MAC performance of an unsaturated IEEE 802.11 WLANS implementing this MUD technique at the physical layer. We have validated our analytical model by ns-2 simulation results. It has been shown that the throughput performance is significantly improved by resolving collisions between two packets while the collision probability is dramatically decreased. We have also derived the closed-form expression for the optimal power distribution which maximizes the throughput.

REFERENCES

TABLE II

<table>
<thead>
<tr>
<th>M = 3</th>
<th>Closed-form Results</th>
<th>Optimization Results</th>
<th>M = 5</th>
<th>Closed-form Results</th>
<th>Optimization Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>$p_i = (0.6333, 0.3333, 0.3333)$</td>
<td>$T = 0.3644$</td>
<td>24</td>
<td>$p_i = (0.32, 0.26, 0.2, 0.14, 0.08)$</td>
<td>$T = 0.4035$</td>
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<tr>
<td>16</td>
<td>$p_i = (0.5333, 0.3333, 0.1333)$</td>
<td>$T = 0.3777$</td>
<td>26</td>
<td>$p_i = (0.28, 0.24, 0.2, 0.16, 0.12)$</td>
<td>$T = 0.4091$</td>
</tr>
<tr>
<td>18</td>
<td>$p_i = (0.4333, 0.3333, 0.2333)$</td>
<td>$T = 0.3862$</td>
<td>28</td>
<td>$p_i = (0.24, 0.22, 0.16, 0.16)$</td>
<td>$T = 0.4091$</td>
</tr>
<tr>
<td>2</td>
<td>$p_i = (0.3333, 0.3333, 0.3333)$</td>
<td>$T = 0.389$</td>
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<td>$p_i = (0.2, 0.2, 0.2, 0.2)$</td>
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<tr>
<td></td>
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<td>$p_i = (0.16, 0.16, 0.2, 0.22, 0.24)$</td>
<td>$T = 0.4091$</td>
</tr>
<tr>
<td>24</td>
<td>$p_i = (0.1333, 0.3333, 0.5333)$</td>
<td>$T = 0.3777$</td>
<td>34</td>
<td>$p_i = (0.12, 0.16, 0.2, 0.24, 0.28)$</td>
<td>$T = 0.4091$</td>
</tr>
<tr>
<td>26</td>
<td>$p_i = (0.0333, 0.3333, 0.6333)$</td>
<td>$T = 0.3644$</td>
<td>36</td>
<td>$p_i = (0.08, 0.14, 0.2, 0.26, 0.32)$</td>
<td>$T = 0.4091$</td>
</tr>
</tbody>
</table>


[17] MATLAB 7.6.0 (R2008a).