Fracture Networks: Analysis with Graph Theory, LBM and FEM

H.Ghaffari

Department of Civil Engineering, Lassonde Institute,
University of Toronto, Toronto, ON, Canada

Embedded fracture networks in rock masses are studied. The fluid flow in fracture networks with respect to variation of connectivity patterns is analyzed. Lattice Boltzmann method is used to show sensitivity of the permeability and fluid velocity distribution to connectivity patterns of generated fracture networks. Furthermore, fracture networks are mapped into the graphs and the characteristics of theses graphs are compared to the main spatial fracture networks. Among these characteristics, node’s degrees, clustering of edges and sub-graphs distribution are investigated. Implemented power law distributions of fracture length -in spatial fracture networks- yield the same node’s degree distribution in transformed networks. Two general spatial networks are considered: random networks and networks with “hubness” properties (both with power law distribution of fracture length). In the first case, the fractures are embedded in a uniformly distributed fracture sets while the second case covers spatial fracture zones. The abnormal change (transition) in permeability is controlled by hub growth rate. Also, comparing LBM results with characteristic mean length of links of transformed networks shows a reverse relationship between the mentioned parameters. In addition, the abnormalities in advection through nodes are presented. As the last part of our research, the cubic law of transmitivity of each fracture is implemented in COMSOL software. The results of the later analysis are compared to LBM and advective flow–based networks equation.

Key words: Fracture Networks, Rock mass, Graph Theory, Modular Networks, Permeability, LBM, FEM
1. Introduction

Fracture networks embedded in different structures are the subject to the wide different research areas. However, the main relevant of fracture networks are propagation of fractures (cracks) in brittle materials. Complex form of linked cracks inspires scientists to develop several theories to capture the general mechanisms of fracture network formation. One of the branches of these theories is associated with single fracture mechanics. As a complementary accomplishment to the first approach, interaction of fractures yield modified stress filed. Then, the concentration of stress or variation of strain satisfies yielding criteria [1]. Inducing disorder into the system (say intact rock) gives the path of propagation of fracture. This branch of fracture propagation covers a wide range of methods as well as linear elastic fracture methods, effective medium theory, finite element or boundary element to tackle with complex geometries, molecular dynamics based methods (i.e., distinct element methods) to give a detail insight into the system, lattice based methods associated with random fuse models or random beam methods and finally the methods based on statistical field theory like interface or string growth methods [2-5].

The second approach corresponds with the collective observations through controllable laboratory tests and field data. Then, obtained patterns are analyzed using pattern recognition techniques and eventually the aim is to organize simple rules to satisfy the same characteristics (features or attributes) of the fracture system. The remarkable simplicity of these methods is originating from statistical methods which unavoidably are associated with the statistical physics. To be specific, the most well known methods and employed methods –under the second approach- employ statistical techniques to fracture networks which try to satisfy general characteristics of fracture networks like distribution law over density, length, directionality, distance, fractal dimension or characteristics of flow propagation like permeability [6-9].

The first steps in analysis structural configurations of fractures included fractality of fracture systems. Having a fractal dimension with small variation indicates the system is a universality property. However, there is not such universal dimensionality in fracture networks as if noticeable researchers have recognized two fractal dimensions for roughness (non-directional) of a single fracture [10]. Furthermore, the recent developments in graph theory [11-15], during the past decade, opened a new chapter in analysis of large interwoven systems (complex systems). This new perspective tackles with another facet of complexities embedded
in fractured materials called structural or topological complexity while the statistical methods investigate uncertainties as the probability or in more advanced form other uncertainties approaches (like fuzzy set theory, rough set theory or evidence theories). The interested topics in this field might be summarized as follows: 1) Transformation of spatial networks in graph forms; 2) information propagation (fluid flow, energy or waves) through networks; 3) Deformation of fractures due to mechanical or hydro-mechanical forces and generally the reaction of a disordered system like fracture system to external forces.

Our focus in this study is to generate different configurations of spatial fracture networks. In this way, we try to use the simplest algorithms to fracture network generation. Then we transform spatial fracture networks in graph forms where we ignore the spatial distribution of fractures. The advantages of these transformation are linking the regular fracture networks to modern graph theory. The fracture zones as a high density of fractures also as the abnormal emergence of fractures are another part of our research. We present a simple algorithm which gives the directionality and effective radius of the hubs while the growth of hubs under a back growth of random joints is taken into account. Advection of information (here fluid flow) through the generated spatial network and transformed networks is the main part of our research.

To analysis fluid flow, three different methods were employed. To obtain the fluid flow patterns, velocity, pressure distribution and permeability, for the spatial fracture networks; we use lattice Boltzmann method (LBM) and finite element method (FEM). In addition to this, advection based network equation is used over transformed networks. The organization of the paper is as follows. The first section presents a simple algorithm to generate fracture networks and the way to map into the graphs. Then in this section we introduce some basic characteristics of graphs. The next section summarizes three methods, LBM, FEM and advection based networks, to model fluid flow (laminar) in fracture networks. The main part of our study will be covered in section 4 where we present the results and discussion on accuracy of the employed methods. Finally, the last section gives the summery and conclusion of the present work.

2. Fracture Networks and Graph Theory

In this part, a simple algorithm is introduced to cover main characteristics of fracture networks. Then, the method to transformation of constructed fracture networks into graphs forms and the distinguished characteristics of graphs are demonstrated.
2.1. Random Fracture Networks

Several algorithms based on the second approach—presented in the introduction part—have been proposed to generate and cover the main statistical properties of natural fracture networks. The simplest algorithms in 2D consider distribution of fracture length, dip direction of joint sets and joint spacing [7-9]. New generations of fracture networks algorithms (or Discrete Fracture Networks: DFN) consider other parameters which increase the accuracy of the estimations like fracture density parameter (as the number of fracture per length/area or volume); fractal dimension where fractallity of the fracture centers is imposed through a hierarchal multiplicative process [9]. Most of the mentioned algorithms (and codes) impose a power law or modified power law distribution to fracture length. Cut-off value in power law distribution shows collapsing data set which shows a same mechanism/signature in the modeled data set. The basic idea in the behind of the power law distribution of fractures is yet a challenging question. However, recent developments in graph theory have tried to give a reasonable answer. The backbone of power law or sub-classes of power law distribution is originated from defection and absorption elements while probability of finding of small defection is much higher than large defections. Based on this idea the mother-daughter algorithm to generate fracture networks has been developed in which the main joint gives birth to “children” [16]. In other words, new weak generations intended to attach to main defection (“Mother”).

In three dimension scenarios each individual fracture is assumed as triangle, circle or any other polygon shapes where the consecutive generations based on the pre-set parameters is completed [17]. Another main attribute in fracture system is density or spatial density of fractures. This property yields the possible concentration of fractures in space as well as fracture tips, around tunnels, the interface between two layers and generally any sharp or disturbed sub-fields within the system. The equivalent definition of such fracture zones may be found in modularity and hubness concept in which the groups, communities or dense clusters, through random links, communicate with each other [18]. Our algorithm captures the power law distribution, directionality of joint sets and “hubness” properties of networks, as it is described as follows (Figure 1).

1. Also, see “A new computer code for discrete fracture network modelling” by Chaoshui Xua and Peter Dowd in *Computers & Geosciences*; Volume 36, Issue 3, March 2010, Pages 292-301.
Our algorithm is based on power law distribution of fracture length \( p(l) \sim l^{-\gamma} \) where \( l \) is fracture length and gamma is the power which controls the connectivity and length. With increasing gamma the probability of finding fractures with long length reduces. Other parameters are number of joint sets, number of fracture zones (hubs), hub growth, background rock joint growth (hereafter we call external links or back joints), maximum diameter of growth in \( x \) and/or \( y \) directions, azimuths of joint sets and mean aperture of each fracture \(^2\). The parameter related to “hub” and “back” growth is reported as reciprocal values. Then large values of growth show slow propagation of links. The spatial distributions of hubs are accomplished by employing a Gaussian distribution. The growth in hub zones and complementary background fractures are based on a power law distribution of fracture length and uniform distribution of dips of joint sets. We show with this implementation the distribution of links – in transformed shape of fracture networks – for a wide range of gamma parameter obeys from power law distribution, indicating scale free networks. In other words, with the described method, we achieved modular networks with power distribution either in transformed networks.

![Code Snippet](https://example.com/code_snippet.png)

Figure 1. Part of the parameters employed in fracture network code-The code was compiled in M-file format/Matlab.

### 2.2. Modern Graph Theory

A network consists of nodes and edges connecting them [11-12]. In order to analyze the network properties of the generated fracture networks, we map into each fracture generation a

\(^2\). Apertures have the same value along a fracture.
corresponding node in a graph space. When two fractures are intersected, two nodes are connected by a link. This procedure is illustrated in Figure 2, and enables us to investigate the networks using the tools from modern network theory discussed in the following.

![Figure 2. Transformation procedure of spatial fracture network in a graph shape with nodes and edges.](image)

Let us introduce some properties of the networks: clustering coefficient \( C \), the degree distribution \( P(k) \) and average path length \( L \). The clustering coefficient describes the degree to which \( k \) neighbors of a particular node are connected to each other. What we mean by neighbors is the connected nodes to a particular node. The clustering coefficient shows the collaboration between the connected nodes, i.e., local structures. Assume that the \( i^{th} \) node to have \( k_i \) neighboring nodes. There can exist at most \( k_i(k_i-1)/2 \) edges between the neighbors.

We define \( C_i \) as the ratio

\[
C_i = \frac{\text{actual number of edges between the neighbors of the } i^{th} \text{ node}}{k_i(k_i-1)/2}
\]  

Then, the clustering coefficient is given by the average of \( C_i \) over all the nodes in the network [12]:

\[
C = \frac{1}{N} \sum_{i=1}^{N} C_i.
\]
For $k_i \leq 1$ we define $C \equiv 0$. The closer $C$ is to one the larger is the interconnectedness of the network. The connectivity distribution (or degree distribution), $P(k)$, is the probability of finding nodes with $k$ edges in a network. In large networks, there will always be some fluctuations in the degree distribution. The large fluctuations from the average value ($<k>$) refers to the highly heterogeneous networks while homogeneous networks display low fluctuations [12-13]. The average (characteristic) path length $L$ is the mean length of the shortest paths connecting any two nodes on the graph. The shortest path between a pair $(i, j)$ of nodes in a network can be assumed as their geodesic distance, $g_{ij}$, with a mean geodesic distance $L$ given as below [12]:

$$L = \frac{2}{N(N-1)} \sum_{i<j} g_{ij},$$

where $g_{ij}$ is the geodesic distance (shortest distance) between node $i$ and $j$, and $N$ is the number of nodes. Based on the mentioned characteristics of networks two lower and upper bounds of networks can be recognized: regular networks and random networks (or Erdős–Rényi networks [12]). Regular networks have a high clustering coefficient ($C \approx 3/4$) and a long average path length. Random networks (construction based on random connection of nodes) have a low clustering coefficient and the shortest possible average path length. However Watts and Strogatz [11-12] introduced a new type of networks with high clustering coefficient and small (much smaller than the regular ones) average path length.

Analyzing the internal structures of networks from another perspective, the granulations of internal structures (local building blocks) through the obtained networks are presented by subgraphs and motifs. The subgraphs are the nodes within the network with the special shape(s) of connectivity together. The relative abundance of subgraphs has been shown to be an index to the functionality of networks with respect to information processing. Also, they correlate with the global characteristics of the networks [19-20]. The network motifs introduced by Milo et al. [19] are particular sub-graphs representing patterns of local interconnections between the nodes in the network. A motif is a sub-graph that appears more than a certain amount (other criteria can be found in literatures). A motif of size $k$ (containing $k$ nodes) is called a k-motif (or generally sub-graph).
3. Fluid Flow in Fracture Networks: Lattice Boltzmann (LBM), Finite Element Methods (FEM) and Advection Based Network Equation

3.1. Finite Element Modeling of Fluid Flow

When flow velocity is low and the fracture surface geometry does not vary too abruptly the Reynolds equation can be used. Assuming that the flow of an incompressible fluid through the fracture follows the cubic law, in a steady state, the governing equation can be written as [21]:

\[
\frac{\partial}{\partial x} \left( T_{xx} \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left( T_{yy} \frac{\partial h}{\partial y} \right) + Q = 0
\]

(4)

where \( Q \) is the source/sink term, and \( T_{xx} \) and \( T_{yy} \) are the fracture transmissivity in x- and y-directions, respectively. In this paper, the local transmissivity at each point is assumed to be equal in x- and y-directions for simplicity and is defined by [21]:

\[
T_{xx} = T_{yy} = T(x, y) = \frac{\rho g b h^3}{12 \mu}
\]

(5)

where \( \mu \) is the dynamic viscosity, \( \rho \) the fluid density, \( g \) the gravitational acceleration, and \( b \) the mean fracture aperture, respectively. To solve Eq. (4), the commercial FEM software, COMSOL Multiphysics [20] was used to simulate flow processes. The local transmissivity of the fracture in our simulation for each generation of fracture is assumed a constants value.

3.2. Lattice Boltzmann Method (LBM)

Chronologically, the LBM is the result of the efforts to improve the Lattice Gas Cellular Automata (LGCA) [22]. The LBM predicts the distribution function of fictitious fluid particles on fixed lattice sites on discrete time steps in discrete directions. Two main steps in the LBM algorithm are the streaming and collision. In the streaming step, particles move to the nearest neighbor along their direction-wise discretized velocities. The streaming process is followed by collision step, where the particles relax towards a local equilibrium distribution function. In the single relaxation time LBM the collision operator is simplified to include only one relaxation time for all the modes. In the D2Q9 LBM, the fluid particles at each node are allowed to move to their eight nearest neighbors with eight different velocities, \( e_i \). The ninth particle is at rest and does not move. The fluid density and macroscopic velocities are calculated by properly
integrating the particle distribution functions on each node. The evolution of distribution function in single-relaxation time LBM [22-24] is given by:

\[ f_i(x + e_i \Delta t, t + \Delta t) = f_i(x, t) - \frac{\Delta t}{\tau} [f_i(x, t) - f_i^{eq}(x, t)] \]  

(6)

where for any lattice node, \( x + e_i \Delta t \) is its nearest node along the direction \( i \); \( \tau \) is the relaxation time; \( f_i^{eq} \) is the equilibrium distribution function in direction \( i \). The macroscopic fluid variables, density \( \rho \) and velocity can be obtained from the moments of the distribution functions as follows:

\[ \rho = \sum f_i; \rho v = \sum f_i \mathbf{e}_i \]  

(7)

while the fluid pressure field \( p \) is determined by the equation of state of an ideal gas:

\[ p = C_s^2 \rho \]  

(8)

where \( C_s \) is the fluid speed of sound and for D2Q9 lattice is equal to \( \frac{1}{\sqrt{3}} \). The relaxation time is characterizing the time-scale behavior of fluid particle collisions and determines the lattice fluid viscosity:

\[ \nu = \frac{1}{3} (\tau - \frac{1}{2}) \]  

(9)

Once the macroscopic velocity field is determined, the permeability of the medium under study can be predicted using Darcy’s law:

\[ \langle \mathbf{v} \rangle = -\frac{K}{\mu} \nabla p \]  

(10)

where \( \langle \mathbf{v} \rangle \), \( K \), \( \nabla p \) and \( \mu \) are the volume averaged flow velocity, permeability tensor, pressure gradient vector and the dynamic viscosity of the fluid, respectively. The fluid flow is driven by pressure (or by some external force) on the boundaries. To extract the correct permeability values, the system should be in the steady state. The computation is in the steady state when the relative change of the average velocity is less than a tolerance variable (here \( 10^{-9} \)). More details about the different numerical and technical aspects of the LBM can be found elsewhere [22-24].
3.3. Advection Based Network Equation

For a given network with \( N \) nodes, the degree of the node and laplacian of the connectivity matrix are defined by [25]:

\[
    k_i = \sum_{j=1}^{N} A_{ij}; L_{ij} = A_{ij} - k_i \delta_{ij}
\]

(11)

where \( k_i, A_{ij}, L_{ij} \) are the degree of \( i \)th node, elements of a symmetric adjacency matrix and the network Laplacian matrix, respectively.

Flow of information (\( u_i \)) is expressed by network mode of diffusion or advection [26]:

\[
    \frac{d}{dt} u_i(t) = f(u_i) + \varepsilon \sum_{j=1}^{N} L_{ij} u_j
\]

(12)

where \( f(u_i) \) and \( \varepsilon \) are local dynamics of each profile (node) and diffusion constant, respectively (in our simulation \( f(u_i) = 0; \varepsilon = 1 \)). This equation is a discrete form of classical diffusion (advection) equation. In steady state case we have \( \varepsilon \sum_{j=1}^{N} L_{ij} u_j = 0 \) which is equivalent with \( \nabla^2 u = 0 \) while each node is connected to \( k \) other nodes instead of 2 or 4 nodes. Application of Eq.q12 as coupled oscillators also has been investigated in synchronization (or reaching to steady state time) of oscillators mounted on each node.

4. Results and Discussion

In this section, the results of our simulation based on the mentioned methods are presented. Two-dimensional fracture networks were generated at the percolation threshold, where there is a continuous path from one side of the system to the other. All of the cases had 270 *270 grids except one case which includes 650*650 grids. The first case (upper row in Figure 3a) was a complex case that included a uniform distribution of apertures for the whole of fractures. However in all of other cases the similar values of opening (aspect ratio) were chosen for generated fractures. As it has been depicted in Figures 3 and 4, increasing the gamma parameter fracture length and increases the number of fractures with small length. This changes
the total permeability and velocity distribution of fluid. Furthermore, Increasing gamma parameter gives a less complex fluid pattern because fewer fractures incorporate in conducting of flow (Figure 4). A broad range of directionality in fracture generations also dramatically changes the complexity of flow. This issue is revealed in curvature of flow paths, i.e., tortuosity. It induces more consumption of energy in order to reach steady state or the flow to drive. The fracture systems with regular rock joint sets\(^3\) exhibit relatively simpler and then more predictable fluid flow paths (Figure 5). Particularly, the relative direction of flow to the dip of fracture sets changes the spread of permeability (the range of variation of permeability - figure 13). As it has been proved, distribution of velocity directly affects the total permeability of the system as well as synchronization time (time to each steady state)\(^4\).

Figure 3. Variation of fracture networks depends on “gamma” parameter, joint sets and density of fractures. The angular distribution of joints and fracture length have been shown as the insets.

\(^3\) Hereafter “regular rock joints” stands for joints with sets (two or three).

\(^4\) For example see: Permeability of Three-D Random Fiber Webs by Koponen et al in PRL-VOL.80-1998
Figure 4. Complex fracture networks with different distribution in fracture length and corresponding fluid flow patterns, obtained by LBM. The velocity field is in logarithmic scale.

Figure 5. Other configurations of complex fracture networks with different distribution in fracture length, the directionality of joint sets with two sets and corresponding fluid flow patterns, obtained by LBM. The velocity field is in logarithmic scale.
The results of the modeling with LBM and FEM (Figures 4, 5 and 7) show distinguished abnormality and differentiated spatial area in velocity distribution. As it has been visualized in figure 4 and 5, the dense fractures resulted from low values of gamma parameters have more complicated discretized areas (blocks) than regular rock joint sets. We detect 4-edges blocks (as loops) and other non-loops 4 point nodes in transformed graphs. We show as if the general configurations of the generated systems vary but the system ensues a nearly same distribution of 4-point loops/non-loops. This is correct for a broad range of variations of parameters in term of gamma parameter.

To model the heterogeneity in fracture density, we set a uniform core for each module (fracture zone) with maximum 10 or 20 units (arbitrary) radius and with star-like directionality (with uniform distribution). Back-ground random links have a uniform distribution of dip in the range of 0-180 (half star). Modeling with LBM (and also with FEM) shows how fracture zones are trapping flow and change pressure distributions (Figure 6 and 7). Generally the fracture zones act as the relaxation of velocity and rapid reduction of velocity (and then gradient of pressure).

---

5 It can be investigated that transformed networks roughly can give sub-graphs with different shapes.
More fracture zones generally reduces permeability of the total system however its reduction magnitude depends on quality of links among modules (back ground rock joint).

Figure 7. Fluid flow modeling with FEM in a) Single hub pattern with gamma =0.75, hub growth=8, back growth =3; b) Multiple hubs with 650 grids and c) two joint sets (0, 90 degree azimuths). The right hand pictures are the pressure distribution with the corresponding networks.

Here we should consider two general points in analysis of permeability: influence of internal links in fracture zones and the effects of external links. Each of these parameters effectively changes the information (fluid) flow. Weak external links yield concentration of particles in especial modules while frequent external links spread (deconcentrate) the flow from
a few modules. Internal fractures in fracture zone impose local complexity and can be assumed as sub-fracture networks in the system. The combination of the mentioned factors gives a complicated situation. We show as if this discloses complicate cases, however due to the same nature and mechanism in fracture generation, some attributes for a broad range of these two parameters are nearly following the same patterns.

We should notice due to the randomness nature of the fractures webs, to get precise answer; accomplishment of Monte Carlo simulation is unavoidable. For LBM, this is time consuming and we could analysis only 5-10 for each case (each constant parameter in the model). For modeling in FEM, we assumed each fracture has the dimension in z direction as the aperture /opening of each joint. The results of simulation with LBM and FEM (5, 6 and 7) shows approximately the same patterns in fluid flow path .We expect the results, for permeability or twistability, will display the same answer for both of the methods.

Figure 8. a) Network with 400 generation per each hub with 4 hubs. Hub growth and back-growth parameters respectively are 3 and 2 with gamma=0.85; b) sub-graphs distribution of 4-points sub-graphs over transformed fracture networks in graphs ;c) distribution of node’s degree; d) scaling of clustering coefficient with number of links; e) distribution of velocity distribution in advection –network system and f) visualization of node’s distance.
Figure 9. a) Network with 400 generation per each hub with 4 hubs. Hub growth and back-growth parameters respectively are 2 and 4 with gamma=0.85; b) sub-graphs distribution of 4-points sub-graphs over transformed fracture networks in graphs; c) distribution of node’s degree; d) scaling of clustering coefficient with number of links; e) distribution of velocity distribution in advection – network system and f) visualization of node’s distance.

Figure 10. a) Network with 400 generation per each hub with 4 hubs And gamma=0.85 with two rock joint sets; b) sub-graphs distribution of 4-points sub-graphs over transformed fracture networks in graphs; c) distribution of node’s degree; d) scaling of clustering coefficient with number of links; e) distribution of velocity distribution in advection – network system and f) visualization of node’s distance.

Now, we map the spatial joint networks into graphs with using the described method in section 2.2. We distinguish how different hubness parameters affect the hierarchical patterns in fractures. Also, using the method described in section 3.3 distribution of velocity over nodes in
steady state is presented and compared them with FEM results. To model Eq.12 we used Euler method in integration while we should consider the stability of solution with respect $\Delta t$ and $\Delta x$ ($\epsilon$ is constant per each fracture which is equivalent with permeability of each joint).

In figures 8 to 10, we have shown five main statistical characteristics of graphs. The part “b” in aforementioned figures shows the distribution of 4-point sub graphs, indicated by indexes 1to 6. As it is clear despite of different configuration in hubness implementation (figures 8 and 9) the frequency of sub-graphs follow the same trend. The falling down the frequency of index 4 as a rectangular loop and jumping index 5 up are the in common features of frequencies. This is not the case in regular joints with perpendicular intersection (Figure 10) in which the index 4 shows slightly increment. Referring to simulation with LBM (or FEM) reveals that regular fracture networks with pre-set joint sets present much uniform distribution in velocity rather than fracture zones with abnormal distribution (all see figure 11 with advection-network equation).

![Figure 11. Distribution of velocity in transformed networks using the steady state solution of advection-network system over different configurations of hub growth (HG) and back fracture growth (BG). The values of growths are the reciprocal of propagations of fracture. Total number (max.) of hubs could be 4. The simulated system had sinks and sources nodes with -10 and 10 values of velocity (or concentration of particles). First 10 and last 10 fracture networks were considered as sources and sinks respectively.](image)

It suggests the index 4 (and somehow 2$^7$) implies for most channelized flow and possibly with low value of twistability. Indexes 3 and 5 indicate such abnormality in fluid flow and index

---

6. The results presented here is the mean results for 5 times realizations. The injection of particles from not-similar points like figures 4-7 also might be questionable but It can be probed that the general distribution would not be affected.

7. We analysed this fact in a single shear fracture ;see H.Ghaflari et al, Fluid flow analysis in a rough fracture (type II) using complex networks and lattice Boltzmann method, PanAm-CGS (2011), Toronto-Canada .
6 is possibly shows the most homogeneity in information flow. Such abnormality in distribution of velocity\(^8\) (or particle concentration) through nodes (fractures) has been modeled with advection based network equation described in section 3.3 and has been illustrated in part "c" of the figures 8 to 10. In our model, we put the source and sink term in first and last 10 generations of fractures (with values with the amount 10 and -10, respectively). Increasing external links (back-ground joints) induces uniformity and homogeneity in pressure and velocity spatial spread. In other words, it suppresses, somehow, the hubness and modularity attribute of the system. Increasing hubness aspect and decreasing external links yield non-uniform flow and concentration of particles in especial nodes.

Figure 12. Distribution of velocity in spatial fracture networks using FEM over 3 configurations of spatial fracture Networks. The simulated systems have been shown in figure 7.

Figure 11 shows the distribution of velocity fields in steady state case for different configurations of fracture zones parameters. The general trend of distribution is Gaussian

\(^8\) . Recent analysis in network theory has highlighted the abnormality of diffusion with random walk theory and stochastic particle simulation for example see PRL 94,248701(2005) and PRE E82,055101(2010).
distribution. For the system with minimum effect of modularity we observed more fluctuation in
distribution with evident peak around zero velocity. This is obvious that decreasing modularity
induces a transition in velocity (and then total permeability) distribution. Deleting or cutting
external linkages forces the system to have power law distribution, i.e., abnormality in pressure
and velocity abundances, with less fluctuation in velocity or pressure frequencies. Exhibition the
distribution with the tailed feature can be approved with plotting spatial joint systems modeled
with FEM as it has been shown in figure 12. As though in transformation of spatial networks, we
lost the spatiality however we could recover approximately the same properties of flow with such
a mapping into the graphs.

The part “c”, in aforementioned figures, shows degree distribution which roughly obeys
from power law distribution. In other words, our fracture algorithm (with modularity or without
it), for a wide range of parameters variations, gives “scale free networks”. Part”d” demonstrates
a space called spectrum of graphs which is associated with clustering coefficient (c) - node’s
degree (k) space. It has been approved networks with power law distribution and having a linear
scaling of c-k present hierarchical property [27]. We observed strong hierarchical patterns in
modular networks and week hierarchical in regular fracture systems (figure 10).

Figure 13. Left) variations of permeability with gamma parameter trough complex fracture network configuration
Right) variations of permeability with the directionality (Azimuth) of joint sets under constant gamma parameter (no
hubness property is induced in fracture patterns).
As the last part of our research, we focus on the possible relation between mean path length of transformed networks and variation of permeability. Part “f” in figures 8-10 shows the distance of nodes where cool colors is showing near neighborhoods or immediate intersected joints and hot colors indicate far neighborhoods or non-connected nodes. Comparing calculating mean path length or diameter of the graphs and the simulated permeability shows a reverse correlation between the two parameters. Roughly speaking rich neighborhood attribute exposes high permeable fractured system and poor nodes from connectivity point of view reveal low amount of permeability (Figure 14).

Figure 14. An approximate relation between the mean path lengths of transformed fracture networks and simulated permeability using LBM. Each point is the result of 5 times realizations.

5. Conclusions

We studied fracture networks in rock mass. The fluid flow in fracture networks with respect in variation of connectivity patterns was analyzed. Lattice Boltzmann method and FEM were used model permeability and fluid velocity distribution. Furthermore, fracture networks were mapped into the graphs and the characteristics of the obtained graphs were compared with the main spatial fracture networks. We distinguished hubness character of fracture as fracture zones. The results showed the fracture zones, internal and external links of damaged zones
dramatically change the transport properties of rock mass. Our study might be confirmed with doing more realizations over several cases.

References


