ADAPTIVE SUPPRESSION OF NARROWBAND DIGITAL INTERFERERS FROM SPREAD SPECTRUM SIGNALS

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ABSTRACT

We consider the application of the minimum-mean-square-error (MMSE) multiuser detection technique to the problem of suppressing the digital narrowband interference (NBI) from spread spectrum signals. The MMSE multiuser detector can be implemented using a blind adaptive method, which is ideally suited for use in the NBI suppression framework. This application requires the treatment of a single narrowband digital signal as a group of related, virtual spread-spectrum signals with very simple spreading codes. This model gives a special structure to the matrices appearing in the optimization problem implied by the MMSE criterion, and this structure is exploited herein to develop and analyze a practical adaptive algorithm. The major contribution of this paper beyond the previous work in the field of NBI suppression is the development of this adaptive algorithm that can exploit the advantages of multiuser detection in suppressing narrowband digital interference from spread-spectrum networks.

1. INTRODUCTION

Code-division multiple-access (CDMA) implemented with direct-sequence spread-spectrum signaling is among the most promising multiplexing technologies for cellular telecommunications services, such as personal communications, mobile telephony, and indoor wireless networks. The advantages of direct-sequence spread-spectrum for these services include superior operation in multipath environments, flexibility in the allocation of channels, the ability to operate asynchronously, increased capacity in bursty or fading channels, and the ability to share bandwidth with narrowband communication systems without undue degradation of either system’s performance.

In the presence of narrowband interference (NBI) caused by co-existence with conventional communications, the performance of spread-spectrum systems can be enhanced significantly through the use of active NBI suppression prior to despreading and demodulating. Over the past two decades, a significant body of research has been concerned with the development of techniques for active NBI suppression in spread-spectrum systems. Early work in this area focused on techniques based on the linear signal processing regimes of adaptive linear transversal filtering, and Fourier-domain filtering [1]. More recently, model-based techniques that employ non-standard signal processing methods, including nonlinear filtering techniques and multiuser detection (MUD) techniques, have been used to enhance the interference mitigation capabilities in CDMA systems [2].

In this paper, we consider the MUD-based approach to the suppression of digital NBI. Although this methodology has the potential for very substantial SNR improvement over other methods [3], a disadvantage is that the currently proposed implementation is not easily amenable to adaptivity. Here, we consider application of the minimum-mean-square-error (MMSE) approach to MUD (see [4]) to the NBI suppression problem. In MMSE multiuser detection, data is detected after passing the received signal (which consists of a desired signal plus multiple-access interference and ambient noise) through a linear filter that is chosen to minimize, within a constraint, the mean-square value of its output. Although this optimization criterion is not directly related to the traditional criteria based on probabilities of error, it apparently yields a detector which performs well within such criteria [4]. Moreover, the minimization of mean-square error in this context can be performed using a blind adaptive method (see [5]), and so the technique is ideally suited for use in the NBI suppression framework.

The MMSE detector has been considered previously in the context of suppressing wideband multiple-access interference in CDMA networks. Its application to NBI suppression has not been considered. This application requires the treatment of a single narrowband digital signal as a group of related, virtual spread-spectrum signals with very simple spreading codes. This model gives a special structure to the matrices appearing in the optimization problem implied by the MMSE criterion, and this structure is exploited herein to develop and analyze a practical adaptive algorithm. The major contribution of this paper beyond the previous work in the field of NBI interference suppression is the development of this adaptive algorithm that can exploit the advantages of multiuser detection in suppressing narrowband digital interference from spread-spectrum networks.

2. MMSE MULTIUSER DETECTOR FOR DIGITAL NBI SUPPRESSION

2.1. System Model

We follow the system model of [3]. Consider a system with one SS signal and one narrowband binary signal in an other-
wise additive white Gaussian noise (AWGN) channel. Each data bit of the SS user is modulated by a pseudo noise (PN) signature sequence (each entry being one chip), which spreads the signal in the frequency domain. We assume for now that the narrowband signal is synchronised with the SS signal. Furthermore, we assume a relationship between the data rates of the two users, i.e., $m$ bits of the narrowband user occur for each bit of the SS user. As shown in Fig.1, the narrowband digital signal can be regarded as $m$ virtual users, each with its virtual signature sequences. The first virtual user's signature sequence is one during the first narrowband user's bit interval, i.e., a virtual chip interval, and zero everywhere else. Similarly, each other narrowband user's bit can be thought of as a signal arising from a virtual user with a signature sequence with only one non-zero entry. It is obvious from this construction that the signature waveforms of the virtual users are orthogonal with each other. However, in general, the $k$-th virtual user has some cross correlation with the spread spectrum user. If we use $\rho$ to denote the vectors formed by the cross correlations, defined explicitly in (2), and we use $\mathbf{I}_m$ to denote the $m \times m$ identity matrix, then the cross correlation matrix $\mathbf{R}$ of this virtual multiuser system has the following simple structure (Note the SS user is numbered 0, and the $m$ virtual users are numbered from 1 to $m$).

\[
\mathbf{R} = \begin{bmatrix}
1 & \rho^T \\
\rho & \mathbf{I}_m
\end{bmatrix}.
\]

We have assumed that the narrowband user had a faster data rate than the SS user (but the rate is still much slower than the chip rate). The opposite case can also hold and our analysis applies to it as well, although we do not discuss this case explicitly. The covariance matrix of the system in this case has the same structure as (1).

Let $T$ be the bit duration of the SS user, then $T/m$ is the bit duration of the narrowband user. Let $N$ be the processing gain of the SS signal, then the chip interval has length $T/N$. By our assumption that the interferer is narrowband, we have $N \gg m$. Let $s_0(t)$ be the normalised signature wave form of the SS user, i.e., $s_0(t)$ is zero outside the interval $(0, T)$ and has unity energy. Similarly, let $s(t)$ be the normalised bit waveform of the narrowband user, i.e., $s(t)$ is zero outside the interval $(0, T/m)$ and has unity energy. Then the normalised signature waveform of the $k$-th virtual user is $s_k(t) = s(t-(k-1)T/m)$. The cross correlation vector mentioned earlier is $\rho = [\rho_1, \rho_2, \ldots, \rho_m]^T$, where $\rho_k$ is the cross correlation between the $k$-th virtual user and the SS user, defined as

\[
\rho_k = \langle s_0, s_k \rangle.
\]

We assume that the SS user and the narrowband user are sending digital data through the same channel characterised by AWGN with variance $\sigma^2$. Let $w_t$ be the received bit energy of the narrowband signal, and $w_0$ be the received bit energy of the SS signal (including the processing gain). We use the notation that the narrow band user data bits during the interval $(0, T)$ are $b = [b_1, b_2, \ldots, b_m]^T$, and the SS bit is $b_0$. When the users are synchronous, it is sufficient to consider the one-shot version of the received signal

\[
t(t) = \sqrt{w_0} b_0 s_0(t) + \sqrt{w_1} \sum_{k=1}^{m} b_k s_k(t) + \sigma n(t), \quad t \in [0, T].
\]

where $n(t)$ is white Gaussian noise with unit power spectral density.

2.2. MMSE Linear Multiuser Detector

A linear detector for user 0 is characterised by the impulse response $c_0 \in L_2[0, T]$, such that the decision on $b_0$ is

\[
b_0 = \text{sgn}(\langle \tau, c_0 \rangle).
\]

The minimum mean-square-error (MMSE) linear multiuser detector for user 0 is defined as the signal $c_0 \in L_2[0, T]$ that minimises the MSE

\[
E[(\langle \sqrt{w_0} b_0 - \langle \tau, c_0 \rangle \rangle)^2].
\]

It can be shown that the MMSE linear detector for the SS user in this system is

\[
c_0(t) = \alpha \left[ \left(1 + \frac{\sigma^2}{w_0}\right) s_0(t) - \sum_{k=1}^{m} \rho_k s_k(t) \right],
\]

where the constant $\alpha = \left[ \left(1 + \frac{\sigma^2}{w_0}\right) \left(1 + \frac{\sigma^2}{w_1}\right) - \rho^T \rho \right]^{-1}$.

The blind adaptive implementation of the MMSE detector (6) is based on the decomposition of the linear detector as the sum of two orthogonal components [5]. One component is equal to the signature waveform of the desired user, which is assumed known and fixed. The other is an orthogonal and adaptive component. For this purpose it is important to introduce the canonical representation for the linear detector (6) of user 0:

\[
\delta_0 = s_0 + \varepsilon_0,
\]

where $\langle s_0, \varepsilon_0 \rangle = 0$. Therefore a MMSE detector of canonical form $\delta_0(t)$ is the one that minimizes the MSE (8) subject to the constraint $\|s_0\|^2 = 1$. It can be shown that

\[
\delta_0(t) = \beta \left[ \left(1 + \frac{\sigma^2}{w_0}\right) s_0(t) - \sum_{k=1}^{m} \rho_k s_k(t) \right],
\]

where the constant $\beta = \left[ \left(1 + \frac{\sigma^2}{w_0}\right) - \rho^T \rho \right]^{-1}$.

\[1 < \pi, y > = \int_0^T \pi(t)y(t)dt.
\]
2.3. Performance Analysis and Comparisons

2.3.1. Probability of Error

Suppose all users transmit binary, equiprobable, antipodal symbols. By symmetry, we may condition on \( b_0 = 1 \) for the purpose of evaluating the error probability \( P(b_0 \neq b_0) \).

The probability of error for the MMSE detector (6) is then

\[
2 P_e(MMSE) = \frac{1}{2^m} \sum_{\mathbf{b}_R \in \{+1,-1\}^m} Q \left( \frac{\sqrt{\frac{u_s}{\sigma^2}} (1 + \frac{\rho^T \rho}{\sigma^2} \rho) - \sqrt{\frac{u_s}{\sigma^2}} b_R^T \rho}{\sqrt{1 + \frac{\rho^T \rho}{\sigma^2} \rho}^2 - 1 + \frac{\rho^T \rho}{\sigma^2} \rho^T \rho + \rho^T \rho} \right).
\]

The canonical form \( \bar{c}_0(t) \) has the same probability of error given by (9). As a comparison, the probability of error of the decorrelating filter for this system is [3]

\[
P_e(\text{decorrelator}) = Q \left( \frac{\sqrt{u_s(1 - \rho^T \rho)}}{\sigma} \right).
\]

A lower bound for the probability of error is the one when there is no narrowband interference present, which is given by

\[
P_e(\text{no interference}) = Q \left( \frac{\sqrt{u_s}}{\sigma} \right).
\]

Fig.2 shows that the probability of error for the MMSE is upper bounded by the probability of error for the decorrelating detector, which is in turn very close to the lower bound.

![Figure 2](image-url) Probability of error for the MMSE detector as a function of the narrowband interference power. \((m = 4, N = 31, w_s/\sigma^2 = 10\text{dB})\).

2.3.2. Signal-to-Interference Ratio (SIR)

Another performance measure of interest is signal-to-interference ratio (SIR). The SIR is defined to be the ratio of the desired SS signal power to the sum of the powers due to noise and narrowband interference at the output of the filter \( c_0(t) \). That is

\[
\text{SIR(MMSE)} = \frac{w_s < c_0, s_0 >^2}{\sigma^2 < c_0, c_0 > + w_s \sum_{k=1}^m < c_0, s_k >^2} = \frac{w_s}{\sigma^2} \left( 1 - \rho^T \rho + \frac{\rho^T \rho}{1 + \frac{\rho^T \rho}{\sigma^2}} \right).
\]

(10)

The SIR of the decorrelating filter for this system is

\[
\text{SIR(\text{decorrelator})} = \frac{w_s}{\sigma^2} (1 - \rho^T \rho).
\]

The signal-to-noise ratio (SNR) of the SS signal when there is no narrowband interference can serve as an upper bound on the achievable SIR, i.e., \( \text{SIR(\text{no interference})} = \frac{w_s}{\sigma^2} \).

Clearly we have \( \text{SIR(\text{decorrelator})} < \text{SIR(MMSE)} < \text{SIR(\text{no interference})} \).

\[Q(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{t^2}{2}} dt.
\]

![Figure 3](image-url) Comparison of the SIR for the MMSE detector with the SIR upper bounds for the linear predictor/subtractor and the linear interpolator/subtractor. The parameters are the same as in Fig.2.

\[\text{SIR(\text{no interference})}. \]

And the SIR of the MMSE detector is at most within a factor of \( (1 - \rho^T \rho) \) of the upper bound.

We next compare the SIR for the MMSE detector with the upper bounds of the SIR's for linear predictor/subtractor structures for suppressing the narrowband interference. The SIR upper bounds for the linear predictor/subtractor and the linear interpolator/subtractor were given in [6] and [7] respectively. In Fig.3 we plot these SIR upper bounds for the two structures together with the exact SIR for the MMSE detector. It can be seen that the MMSE detector greatly outperforms the other two in suppressing the digital narrowband interference.

3. BLIND ADAPTIVE SUPPRESSION OF DIGITAL NBI

3.1. Blind Adaptation Algorithm

We consider the linear detector in canonical form \( \bar{c}_0 = s_0 + x_0 \), where \( < s_0, x_0 >= 0 \). It can easily be shown that the mean square error and the output energy differ by a constant in terms of the canonical representation of the linear detector [5]. Thus, the MMSE linear detector minimises a signal to which the receiver has access, and therefore it can be found adaptively for the SS user with knowledge of only its signature waveform \( s_0 \); that is, the MMSE detector admits an adaptive implementation that uses only the knowledge required by the conventional detector.

Henceforth we assume a vector representation for the user waveforms, which results from projection onto a finite basis. For instance, the signature signals assigned to the SS user and each virtual user can be viewed as vectors of samples of chip-matched filter outputs within a SS symbol period. We use lower case bold variables to denote vectors in \( \mathbb{R}^N \).

Denote the received waveform in the \( n \)-th SS symbol interval \( [nT_s, (n+1)T_s] \) by \( r_n \). The output of the linear detector is \( b_0 = \text{sgn} \left( (s_0 + x_0)^T r_n \right) \). The stochastic gradient adaptation rule for updating the orthogonal component \( x_0 \) is [5]

\[
x_0^n = x_0^{n-1} - \mu \left( (s_0 + x_0^{n-1})^T r_n \right) \left[ r_n - (s_0^T r_n) s_0 \right]. \tag{11}
\]

\[\text{Figure 3. Comparison of the SIR for the MMSE detector with the SIR upper bounds for the linear predictor/subtractor and the linear interpolator/subtractor. The parameters are the same as in Fig.2.}\]
3.2. Convergence Analysis

3.2.1. Convergence of the Mean Tap Weight Vector

We consider the trajectory for the mean tap weight vector $E[c_n]$. The optimum MMSE detector of canonical form $c_0$ is given in (8). Now define the tap error vector $e^n = c^n - c_0$. Also define the following quantities: $R_{rr} = E[e^r e^r'] = (\lambda_1 \delta_0 \delta_0^T + \lambda_2 \sum_{k=m}^{m+n} \delta_0 \delta_0^T + \sigma^2 I_k)$, $R_{rw} = (1 - \delta_0 \delta_0^T) R_{rr}$, $R_{wv} = R_{wv}^T$, and $R_{vv} = (1 - \delta_0 \delta_0^T) R_{rr} (1 - \delta_0 \delta_0^T)$. It can be shown that

$$E[e^n] = (I - \mu R_{rr}) E[e^{n-1}]$$

Hence from (12) we conclude that $c^n_0$ converges to $c_0$ along $N$ modes, each of which decays exponentially with parameter $1 - \mu \lambda_l$, where $\lambda_l$ is the $l$-th eigenvalue of $R_{rr}$. It can be shown that the eigenvalues of $R_{rr}$ are

$$\lambda_l = \begin{cases} 0, & k = 1, \\ (1 - \rho^2 \rho) w_i + \sigma^2, & k = 2, \\ w_i + \sigma^2, & k = 3, 4, \ldots, m + 1, \\ \sigma^2, & k = m + 2, \ldots, N. \end{cases}$$

For stability, we must have

$$0 < \mu < \frac{2}{\max_{|l|} |\lambda_l|} = \mu < 2/(w_i + \sigma^2).$$

3.2.2. Tap Weight Error Correlation Matrix

Let $K_n$ be the correlation matrix of the tap weight error vector $e^n$, $K^n = E[e^n e'^n]$. Let $\xi_0$ be the mean output energy of the optimum detector (8), i.e., $\xi_0 = E[\xi_0^2]$. We can derive a recursive relationship for the time evolution of $K^n$ as $K^n = K^{n-1} - \mu \left[ R_{rr} K^{n-1} + K^{n-1} R_{rr} \right] + \mu^2 \left[ R_{rr} (K^{n-1} + K^{n-1} R_{rr}) R_{rr} + R_{rr} (K^{n-1} R_{rr}) + \sigma^2 \xi_0 R_{rv} \right]$, where we have dropped the transient terms in (15). The term $\mu^2 \xi_0 R_{rv}$ in (15) prevents $K^n = 0$ from being a solution to this equation. Therefore the tap-error vector $e^n$ only approaches zero, but then fluctuates around zero.

3.2.3. Convergence of the MSE

It can be shown that the condition for convergence in the mean square error at the output of the detector is

$$\mu < \frac{2}{w_s + mw_i + N \sigma^2}.$$  

(16)

Let $\xi$ be the mean output energy of the linear detector in steady state, then it can be shown that

$$\xi = \frac{\xi_0}{1 - \frac{\mu}{\xi_0} (w_s + mw_i + N \sigma^2)}.$$

(17)

Therefore the adaptation process produces asymptotic excess output energy $\xi - \xi_0$.

3.3. SIR in Steady State

It can be shown that at any time $n$ the portion of the output power that is due to the desired SS signal (i.e., user 0) is always $w_s$. Therefore the portion of output power in steady state that is due to interference and noise is $\xi - w_s$. The SIR in steady state is then

$$SIR = \frac{w_s}{\xi - w_s}$$

(18)

The SIR of the optimum detector $c_0$ is $w_s/(\xi_0 - w_s)$. Therefore the steady state SIR is lower than the optimum SIR because $\xi > \xi_0$.

We can do the analogous analysis for the conventional LMS algorithm with a training sequence. It can be shown that the LMS algorithm with a training sequence has a higher SIR in the steady state than the blind adaptation algorithm, because there is excess SS signal power as well in steady state. Hence it is best to switch to a decision directed algorithm as soon as possible (see, also [5]).

4. SUMMARY

In this paper, we have examined the application of the MMSE multiuser detection technique to the digital NBI suppression problem. In particular, we have derived the closed-form expressions for the MMSE detector for digital NBI suppression, and expressions for its probability of error and signal-to-interference ratio (SIR). We have shown from this that the performance of the MMSE detector is very close to the situation when there is no narrowband interference present.

We have also analyzed a blind adaptive version of the MMSE detector. In particular, we have derived expressions for the trajectories of the mean tap vector and the MSE. We have also derived the asymptotic MSE value in the steady state, together with the stability conditions, as well as the expression for the output SIR in steady state.

REFERENCES