A New, Simple, and Exact Union Bound for Reference-Based Predetection and Postdetection Diversity TCM-MPSK Systems in Rayleigh Fading

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Abstract—Trellis-coded modulation (TCM), when combined with interleaving of sufficient depth, is known to provide some form of time diversity that allows for the achievement of good error performance in fading environments. Also, space diversity reception, in which several signals received at different antennas are combined, is a well-known method that can be used to combat the effects of fading in wireless systems. In this paper, we consider the analysis of the error performance of reference-based predetection and postdetection diversity systems when used in conjunction with trellis-coded modulation techniques. Based on the combination of the transfer function upper bounding technique with the exact calculation of pairwise error event probabilities, we present a new and simple method for exact calculation of the union bound on the average bit error rate (BER) performance of these systems in correlated Rayleigh fading channels. This method has the same complexity as the union-Chernoff bound, and some examples are given to illustrate its application. Monte-Carlo simulation results, which are more indicative of the exact system performance, are also shown.

I. introduction

In the last few years, considerable attention has been devoted to the study of adequate transmission techniques for wireless mobile communication systems. Although trellis-coded modulation (TCM) was originally developed for telephone channels [1], it has been considered as a valid transmission scheme to mitigate the effect of multiplicative fading in the mobile channel [2]–[9]. The primary advantage of TCM over transmission schemes using traditional error correction coding is its ability to achieve increased power efficiency without the customary expansion of bandwidth introduced by the coding process. Furthermore, when combined with interleaving of sufficient depth, TCM is known to provide some form of time diversity that allows the error rate to decrease with signal-to-noise ratio (SNR) faster than the inverse law commonly found on Rayleigh fading channels. On the other hand, space diversity techniques, in which several signals received on different antennas are combined [10], [11], provide a simple way to significantly improve the bit error rate (BER) performance of a communication system when transmitting through a fading propagation medium. To assist with the demodulation/decoding process and perform space diversity combining in a fading environment, it is common practice to incorporate a reference signal (pilot symbols [12] or pilot tones [13]) alongside the transmitted data symbols. The use of reference-based systems allows the random FM noise and envelope fluctuations caused by multipath fading to be accurately tracked and eliminated, thus overcoming the error floor commonly associated with data transmission over fading channels.

There are different approaches that are usually followed for the average BER performance evaluation of TCM schemes over fading channels. Chernoff upper bound combined with the pair state generalization transfer function bound approach or the modified state transition diagram transfer function bound approach has been widely used. However, although readily determined, this bound is too loose over normal SNR ranges of interest. Other approaches, based on the exact calculation of the pairwise error event probabilities, are often too cumbersome, can only be calculated for a finite number of dominant error events, or produce expressions that cannot be reduced to closed form except for systems with ideal channel state information (CSI) and uncorrelated fading channels.

In this paper, we consider the BER performance evaluation of a reference-based predetection space diversity system when used in conjunction with trellis-coded modulation schemes over correlated fading channels. As a baseline performance measure, we also include the analysis of postdetection space diversity TCM schemes based on conventional differential detection. Based on the statistical properties of correlated complex Gaussian random variables, we describe a simple way to combine the exact calculation of the pairwise error event probability with the transfer function bounding techniques. This approach, having the same complexity as the union-Chernoff bound, provides a new method to evaluate the exact union bound on the average bit error probability.

This paper is organized as follows. The system and analysis models used in this study are described in Section II. Considering space diversity on correlated Rayleigh fading channels, average bit error probability bounds for both pilot tone-aided predetection diversity coherent systems and postdetection diversity differential detection systems are then derived in Section III. Section III also illustrates the use of our analytical expressions with some examples. Analytical results are compared with those obtained through Monte-Carlo simulation. Finally, discussion and the conclusions of this study are given in Section IV.

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II. SYSTEM AND ANALYSIS MODELS

The block diagram of the baseband system model under investigation is given in Fig. 1. A sequence of binary digits at rate \( R_b \) is encoded by a rate \( k/(k + 1) \) trellis encoder producing a sequence of coded MPSK (\( M = 2^{k+1} \)) symbols. The sequence of coded symbols is passed to an interleaver to randomize the distribution of symbols affected by amplitude fading of duration greater than one symbol period. Following the interleaving process a phase reference signal is added to the data bearing signal. The composite signal is then pulse shaped and transmitted. The transmitted signal is faded and corrupted by additive white Gaussian noise (AWGN) passing through the Rayleigh fading channels. The received signal in each diversity branch is simultaneously passed to a matched filter and to a channel estimator. In-phase and quadrature components are combined, demodulated, quantized for soft decision and block deinterleaved. Using these quantized symbols, the Viterbi decoder detects the transmitted sequence based on maximum likelihood estimation.

For a received sequence of length \( N \), namely \( Y_N = (y_1, y_2, \ldots, y_N) \), the metric between \( Y_N \) and any transmitted signal sequence \( X_N = (x_1, x_2, \ldots, x_N) \) is of the form \( m(Y_N, X_N; Z_N) \) if side information is available and \( m(Y_N, X_N) \) if it is not. The metric is used by the decoder to make decisions as to which sequence was transmitted given the corresponding channel output sequences. Whatever metric is selected, to simplify the decoder processing complexity, it is required to have an additive property where, for sequences of symbols, the total metric of the sequences is the sum of the metrics of each channel input and output pair of the sequence [14], i.e.,

\[
m(Y_N, X_N; Z_N) = \sum_{n=1}^{N} m(y_n, x_n; z_n).
\]

The decoder incorrectly decides that the transmitted coded sequence is \( \hat{X}_N \neq X_N \) when

\[
m(Y_N, X_N; Z_N) \leq m(Y_N, \hat{X}_N; Z_N)
\]

with a probability \( P(X_N \rightarrow \hat{X}_N) \) which is called the pairwise error probability. By our previous assumptions

\[
P(X_N \rightarrow \hat{X}_N) = \Pr(f \geq 0 | X_N)
\]

where

\[
f = m(Y_N, \hat{X}_N; Z_N) - m(Y_N, X_N; Z_N).
\]

An upper bound on the average bit error probability is obtained from \( P(X_N \rightarrow \hat{X}_N) \) as

\[
P_b \leq \sum_{N=L_s}^{\infty} \sum_{X_N} \sum_{\hat{X}_N} a(X_N, \hat{X}_N) p(X_N) P(X_N \rightarrow \hat{X}_N)
\]

where \( a(X_N, \hat{X}_N) \) is the number of bit errors occurring when \( X_N \) is transmitted and \( \hat{X}_N \) is chosen by the decoder, \( p(X_N) \) is the a priori probability of transmitting \( X_N \) and \( L_s \) is the Hamming distance of the shortest error event.

III. AVERAGE BIT ERROR PROBABILITY BOUNDS

A. Pilot Tone-Aided Predetection Diversity Coherent Systems

In order to have an accurate channel estimation at the receiver, a pilot signal can be sent along with the data bearing signal. The pilot signal can be a tone (or multiple tones) or it can be a sequence of symbols inserted periodically into the data bearing signal. Anyway, as shown in [5], both systems provide
the same performance for a given system power and information throughput. With a reference pilot tone signal, the baseband equivalent of the transmitted signal is

\[ x(t) = B + \sum_i x_i q(t - iT) \]  

(6)

where \( B \) represents the reference signal, \( x_i = A e^{j\phi_i} \), \( T \) is the symbol period, and \( q(t) \) represents the complex impulse response of a pulse shaping filter that satisfies Nyquist’s criterion for zero intersymbol interference. We normalize the energy of the pulse \( q(t) \) such that \( \int_{-\infty}^{\infty} |q(t)|^2 \, dt = 1 \). It is assumed that the power spectrum of the data bearing signal has a spectral null that allows for the insertion and the extraction of the pilot.

Assuming, in general, a \( K \) branch diversity system, the baseband equivalent of the received signal on the \( k \)th receiver \((k = 1, 2, \ldots, K)\) can be expressed as

\[ r_k(t) = \chi_k(t) x_k(t) + \nu_k(t) \]  

(7)

where \( \nu_k(t) \), which represents the additive thermal noise at the receiver front end, is a zero-mean complex Gaussian noise process with single-sided power spectral density \( N_0 \) and \( \chi_k(t) \), which represents the multiplicative Rayleigh fading characteristic of the channel, is a normalized (unit mean-square value), stationary, and zero-mean complex Gaussian process. Under the assumption of nonindependence between diversity branches, the correlation function between the complex channel weights of two of the antennas can be modeled with the following Bessel model [11]:

\[ E\{x_i(t) x_j^*(t)\} = J_0(2\pi d_{i,j}) \]  

(8)

where \( J_0(\cdot) \) is the Bessel function of order zero and \( d_{i,j} \) is the normalized distance between antennas \( i \) and \( j \) in the diversity array. As shown in Fig. 1, the received signal is simultaneously passed to a matched filter with an impulse response equal to \( q^*(-t) \) and to a channel estimator that, in this case, is simply a pilot extraction filter with a frequency response

\[ H(f) = \begin{cases} 1, & -B_p/2 \leq f \leq B_p/2 \\ 0, & \text{otherwise.} \end{cases} \]  

(9)

Note that the bandwidth of the filter must be wide enough to allow the fading to pass through undistorted, that is, it must be at least twice the maximum Doppler shift \( f_d \). In the following analysis, it will be assumed that a filter with bandwidth \( B_p \geq 2f_d \) is used. Thus, the data bearing signal and the pilot signal at the output of the matched filter and pilot extraction filter can be expressed as

\[ u_k(t) = \chi_k(t) \sum_i x_i h(t - iT) + n_k(t) \]  

(10)

and

\[ p_k(t) = \chi_k(t) B + \nu_k(t) \]  

(11)

respectively, where \( h(t) = q(t) \ast q^*(-t) \) represents the overall impulse response of the system for a perfect nonselective transmission medium and \( n_k(t) \) and \( \nu_k(t) \) are zero mean complex Gaussian noise processes. Note that \( n_k(t) \) is independent of \( u_k(t) \) due to the fact that they are output noise processes of two filters whose frequency responses do not overlap. Using the pilot tone signals to co-phase and weight the \( K \) data bearing signals, maximal ratio combining (MRC) results. Thus, the signal at the output of the diversity combiner will be given by

\[ g(t) = \sum_{k=1}^{K} u_k(t) p_k^*(t). \]  

(12)

The signal at the output of the diversity combiner is sampled by an A/D converter at time \( t_i = iT + \tau \), where \(-T/2 \leq \tau \leq T/2\) determines the sampling instant. Assuming a perfect clock recovery, \( \tau = 0 \), the complex sample at the output of the deinterleaver will be given by

\[ y_i = \sum_{k=1}^{K} u_k(t) d_k^i(t) \]  

(13)

where, for simplicity of notation, we have dropped the delay introduced by the interleaving/deinterleaving process.

A soft Viterbi decoder performing maximum-likelihood sequence estimation (MLSE) will select the codeword for which the \( a \) posteriori probability is the largest. With the assumption of equally probable MPSK symbols and ideal interleaving, the maximum-likelihood (ML) branch metric is

\[ m(y_i, x_i; z_i) = -|y_i - x_i|^2. \]  

(14)

Substituting (14) into (4), the decision variable \( f \) can be expressed as

\[ f = \sum_{i=1}^{N} \sum_{k=1}^{K} 2 \Re\{w_{k,i} d_k^i (x_i - x_i^*)\}. \]  

(15)

Now, let us expand the product \( \omega_{k,i} d_k^i \) that appears in (15) as

\[ \omega_{k,i} d_k^i = B x_i |\chi_{k,i}|^2 + B x_i \chi_{k,i} \nu_{k,i}^* + x_i |\chi_{k,i}| \nu_{k,i}^* + |\nu_{k,i}| \nu_{k,i}^*. \]  

(16)

By conditioning on \( X_i = (\chi_{1,i}, \chi_{2,i}, \ldots, \chi_{K,i}) \), the complication in determining the PDF for the variable decision \( f \) is the term \( \eta_{k,i} \nu_{k,i}^* \), which is the product of two complex-valued Gaussian random variables. However, at SNR’s of practical interest and assuming a slow Rayleigh fading, the term \( \eta_{k,i} \nu_{k,i}^* \) is small relative to the other noise dominant terms [15]. Thus, neglecting this term the new decision variable is a Gaussian random variable with mean and variance given by

\[ \mu_f = -A^2 B \sum_{i=1}^{N} \gamma(X_i) d_i^2 \]  

and

\[ \sigma_f^2 = 4(A^4 N_0 B_p + A^2 B^2 N_0) \sum_{i=1}^{N} \gamma(X_i) d_i^2 \]  

(18)
respectively, where
\[ \gamma(x_i) = \sum_{k=1}^{K} |\chi_k x_i|^2 \]
and \( d^2_{x_k} \) is the normalized squared Euclidean distance defined as
\[ d^2_{x_k} = \frac{|x_i - \hat{x}_i|^2}{A^2} = 2[1 - \cos(\phi_i - \phi_k)]. \]
Thus, when conditioned on the transmitted signal sequence \( X_N = (x_1, x_2, \ldots, x_N) \) and the channel fading \( \Gamma_N(\chi) = (\gamma(\chi_1), \gamma(\chi_2), \ldots, \gamma(\chi_N)) \), the conditional pairwise error probability will be given by
\[ P(X_N \rightarrow \hat{X}_N | \Gamma_N(\chi)) = \frac{1}{2} \text{erfc}\left( -\frac{\mu_f}{\sqrt{2}\sigma_f} \right). \]
In [16], J.W. Craig shows that the complementary error function can be expressed as
\[ \text{erfc}(x) = \frac{2}{\pi} \int_0^{\pi/2} \exp\left( -\frac{x^2}{\sin^2 \Phi} \right) d\Phi, \quad x \geq 0 \]
which, in addition to the advantage of having finite integration limits, has its argument contained in the integrand rather than in the integration limits and can be used to write (21) in product form as
\[ P(X_N \rightarrow \hat{X}_N | \Gamma_N(\chi)) = \frac{1}{\pi} \prod_{\phi \in \eta} \exp\left( -\frac{\alpha_k \gamma(x_i)}{\sin^2 \Phi} \right) d\Phi. \]
where
\[ \alpha_k = \frac{A^2 B^2 d^2_{x_k}}{8(A^2 N_0B_0 + B^2 N_0)} \]
and \( \eta = \{ i : x_i \neq \hat{x}_i \} \). The unconditional pairwise error probability is obtained by averaging (23) over the probability density function (PDF) of \( \Gamma_N \). Since we have assumed that the interleaving/deinterleaving process makes the \( \gamma(x_i) \)s independent, then the average over \( \Gamma_N \) can be computed as the product of averages. That is,
\[ P(X_N \rightarrow \hat{X}_N) = \frac{1}{\pi} \prod_{\phi \in \eta} \int_0^{\pi/2} \exp\left( -\frac{\alpha_k \gamma(x_i)}{\sin^2 \Phi} \right) d\Phi. \]
By defining the moment generating function (MGF) of a random variable \( x \geq 0 \) as [17]
\[ \mathcal{M}_x(s) = \mathbb{E}\{ e^{sx} \} = \int_{-\infty}^{\infty} e^{sx} p_x(z) \, dz \]
(25) can be written in terms of the MGF of the random variable \( \gamma(x_i) \) as
\[ P(X_N \rightarrow \hat{X}_N) = \frac{1}{\pi} \int_0^{\pi/2} \prod_{\phi \in \eta} \mathcal{M}_\gamma \left( -\frac{\alpha_k}{\sin^2 \Phi} \right) d\Phi. \]
This expression can be easily evaluated to any degree of accuracy by several numerical integration methods. In particular, the change of variables \( x = \cos(2\Phi) \) enables us to use the Gauss–Chebyshev quadrature rules [18], which have the advantage that their abscissas and weights admit a closed-form expression. In this way, after some algebraic manipulations, we obtain [19]
\[ P(X_N \rightarrow \hat{X}_N) = \frac{1}{2\pi} \int_0^{\pi/2} \prod_{\phi \in \eta} \mathcal{M}_\gamma \left( -\frac{\alpha_k}{\sin^2 \left( \frac{\cos^{-1} x}{2} \right)} \right) \sqrt{1-x^2} \, dx = \frac{1}{2n} \sum_{l=1}^{n} \prod_{\phi \in \eta} \mathcal{M}_\gamma \left( -\frac{\alpha_k}{\sin^2 \left( \frac{\phi_i}{2(1-\pi/4n)} \right)} \right) + R_n. \]
for any positive integer \( n \), with the remainder term \( R_n \rightarrow 0 \) as \( n \rightarrow \infty \).
For the Rayleigh fading channel, the problem of determining the MGF of \( \gamma(x_i) \) is tantamount to the computation of a sum of squares of nonindependent Rayleigh random variables [20]. Let us introduce the multivariate Gaussian random variable \( \chi_i = (\chi_{1,i}, \chi_{1,i}, \chi_{2,i}, \chi_{2,i}, \ldots, \chi_{K,i}, \chi_{K,i}) \), where \( \chi_{k,i} \) and \( \chi_{k,i}^* \) denote the zero-mean real and imaginary random variable components of \( \chi_{k,i} \). With the above definition, the random variable \( \gamma(x_i) \) can be written as
\[ \gamma(x_i) = \chi_i^T \cdot x_i^T \]
and its MGF is given by [20]
\[ \mathcal{M}_\gamma(s) = \int_0^\infty \exp \left( -\frac{1}{2} \chi_i^T \left( \text{cov}(\chi_i) \right)^{-1} - 2s \chi_i^T \right) \left( 2\pi \right)^{-K/2} \left| \text{cov}(\chi_i) \right|^{1/2} \, d\chi_i, \]
In [20] and [17] it is shown that (30) has the closed form
\[ \mathcal{M}_\gamma(s) = \left[ I - 2s \text{cov}(\chi_i) \right]^{-1/2} = \left[ I - s \text{cov}(\chi_i) \right]^{-1} \]
which, since \( \text{cov}(\chi_i) \) is Hermitian, can be written as
\[ \mathcal{M}_\gamma(s) = \prod_{k=1}^{K} (1 - s \lambda_k)^{-1} \]
where \( \lambda_k, k = 1, 2, \ldots, K \), are the eigenvalues of \( \text{cov}(\chi_i) \).

\footnote{By letting \( \sin^2(\phi) = 1/(\phi^2 + 1) \), the complementary error function can be expressed as (4) in [8].}
Finally, by substituting (32) into (28), we get the following expression for the union bound on the average bit error probability:

$$P_b \leq \frac{1}{2n} \sum_{i=1}^{n} \sum_{N=L_o}^{\infty} \sum_{X_N}^{\infty} \sum_{X_N}^{\infty} a(X_N, \hat{X}_N) p(X_N)$$

$$\times \prod_{i \in \mathcal{O}} \prod_{k=1}^{K} \left(1 + \Delta_{t,i,k}\right)^{-1}$$

(33)

where

$$\Delta_{t,i,k} = \frac{E_s}{N_0} \frac{d_t^2 \lambda_k}{4(B_p T + r)(1 + r) \sin^2((2l - 1)\pi/4)}$$

(34)

with $E_s = (A^2 + B^2 T)/2$ and $r = B^2 T/A^2$ being the equivalent energy per MPSK symbol and the pilot to signal power ratio, respectively. Optimizing (33) over $r$ we get

$$r_{\text{opt}} = \left(\frac{B_p T}{1}\right)^{1/2}.$$  

For the particular case of independent diversity branches, $\text{cov}(\mathbf{X}_t) = \mathbf{I}$, and hence its eigenvalues $\lambda_k$, $k = 1, 2, \ldots, K$, are all equal to one. Nevertheless, as shown in Appendix I, it is possible to obtain an exact expression for the union bound on the average BER as

$$P_b \leq \frac{1}{2n} \sum_{i=1}^{n} \sum_{N=L_o}^{\infty} \sum_{X_N}^{\infty} \sum_{X_N}^{\infty} a(X_N, \hat{X}_N) p(X_N)$$

$$\times \prod_{i \in \mathcal{O}} \prod_{k=1}^{K} \left(1 + \Delta_{t,i,k}\right)^{-K}$$

(35)

where $\Delta_{t,i,k}$ is defined as

$$\Delta_{t,i,k} = \frac{r}{(1+r^2) \left(\frac{E_s}{N_0} \left(\frac{r + B_p T}{B_p T(1 + r)}\right) \sin^2((2l - 1)\pi/4)\right)}.$$  

(36)

Optimizing (35) over $r$ we get

$$r_{\text{opt}} = \left(\frac{E_s}{N_0} \left(\frac{E_s}{N_0} \left(\frac{B_p T}{1}\right)^{-1} + 1\right)^{1/2}.$$  

(37)

Expression (35) shows that two factors determine the performance of the signaling scheme over a slow Rayleigh fading. The first, and more significant one, is the Hamming distance profile between all pairs of possible encoded sequences. In fact, it can be seen that the BER asymptotic performance is inversely proportional to the $L_o K$th power of the SNR. This is the combined effect of space diversity and intrinsic time diversity of TCM. The second parameter affecting performance represents the product of normalized squared Euclidean distances between distinct symbols in all possible pairs of encoded sequences. The effect of using pilot tone-aided detection is to reduce the ratio $E_s/N_0$ by a factor that is asymptotically equal to $(1 + \sqrt{B_p T})^{-2}$. Comparing (35) with (33), it can be seen that correlation between diversity branches has the effect of modifying the normalized squared Euclidean distances associated to the error events by factors equal to the eigenvalues of the diversity array covariance matrix. Provided that none of the $\lambda_k$ is equal to zero, the intrinsic code diversity order is still increased by a factor $K$. However, for strongly correlated diversity branches, some of $\lambda_k$ may be vanishing, so that the space diversity order may be reduced. For the limiting case in which $d_{ij} = 0$, $\forall i, j$, $\text{cov}(\mathbf{X}_t) = \mathbf{U}$, where $\mathbf{U}$ is a $K \times K$ all-ones matrix, and hence its eigenvalues are $\lambda_1 = K$, $\lambda_k = 0$, $k = 2, \ldots, K$, so space diversity order is equal to one.

Using the Gauss–Chebyshev quadrature rule to perform integration, we arrive at a transfer function upper union bounds on the average BER for the Ungerboeck’s eight-state 8PSK trellis code [1]. Fig. 2 shows simulation and analytical results with the number of diversity branches as parameter. A normalized fading rate of $f_d T = 0.01$ and a regular linear array receiving antenna consisting of $K$ dipoles with a normalized distance between two consecutive antenna elements $d$ is 0.3.

Fig. 3 illustrates the effect of varying the normalized fading rate $f_d T$ on the performance of the system. A two-branch diversity system with a normalized distance between antennas $d = 0.3$ has been considered. Fig. 4 shows the BER performance of the dual diversity MRC reception scheme, for a normalized fading rate $f_d T = 0.01$, illustrating the effect produced by the variation of the normalized distance between antenna elements $d$ on the BER performance of the system. As expected, as the correlation between the two branch diversity signals increases, the BER performance decreases. However, for a nor-
Fig. 3. BER performance of a pilot tone-aided dual MRC predetection diversity receiver with the normalized fading rate $f_aT$ as parameter. The normalized distance between antenna elements $d$ is 0.3.

Fig. 4. Effect of normalized distance between antenna elements variation on BER performance of a pilot tone-aided dual MRC predetection diversity receiver. The normalized fading rate $f_aT$ is 0.01.

normalized antenna distance as low as $d = 0.3$ much of the diversity gain can still be obtained. That is, even when antennas are very close there is a dramatic improvement due to the use of space diversity. In the limiting case $d = 0$, it is confirmed that there is a 3-dB diversity gain.

B. Postdetection Diversity Differential Detection Systems

In this case, the $i$th element of the interleaved trellis-coded symbol sequence $X_N$, namely $x_i$, is the phasor representation of the MPSK-coded symbol $\Delta \phi_k$ assigned by the mapper in the $i$th transmission interval. It can be written as

$$x_i = e^{j \Delta \phi_i}.$$  

Before transmission over the channel, the mapper output symbol sequence $X_N$ is differentially encoded producing the sequence $V_N$. In phasor notation, the MDPSK-coded symbol in the $i$th transmission interval can be expressed as

$$v_i = v_{i-1}x_i = A e^{j(\phi_{i-1} + \Delta \phi_i)} = A e^{j\phi_i}$$  

and the baseband equivalent of the transmitted signal is

$$v(t) = \sum_i v_i g(t - iT).$$  

Assuming a $K$ branch diversity system and the use of an ideal automatic frequency control (AFC), that is, a perfect compensation of frequency offsets between emitter and receiver local oscillators, the complex envelope of the received signal and the signal at the output of the reception filter in the $k$th receiver ($k = 1, 2, \cdots, K$) can be expressed as

$$r_k(t) = \chi_k(t) x(t) + n_k(t)$$  

and

$$w_k(t) = \chi_k(t) \sum_i v_i h(t - iT) + n_k(t)$$  

respectively. In a postdetection diversity receiver, including the differential detection function [21], each branch input signal is multiplied by its delayed replica, with the time delay $t_d = T$ for differential detection. The combiner’s output is sent to the demodulator to obtain in-phase and quadrature channel outputs. After filtering, these signals are sampled by an A/D converter. Assuming a perfect clock recovery, the complex sample at the deinterleaver output, that is, the $i$th element of the output sequence $Y_N$ corresponding to the input sequence $X_N$ will be given by

$$y_i = \sum_{k=1}^{K} K_i w_{k,i-1} u_{k,i}$$  

where for simplicity of notation we have dropped the delay introduced by the interleaving/deinterleaving process. Using these quantized symbols, the trellis decoder, implemented as a Viterbi algorithm with a metric depending upon whether or not channel state information (CSI) is provided, detects the transmitted symbol sequence based on maximum likelihood estimation. The optimum (maximum-likelihood) metric depends on the joint two-dimensional (amplitude and phase) statistics of a received sequence [22]. Assuming a soft Viterbi decoding based on the much simpler Gaussian metric expressed in (14), the differential detection system can be analyzed as the coherent one by using the previous received symbol as a reference. Thus, the decision variable $f$ can be expressed as

$$f = \sum_{i=1}^{N} \sum_{k=1}^{K} 2 \mathcal{R}\{w_{k,i}u_{k,i-1}(x_i - x_i)^*\}$$  

and by expanding the product $\omega_{k,i}u_{k,i-1}$ we have

$$\omega_{k,i}u_{k,i-1} = A^2 x_i \chi_{k,i} x_{k,i-1} + v_k \chi_{k,i} n_{k,i-1} + u_{k,i} \chi_{k,i} x_{k,i-1} + n_{k,i} \chi_{k,i} n_{k,i-1}.$$

By conditioning on $X_{i-1}$, the term $n_{k,i} \chi_{k,i} n_{k,i-1}$, which is a product of two complex-valued Gaussian random variables, makes the PDF for the variable decision $f$ difficult to compute. Nevertheless, given $X_{i-1}$ and assuming slow Rayleigh fading channels and distances between antennas of practical interest, the complex-valued Gaussian
random variable \( \chi_{k,i} \) has mean and variance that can be roughly approximated by 
\[ \mu_{\chi_{k,i}} \approx J_0(2\pi f_d T) \chi_{k,i} \quad \text{and} \quad \sigma^2_{\chi_{k,i}} \approx \left[ 1 - J_0^2(2\pi f_d T) \right]/2, \]
respectively. Thus, neglecting the term \( n_{k,i} \) in front of the other noise dominant terms [15], the new decision variable is a Gaussian random variable with mean and variance

\[ \mu_f \approx -A^2 J_0(2\pi f_d T) \sum_{k=1}^{N} \gamma(\chi_{k-1}) d_k^2 \] (46)

and

\[ \sigma^2_f \approx 4(A^4 \left[ 1 - J_0^2(2\pi f_d T) \right] + 4A^2 N_0) \sum_{k=1}^{N} \gamma(\chi_{k-1}) d_k^2 \] (47)

respectively. Thus, using the approach described in Section III-A leads to the following expression for the union bound on the average bit error probability:

\[ P_b \leq \frac{1}{2n} \sum_{k=1}^{n} \sum_{N \in L_e} \sum_{X_N} \sum_{X_N} a(X_N, \hat{X}_N) p(X_N) \times \prod_{i \in \eta} \prod_{k=1}^{K} (1 + \Theta_{i,k})^{-1} \] (48)

where

\[ \Theta_{i,k} = \frac{E_s N_0 J_0^2(2\pi f_d T) d_k^2 \lambda_k}{4 \left[ \frac{E_s}{N_0} \left[ 1 - J_0^2(2\pi f_d T) \right] + 2 \right] \sin^2((2L - 1)\pi/4n)} \] (49)

and \( E_s = A^2/2 \) is the equivalent energy per MDPSK symbol.

For the case of independent diversity branches, as shown in Appendix II, it is possible to obtain an exact expression for the union bound on the average BER as

\[ P_b \leq \frac{1}{2n} \sum_{k=1}^{n} \sum_{N \in L_e} \sum_{X_N} \sum_{X_N} a(X_N, \hat{X}_N) p(X_N) \times \prod_{i \in \eta} (1 + \Omega_{i,k})^{-K} \] (50)

where

\[ \Omega_{i,k} = \frac{\left( \frac{E_s}{N_0} \right)^2 J_0^2(2\pi f_d T) d_k^2 \csc^2((2L - 1)\pi/4n)}{4 \left[ \left[ \frac{E_s}{N_0} \right]^2 \left[ 1 - J_0^2(2\pi f_d T) \right] + 2 \left( \frac{E_s}{N_0} \right) + 1 \right]} \] (51)

As in pilot tone-aided systems, the BER performance of differential detection schemes over slow Rayleigh fading channels is rightly dependent on two factors, namely, the Hamming distance profile between all pairs of possible encoded sequences and the product of normalized squared Euclidean distances between distinct symbols in all possible pairs of encoded sequences. Also, correlation between diversity branches has the effect of modifying the normalized squared Euclidean distances associated to the error events by factors equal to the eigenvalues of the diversity array covariance matrix. It can also be seen that the BER performance of the system deteriorates as the normalized fading rate increases and an average bit error floor is clearly shown as the normalized fading rate increases. In fact, by letting \( E_s/N_0 \to \infty \) in (48) we obtain, for slow Rayleigh fading channels and distances between antennas of practical interest

\[ \lim_{(E_s/N_0)\to\infty} P_b \approx \frac{1}{2n} \sum_{k=1}^{n} \sum_{N \in L_e} \sum_{X_N} \sum_{X_N} a(X_N, \hat{X}_N) p(X_N) \times \prod_{i \in \eta} \prod_{k=1}^{K} (1 + \lim_{(E_s/N_0)\to\infty} \Theta_{i,k})^{-1} \] (52)

where

\[ \lim_{(E_s/N_0)\to\infty} \Theta_{i,k} = \frac{J_0^2(2\pi f_d T) d_k^2 \lambda_k}{4 \left[ 1 - J_0^2(2\pi f_d T) \right] \sin^2((2L - 1)\pi/4n)} \] (53)

Thus, provided that none of the \( \lambda_k \) is equal to zero, the error floor of this transmission scheme depends exponentially on the product diversity \( L_d K \), and it is expected that, with differential detection, powerful codes (large \( L_s \)) provide low error floors. Moreover, uncorrelated diversity effectively lowers the error floor and, asymptotically as \( K \to \infty \), removes it. However, for strongly correlated diversity branches, some of \( \lambda_k \) may be vanishing, so that the space diversity order may be reduced and, thus, the error floor increased.

As in the case of pilot tone-aided predetection diversity systems, we consider the error performance of Ungerboeck’s eight-state 8DPSK scheme in correlated Rayleigh fading channels. Fig. 5 shows simulation and analytical results on the average BER with the number of diversity branches as parameter. A normalized fading rate of \( f_d T = 0.01 \) and a regular linear array receiving antenna consisting of \( K \) dipoles with a normalized distance between two consecutive antenna elements of \( d = 0.3 \),
have been considered. Fig. 6 illustrates the effect of varying the normalized fading rate $f_d T$ on the performance of the system. A two-branch diversity system with a normalized distance between antennas $d = 0.3$ has been considered. Fig. 7 shows the BER performance of the dual diversity MRC reception scheme, for a normalized fading rate $f_d T = 0.01$, illustrating the effect produced by the variation of the normalized distance between antenna elements $d$ on the BER performance of the system. Observing these results, one obvious conclusion is that, as in the pilot tone-aided predetection diversity systems, relative diversity gains decrease as space diversity orders increase. Also, as the correlation between the diversity signals increases, the BER performance decreases. However, for a normalized antenna distance as low as $d = 0.3$ much of the diversity gain can still be obtained. As expected, the system performance deteriorates as the normalized fading rate increases and an average bit error floor is clearly shown for fast Rayleigh fading. Comparing the error floor obtained without diversity with that obtained with a two or three antenna diversity, we conclude that space diversity improvement applies to the error floor as well. Our bound is very accurate for slow fading rates and high space diversity orders and becomes less accurate as the fading rate increases, especially in the error floor region.

IV. CONCLUSION

The error performance of reference-based predetection and postdetection diversity systems when used in conjunction with trellis-coded MPSK modulation techniques over correlated Rayleigh fading channels has been analyzed. Based on the statistical properties of correlated complex Gaussian random variables, a new, simple and exact expression for the upper union bound on the average bit error probability has been derived. Monte-Carlo simulation results, which are more indicative of the exact system performance, show that our bound, having the same complexity as the union-Chernoff bound, is extremely tight when compared with other bounds in the literature. It is found that with differential detection the nondiversity system performs quite well for low normalized fading rates. For high normalized fading rates, the error floor produced by differential phase jitter introduced by fading becomes unacceptable. Nevertheless, as confirmed by simulations, the error floor is considerably lower when space diversity reception is used. The problem of the error floor associated with differential detection is fully eliminated at the expense of using a pilot tone-aided predetection diversity coherent system. In fact, with an appropriate choice of the bandwidth of the pilot filter extractor, the performance of the pilot tone-aided scheme is quite close to that of the idealized coherent system ($f_d T \to 0$) for normalized fading rates less than 0.03. It is also remarkable that using space diversity dramatically improves the performance of the system, even when antenna elements are very close. In fact, a normalized antenna distance between consecutive elements of a regular linear array of $d = 0.3$ seems to be enough to obtain a BER performance quite similar to that obtained with uncorrelated antennas.

APPENDIX I

PILOT TONE-AIDED SYSTEMS OVER UNCORRELATED RAYLEIGH FADING CHANNELS

In a Rayleigh fading channel, when conditioned on the codeword, $w_{k,i}$ and $p_{k,i}$ are zero-mean complex Gaussian random variables with variances $\sigma_{w}^2$ and $\sigma_{p}^2$, respectively, and covariance $\sigma_{wp}$, that can be written as

$$\sigma_{w}^2 = \frac{1}{2}E\{|w_{k,i}|^2\} = \frac{A^2}{2} + N_0 \quad (54)$$

$$\sigma_{p}^2 = \frac{1}{2}E\{|p_{k,i}|^2\} = \frac{B^2}{2} + N_0 B_p \quad (55)$$

$$\sigma_{wp} = \frac{1}{2}E\{w_{k,i}p_{k,i}^*\} = \frac{B x_i}{2}, \quad (56)$$

Fig. 6. BER performance of a differential dual MRC postdetection diversity receiver with the normalized fading rate $f_d T$ as parameter. The normalized distance between antenna elements $d$ is 0.3.

Fig. 7. Effect of normalized distance between antenna elements variation on BER performance of a differential dual MRC postdetection diversity receiver for the case of a normalized fading rate of $f_d T = 0.01$. 
The corresponding correlation coefficient is
\[ \rho = \frac{\sigma_{wp}^2}{\sigma_u^2 \sigma_p^2} = \frac{B x_i}{(A^2 + 2N_0)(B^2 + 2N_0 B_p)}. \] (57)

This implies that given \( p_{h,i}, w_{k,i} \) is a complex Gaussian random variable with mean of
\[ \mu_{wp} = \rho \frac{\sigma_u}{\sigma_p} = \frac{B x_i p_{h,i}}{B^2 + 2N_0 B_p} \] (58)
and variance of
\[ \sigma_{wp}^2 = (1 - |\rho|^2) \frac{\sigma_u^2}{\sigma_p^2} = \frac{A^2 B^2 N_0 B_p + 2A^2 N_0 + 2 B^2 N_0^2 B_p}{B^2 + 2N_0 B_p}. \] (59)

Therefore, given \( P_i = (p_{h,i}, p_{2,i}, \cdots, p_{K,i}), i = 1, 2, \cdots, N, \) and assuming uncorrelation between diversity antennas, \( f \) is a Gaussian random variable with mean of \( \mu_f \) and variance of \( \sigma_f^2 \) which can be shown to be given by
\[ \mu_f = \sum_{i=1}^{N} \sum_{k=1}^{K} 2 \text{Re} \left\{ \frac{\sigma_u}{\sigma_p} p_{h,i} x_i^* \left( x_i - \hat{x}_i \right) \right\} \]
\[ = - \frac{A^2 B}{B^2 + 2N_0 B_p} \sum_{i=1}^{N} \gamma(P_i) d_i^2 \] (60)
and
\[ \sigma_f^2 = \sum_{i=1}^{N} \sum_{k=1}^{K} 2(1 - |\rho|^2) \frac{\sigma_u^2}{\sigma_p^2} p_{h,i} x_i^2 \left( x_i - \hat{x}_i \right) \]
\[ = 4 \frac{A^4 N_0 B_p + A^2 B^2 N_0 + 2A^2 N_0^2 B_p}{B^2 + 2N_0 B_p} \sum_{i=1}^{N} \gamma(P_i) d_i^2 \] (61)
respectively. Now, by substituting (60) and (61) into (21) and by applying the approach described in Section III-A, the exact union bound on the average bit error probability is obtained as given by expression (35).

**APPENDIX II**

**DIFFERENTIAL DETECTION SYSTEMS OVER UNCORRELATED RAYLEIGH FADING CHANNELS**

In this case, expressions (57)–(61) and (21) can be used with
\[ \sigma_u^2 = \frac{1}{2} E \{ |w_{k,i}|^2 \} = \frac{A^2}{2} + N_0 \] (62)
\[ \sigma_p^2 = \frac{1}{2} E \{ |w_{k,i} - 1|^2 \} = \frac{A^2}{2} + N_0 \] (63)
\[ \sigma_{wp}^2 = \frac{1}{2} E \{ w_{k,i} w_{k,i}^* \} = \frac{A^2}{2} x_i J_0(2 \pi f_d T). \] (64)

Then, using the approach described in Section III-A, the exact union bound on the average bit error probability is obtained as given by expression (50).

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