A Note on Heterogeneity, Inefficiency, and Indeterminacy with Ricardian Preferences

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The main issue raised in this note is the nonequivalence between the infinite-horizon model where agents are infinitely lived and the successive generations model with altruistic finitely lived agents: in the presence of a nonnegative bequest requirement, endowment heterogeneity imposes a revision of the acritical adoption of the infinitely lived agent representation in modern macro-economics. By analysing nonstationary monetary equilibria in a reinterpretation of Townsend's "turnpike" model, this paper shows how the traditional issues of market inefficiency and indeterminacy of overlapping generations models carry over into modern macroeconomics through the natural finiteness of human lives despite "well behaved" Ricardian altruism. Journal of Economic Literature Classification Numbers: D31, D91, E13, E40. © 2001 Academic Press

1. INTRODUCTION

A large body of modern macroeconomics works under the assumption of infinitely lived agents. If a first year student asks a modern macroeconomist whether or not he or she really believes that people live forever the macroeconomist will typically respond that people have finite lives, but that they have children, that they care about their welfare, and that since Barro [3] we know that this renders a model of finitely lived successive or overlapping generations formally equivalent to a model with infinitely lived agents.

After that, the student will be taught the tool kit of infinite horizon problem solving and will never more question the legitimacy of explaining the "real world" using the assumption of eternal individuals.

Recently, macroeconomists have started to realize more that people are not totally identical, at least in their endowments, as witnessed by a

1 I thank Karl Shell and Neil Wallace for useful discussions and an associate editor and a referee of this journal for very helpful remarks.
growing literature on the causes and the effects of inequality.² It is perhaps legitimate to ask again: are we sure that the infinitely lived agent is equivalent to the infinitely lived family with perishable and altruistic individuals?

Barro [3] correctly warned the reader that agents should not be allowed to leave negative bequests. This is not a problem in a representative agent model, because in the perfectly symmetric equilibrium people always make equal choices and consumption loans are zero. However, if heterogeneity is present the infinitely lived family problem can partition itself into an infinite number of finitely lived agent problems. But then, as Shell [9] warned us, the resulting double infinity of traders and commodities gives scope for substantial departures from both optimality and local uniqueness of competitive equilibria.

This paper shows this by reinterpreting Townsend’s [10] framework as a model of heterogeneous successive generations with Ricardian preferences: in this case the absence of long term credit emerges naturally from the nonnegative bequest requirement. As shown in Section 2, the market equilibria easily become inefficient if agents are not identical and the often claimed equivalence between the economy with infinitely lived families with altruistic members and the economy with literally infinitely lived individuals fails.

Section 3 introduces the social contrivance of outside money into the model. It shows that whenever a monetary steady state exists it will only partially cure the inefficiency of the equilibria, and under mild assumptions a continuum of inflationary monetary equilibria exists. This was ignored in Townsend’s [10] analysis but can be worthwhile in light of the recent renewed interest in indeterminacy in infinitely lived agent models (see the JET special issues on indeterminacy and sunspots, 1994, 1998). A simple explicit example is worked out in Section 4.

The main message of this exercise is that in the presence of unequal endowments the most “well behaved” intergenerational altruism can be insufficient to rule out inefficiency, indeterminacy, and sunspot equilibria, even in the absence of any market failure other than incomplete market participation (see [5]): in our model Ricardian preferences are not sufficient to restore efficiency and local uniqueness unless agents are assumed identical.

Therefore if economists want to seriously analyze heterogeneous populations they should be careful not to apply without reflection the infinitely lived agent metaphor they have learned in the identical agents case, unless literally believing in eternal individual worldly life.

² See Aghion et al. [2], Benabou [4], and Aghion and Howitt ([1, Chap. 9] for very good surveys.
2. THE MODEL WITHOUT MONEY

Assume an infinite sequence of successive generations of individual members of two types of dynasties: A and B. Total population is constant and is normalized to the unit interval, and each type A or B is half of the population. Time is discrete and indexed by \( t = 0, 1, 2, \ldots \) and people live only one period. There is a unique perishable consumption good, no outside money, but individuals can issue private debt (inside money).

The sequence of individual endowments of the consumption good \( \{ \omega^H_t \}_{t=0}^\infty, \ H = A, B \) is such that \( \omega^A_t = 1 - \varepsilon \) in even periods and \( \omega^A_t = \varepsilon \) in odd periods, whereas \( \omega^B_t = 1 - \varepsilon \) in odd periods and \( \omega^B_t = \varepsilon \) in even periods, where \( 0 < \varepsilon < \frac{1}{2} \). Hence members of families of type A are “richer” if they are alive in even periods and “poorer” if they are alive in odd periods, while type B family members are “richer” if they are alive in odd periods and “poorer” if they are alive in even periods.

People are altruistic in a way that is traditionally considered able to make the finitely lived individual decision formally equivalent to the infinitely lived individual; that is, the utility, \( U^H_t \), of a member of a family of type \( H = A, B \) borne at time \( t \) satisfies

\[
U^H_t = u(c^H_t) + \delta U^{H+1}_t,
\]

where \( u(c^H_t) \) is the one period utility of contemporaneous consumption, \( u(\cdot) \) is assumed to be strictly increasing, to be strictly concave, and to satisfy Inada’s conditions, and \( 0 < \delta < 1 \) measures intergenerational altruism.

To compare the two frameworks let us now turn our attention to a setting where individuals are literally infinitely lived: the superscript now denotes an individual’s type, assumed to have the same endowment stream as the previous “family” of the corresponding type.

**Definition 1.** A perfectly competitive equilibrium with infinitely lived individuals (IL) is a sequence of bounded nonnegative futures prices \( \{ q_t \}_{t=0}^\infty \) and of agents’ consumption plans \( \{ c^H_t \}_{t=0}^\infty, \ H = A, B \), such that

(i) market clearing

\[
c^A_t + c^B_t = 1
\]

holds at all dates \( t = 0, 1, \ldots \); and

(ii) \( \{ c^H_t \}_{t=0}^\infty \) solves

\[
\max \sum_{t=0}^{\infty} \delta^t u(c_t) \quad \text{subject to} \quad (\text{IL})
\]

\[
\sum_{t=0}^{\infty} c_t q_t \leq \sum_{t=0}^{\infty} \omega^H_t q_t
\]

and \( c_t \geq 0 \), for all \( t = 0, 1, \ldots \), and \( H = A, B \).
Given the strict monotonicity of the utility functions all equilibrium prices and quantities are strictly positive, as is easily proved. Moreover, a very slight extension of Townsend’s [10] model\(^3\) case shows that the competitive equilibrium in the infinitely lived individual case is unique and Pareto optimal, and there is perfect consumption smoothing despite very uneven endowment patterns, with type \(A\) agents always consuming 
\[
   c^A_t = \frac{(1 - \varepsilon(1 - \delta))/1 + \delta}{1 + \delta} 
\]
and type \(B\) agents always consuming 
\[
   c^B_t = \frac{(\delta + \varepsilon(1 - \delta))/1 + \delta}{1 + \delta}; \quad \text{and the real interest rate is equal to the subjective rate of time preference. Notice that individuals save in periods in which their endowment is } 1 - \varepsilon \text{ and dissave when their endowment is } \varepsilon. \text{ As } \varepsilon \text{ tends to } \frac{1}{2}, \text{ also } c^A_t \text{ and } c^B_t \text{ tend to } \frac{1}{2}; \text{ that is, the allocations approach the representative agent ones, } c^A_t = c^B_t = \frac{1}{2} \text{ for all } t = 0, 1, \ldots.
\]

Let us now go back to our economy with infinitely lived families of finitely lived individuals with Ricardian altruism (FLA), in which bequests have to be nonnegative and in which no outside money exists.

**Definition 2.** A competitive equilibrium with infinitely lived families of finitely lived individuals with altruism and no money (FLA) is a sequence of bounded nonnegative futures prices \(\{q_t\}_{t=0}^{\infty}\) and of consumption plans \(\{c^H_t\}_{t=0}^{\infty}\), \(H = A, B\), such that

(i) market clearing 
\[
c^A_t + c^B_t = 1 
\]
holds at all dates \(t = 0, 1, \ldots\); and

(ii) \(\{c^H_t\}_{t=0}^{\infty}\) solves
\[
\max \sum_{t=0}^{\infty} \delta^t u(c_t) \quad \text{subject to (FLA)}
\]
\[
\sum_{t=0}^{\infty} c_t q_t \leq \sum_{t=0}^{\infty} \omega^H_t q_t, \quad 0 \leq c_t q_t \leq \omega^H_t q_t + \sum_{j=0}^{t-1} (\omega^H_j - c_j) q_j 
\]
and \(c_t \geq 0\), for all \(t = 0, 1, \ldots\), and \(H = A, B\).

Notice that the nonnegative bequest requirement now constrains every generation’s consumption not to exceed the sum of their own endowment

\(^3\) First show that the first order conditions and market clearing imply 
\[
   u'(c^H_t)u'(c^H_{t+1}) = u'(1 - c^H_t)u'(1 - c^H_{t+1}) = \delta q_t q_{t+1}, \quad \text{for all } t = 0, 1, \ldots \text{ and } H = A, B. \text{ These in turn imply } c^H_t = c^\delta \text{ constant and equilibrium real gross interest rate equal to } \delta^{-1}. \text{ Finally plug these into the intertemporal budget conditions and simplify the algebra.}
\]

To show Pareto optimality follow similar steps to Sargent [8, pp. 201–202] with obvious reinterpretations.
and the cumulated savings of their predecessors. In both IL and FLA cases inside money (i.e., private debt) issuance is being allowed; hence, the IL economy and the FLA economy have the same trading institutions, since consumption and inside money are traded over time. The difference is that inside money obligations do not have to be honored by subsequent generations (the nonnegative bequest constraint). Therefore the difference between an infinitely lived agent and an altruistic finitely lived agent is that inside money is far more useful to the infinitely lived agent.

Clearly, generation zero begins with no bequest and therefore (given strict monotonicity of preferences) market clearing implies $c_0^H = \omega_0^H$, $H = A, B$. Hence the next generation is borne at $t = 1$ with no bequest, thus leaving nothing for the generation borne at date $t = 2$, and so on. Therefore there exists a unique equilibrium allocation, and this implies $c_t^H = \omega_t^H$ for all $t = 0, 1, \ldots$ and $H = A, B$; such consumption stream does not coincide with the IL case and it is not Pareto optimal. As a result the representative agent metaphor does not hold unless agents are literally identical, which happens if and only if $\varepsilon = \frac{1}{2}$.

To summarize our discussion so far:

**Proposition 1.** The competitive equilibrium allocations achieved in the infinitely lived individuals economy (IL) cannot be attained as competitive equilibrium allocations of the corresponding economy with infinitely lived families of altruistic finitely lived individuals with no money (FLA) for all $\varepsilon \in [0, \frac{1}{2})$. The two allocations coincide if and only if $\varepsilon = \frac{1}{2}$, that is if and only if there is no heterogeneity at all.

The model with infinitely lived individuals is not equivalent to the model with successive generations within altruistic families, and the assumption of nonnegative bequest is the main “engine” behind this result: the infinitely lived individual is allowed to commit to delivering future commodities, but a member of a dynasty cannot credibly commit to commodities being delivered by his or her descendants. In other words, inequality can generate inefficiency despite Ricardian altruism and no market failures other than human mortality.

3. THE MODEL WITH MONEY

That the perfectly competitive equilibrium with nonnegative bequest requirement generically implies a unique Pareto inefficient equilibrium in the absence of outside money leaves us hope for monetary equilibria to improve on the no money case. As in Samuelson [7], the unpleasant “no
trade” equilibrium result suggests to us to look for other equilibria characterized by the presence of the “social contrivance of money.” One can expect that since money is able to partially “cure” the extreme heterogeneity of the \( \varepsilon = 0 \) case—as we already know from Townsend’s original [10] analysis—it might be able to completely cure more intermediate cases.

In the IL economy outside money is irrelevant because inside money obligations are honored, but in the FLA case one can expect outside money to help the individual to enlarge the set of permissible trades. Since we only analyze a laissez faire economy, we rule out interventionist government policies following the introduction of the initial stock of money.\(^4\)

Assume that type \( B \) agents at time \( t = 0 \) are endowed\(^5\) with \( m > 0 \) units of nonperishable money.

**Definition 3.** A perfectly competitive equilibrium with money is a sequence of nonnegative shot prices \( \{ p_t \}_{t=0}^{\infty} \) and of agents’ \( H = A, B \) consumption and money demand plans, \( \{ c^H_t \}_{t=0}^{\infty} \) and \( \{ m^H_{t+1} \}_{t=0}^{\infty} \), such that

(i) market clearing

\[
c^A_t + c^B_t = 1, \quad \text{and} \quad m^A_t + m^B_t = 1,
\]

holds at all dates \( t = 0, 1, \ldots \); and

(ii) \( \{ c^H_t \}_{t=0}^{\infty} \) and \( \{ m^H_{t+1} \}_{t=0}^{\infty} \) solve

\[
\max \sum_{t=0}^{\infty} \delta u(c_t), \quad \text{subject to (M)}
\]

\[
c_t + p_t m_{t+1} = \omega^H_t + p_t m_t, \quad \text{and}
\]

\[
c_t \geq 0, m_t \geq 0, \quad \text{for all } t = 0, 1, \ldots, \text{ and } H = A, B, \text{ and}
\]

\[
m^A_0 = 0, \quad \text{and} \quad m^B_0 = m,
\]

where \( m_{t+1} \) is the demand for money by a member of a family alive at date \( t \) to bequeath to a member of the same family who is alive in period \( t + 1 \).

Notice that \( p_t \) now indicates the price of money in terms of the consumption good at time \( t \), and the whole sequence \( \{ p_t \}_{t=0}^{\infty} \) is perfectly known and taken as given by the individual agents.

Unfortunately a monetary steady state is not always sustainable in this economy, and the reason is very simple. Similarly to Townsend’s [10]

\(^4\) The reader may think of such “money” as of a fixed stock of “shells” available to the private families.

\(^5\) This assumption is innocent: any initial distribution of money will generate the same outcome after two periods.
economy, in a monetary steady state—with constant and positive real value of money \( p_t = p > 0 \)—consumption paths cannot be constant, because otherwise the marginal rates of intertemporal substitution would be equal to \( \frac{1}{\delta} \) which differs from 1, that is from the marginal rate of economic transformation. A nonconstant consumption pattern in turn implies a corner solution with \( c^H_t = \omega^H_t \) when \( \omega^H_t = \varepsilon \), and \( \delta u'(\varepsilon) \geq u'(1-\varepsilon) \), that is money demand is zero in low income states. But this cannot hold if \( \varepsilon \) is too close to one half, because \( \delta < 1 \). Hence, let us define

\[
\varepsilon \equiv \min\{ \varepsilon \in \mathbb{R}_+ : \delta u'(\varepsilon) \leq u'(1-\varepsilon) \};
\]

note that \( \varepsilon \) exists under our assumptions on \( u(\cdot) \), and that \( 0 < \varepsilon < \frac{1}{2} \). Then we know that a monetary steady state exists if and only if \( 0 \leq \varepsilon \leq \varepsilon \).

If \( \varepsilon > \varepsilon \) the agent wants to borrow when he or she is “rich” and repay when her child is “poor,” and since this is not allowed the agent will consume his or her whole endowment, exchanging nothing with the poor people of his or her own generation. Since the poor wants to anticipate consumption too, there will always be excess supply of money, with its real value, \( p_t \), becoming zero.

By an argument similar to that in the previous discussion we rule out inflationary equilibria. Hence if \( \varepsilon > \varepsilon \) we are left with deflationary equilibria with deflation rates uniformly bounded below by \( 1 - (\delta u'(\varepsilon)/u'(1-\varepsilon)) > 0 \)—with higher deflation the closer \( \varepsilon \) to \( \frac{1}{2} \)—which is inconsistent with constant nominal money supply. Therefore we can state:

**Proposition 2.** A monetary steady state exists if and only if \( \varepsilon \in [0, \varepsilon] \).

As we “approximate” the representative agent economy with a heterogeneous agent’s economy we lose all monetary equilibria and we are left with the inefficient “no trade” steady state with worthless money.

For values of \( \varepsilon \) that are lower than \( \varepsilon \) a monetary steady state exists, but under mild assumptions on preferences it is neither the unique competitive equilibrium in which money is valued nor determinate; as we will shortly see a continuum of nonstationary monetary equilibria arise.

Since in a monetary equilibrium prices are positive and finite we can define \( p_{t+1} = p_{t+1}/p_t \). We will look for equilibria satisfying\(^6\) \( \delta p_{t+1} < 1 \).

The first order conditions for the maximization problem of agent \( H = A, B \) in a monetary equilibrium \( M \) are:

\(^6\)This is not an assumption, because it involves endogenous variables, but a restriction on the kind of equilibria we are interested in. In particular such a restriction has to hold at monetary steady states. Hence what we are doing is ignoring equilibria with deflation whose rate is at least as high as \( 1 - \delta \) at some date.
\begin{equation}
-u'(c^H_t) + \delta u'(c^H_{t+1}) \rho_{t+1} \leq 0
\end{equation}

\[
[ -u'(c^H_t) + \delta u'(c^H_{t+1}) \rho_{t+1} ] \ m^H_{t+1} = 0
\]

\[m^H_{t+1} > 0.
\]

If the first inequalities are strict then \(m^H_{t+1} = 0\): this cannot hold for both types of agents at any date, otherwise we would not be in a monetary equilibrium. Neither can the first inequalities both be equalities: in fact this would imply \(u'(c^H_t) / u'(c^H_{t+1}) = \delta \rho_{t+1} < 1\) for both \(H = A, B\), and therefore \(c^A_{t+1} + c^B_{t+1} < c^A_{t} + c^B_{t}\) which is inconsistent with market clearing at all dates.

Hence our last possibility is to have at each date one strict inequality and one equality. The strict inequality cannot be that of the richer agent in \(t\), because \(m^H_{t+1} = 0\) and the constraint in \(M\) would imply the absurdity

\[
0 > -u'(1 - \varepsilon + p_t m^H_t) + \delta u'(\varepsilon - p_{t+1} m^H_{t+1}) \rho_{t+1}
\]


under \(0 \leq \varepsilon < \frac{1}{2}\). Therefore the agent who is poor at date \(t\) must be the only one at the corner solution and he or she will demand zero money to bequeath to his or her child.

Hence the following holds for \(H = A\) and \(t\) even, and for \(H = B\) and \(t\) odd:

\begin{equation}
-u'(c^H_t) = \delta u'(1 - \varepsilon - c^H_{t}) \rho_{t+1} + \varepsilon \rho_{t+1}
\end{equation}

\[
(1 - \varepsilon - c^H_{t}) \rho_{t+1} = 0.
\]

One and only one solution \(c^H_{t}\) to the first equation in (2) exists for all \(\rho_{t+1} \in (0, \delta^{-1})\) and in particular at the monetary steady state value \(\rho_{t+1} = 1\). Note that in (2) \(c^H_{t} \in (0, 1 - \varepsilon)\). Hence, system (2) implicitly defines consumption at any date \(t\) as a function of \(\rho_{t+1}\) for the family that is richer at \(t\) and as a function of \(\rho_{t}\) for the family that is poorer at \(t\). Let us call \(c^H_{\text{rach}}(\rho_{t+1})\) the first one and \(c^H_{\text{pool}}(\rho_{t})\) the second. When an agent is rich he or she makes a positive decision on his or her child’s consumption, while if the agent is poor he or she will not play an active role in transmitting a bequest to the future. Therefore the infinite horizon family problem is partitioned into an infinite sequence of finite horizon problems. Due to income inequality the infinitely lived family of altruist generates the same “double infinity” (Shell [9]) feature of the traditional overlapping generation model.

\footnote{In fact, if the rich \(H\) demands no money, the poor \(K \neq H\) will be demanding the whole amount \(m\) in equilibrium, and therefore: \(-u'(1 - p_t m) + \delta u'(1 - \varepsilon + mp_{t+1} - m_{t+2}^K \rho_{t+1}) \rho_{t+1} = 0\). Since in equilibrium \(m_{t+2}^K \leq m\), the last inequality in the text follows.}
models with no altruism. It thus becomes natural to investigate the possibility of indeterminacy of monetary equilibria in such a framework. This is what will follow in the rest of this section.

Notice that the subscripts rich and poor do not refer to families that are always richer and families that are always poorer, but to different temporary locations of family members in a particular generation’s income distribution. In other words, in our economy there is perfectly symmetric social mobility across the two types of families, with their individual members alternating their worldly existence in a poorer and in a richer status.

Implicitly differentiating Eq. (2) yields:

\[
\begin{align*}
    c_{\text{rich}}'(& \rho_{t+1}) \\
    &= -\delta u'((1 - \varepsilon - c_{\text{rich}}^H) \rho_{t+1} + \varepsilon) + (1 - \varepsilon - c_{\text{rich}}^H) u'(1 - \varepsilon - c_{\text{rich}}^H) \rho_{t+1} + \varepsilon) \\
    &= u'(1 - \varepsilon - c_{\text{rich}}^H) \rho_{t+1} + \varepsilon) \\
    &= (1 - \varepsilon - c_{\text{rich}}^H) \rho_{t+1} + \varepsilon).
\end{align*}
\]

Hence \( c_{\text{rich}}'(\rho_{t+1}) < 0 \) if the numerator is positive, i.e., the intertemporal substitution effect dominates. For the rest of this section we will assume this to hold locally:

A1. Let \( u(\cdot), \delta, \) and \( \varepsilon \) be such that \( u'(x) < -(x - \varepsilon) u''(x) \) for the value of \( x \) that solves \( u'(x) = \delta u'(1 - x) \).

Notice that Assumption A1 is a mild assumption on preferences and assumes that the endowment of the poor is not too large.

Under A1 it follows that in a neighbourhood of the monetary steady state \((\rho_1 = 1)\):

\[
    c_{\text{poor}}'(\rho_1) = (1 - \varepsilon - c_{\text{rich}}^H) - \rho_1 c_{\text{rich}}'(\rho_1) > 0.
\]

Market clearing at every date implies

\[
    c_{\text{rich}}(\rho_{t+1}) + c_{\text{poor}}(\rho_t) = 1,
\]

which implicitly defines \( \rho_{t+1} = \bar{\chi}(\rho_t) \), where \( \bar{\chi}(\cdot) = (c_{\text{rich}}')^{-1}(1 - c_{\text{poor}}(\cdot)) \) is a decreasing function satisfying \( \bar{\chi}(1) = 1 \). Therefore the dynamical system

\[
    \rho_{t+1} = \bar{\chi}(\rho_t)
\]

\[
    0 < \rho_0 \leq 1
\]
yields a continuum of monetary equilibria parameterized by the initial real gross interest rate $\rho_0$ in a neighbourhood of the monetary steady state.

Therefore we can summarize this section in the following:

**Proposition 3.** Monetary equilibria are possible only if endowment heterogeneity is strong enough, but they do not restore Pareto efficiency. Moreover, whenever a steady state exists, under Assumption A1 there exists a continuum of nonstationary equilibria; that is, we have indeterminacy.

4. A SIMPLE EXAMPLE

In what follows we conduct a global and explicit analysis by working with the CES one period utility function $u(c_H^t) = (c_H^t)^{1-\gamma} - 1/\gamma$, $\gamma > 0$. It is easy to see that $\hat{\rho} = \delta^{1/\gamma} [1 + \sigma^{1/\gamma}] < 1/2$. However, here we will assume that the endowments are exactly as in the basic Townsend [10] model, that is, $\hat{\rho} = 0$.

To look for monetary equilibria, assume that type $B$ agents at time $t = 0$ are given $m > 0$ units of nonperishable money. Following the same steps as in the previous section we obtain that type $B$ families consume $c_A^t = 1/(1 + \delta^{1/\gamma} \rho_{t+1}^{1-\gamma} \rho_t^{1/\gamma})$ in even periods and $c_A^t = (\rho_t^{1/\gamma} \rho_{t+1})/(1 + \delta^{1/\gamma} \rho_{t+1}^{1-\gamma} \rho_t^{1/\gamma})$ in odd periods, while families of type $B$ consume $c_B^t = 1/(1 + \delta^{1/\gamma} \rho_{t+1}^{1-\gamma} \rho_t^{1/\gamma})$ in odd periods and $c_B^t = (\delta^{1/\gamma} \rho_{t+1})/(1 + \delta^{1/\gamma} \rho_{t+1}^{1-\gamma} \rho_t^{1/\gamma})$ in even periods, where $\rho_{t+1} = \rho_{t+1}/\rho_t$, where as before $\rho_t$ denotes the real price of outside money at date $t$.

Markets clear at the previous individual maximizing choices, hence:

$$\frac{1}{1 + \delta^{1/\gamma} \rho_{t+1}^{1-\gamma} \rho_t^{1/\gamma}} + \frac{\delta^{1/\gamma} \rho_{t+1}^{1/\gamma}}{1 + \delta^{1/\gamma} \rho_{t+1}^{1-\gamma} \rho_t^{1/\gamma}} = 1$$

for all $t = 0, 1, 2, \ldots$, which implies

$$\rho_{t+1} = \frac{\rho_t^{1/(1-\gamma)}}{[1 + \delta^{1/\gamma}(\rho_t^{1-\gamma} \rho_t^{1/\gamma} - \rho_t^{1/\gamma})]^{1/(1-\gamma)}}. \quad (3)$$

Notice that the monetary steady state is at $\rho_t = 1$ and that $\rho_t < \delta^{-1}$ holds for all sequences starting at some $\rho_0 \in (0, 1]$.

Let us remind the reader that none of these monetary equilibria are Pareto efficient. Pareto efficiency in fact requires that the intertemporal marginal rate of substitution always equal $\delta^{-1}$, and therefore a Pareto optimal monetary equilibrium would imply $\rho_{t+1} = \delta^{-1}$, that is, constant deflation and real monetary balances exceeding aggregate resources in finite time, unless a suitable calibrated government intervention takes place in every period.

Notice that Assumption A1 is satisfied here.
The first derivative of $p_{t+1}$ with respect to $p_t$ evaluated $p_t = 1$ is $(1 + \gamma \delta^{1/\gamma})/1 - \gamma$ and straightforwardly implies the local analysis. Global analysis follows from the study of Eq. (3); in particular for $0 < \gamma < 1$ the monetary steady state is globally unstable: similar to Gale’s [6] Samuelson case if inflation starts it will increase over time and the system will converge in an infinite number of periods to the no trade steady state. For $\delta < (1 - 2/\gamma)^2$ the monetary steady state is locally stable (Gale’s classical case), with dampering oscillations.

Let us remark that the formal similarities with the traditional overlapping generations models should not mislead the reader to forget that in this economy families are well behaved infinite horizon decision makers, whose some members have to behave as if they were myopic and nonaltruistic due to their particular kind of mobility up and down the social ladder.

5. FINAL REMARKS

In the simple model previously analyzed heterogeneity in endowments breaks the chain of intergenerational altruism that with perfectly identical altruistic families would imply the equivalence with the infinitely lived individual case. With heterogeneous endowments the intergenerational links are cut by the inability of the member of a dynasty to commit to commodities being delivered by their descendants.

Despite the peculiar form of intergenerational altruism that is supposed to make us accept the infinite life metaphor as a tool for understanding reality, the nonnegative bequest requirement can make the fictitious infinite horizon problem partition itself into an infinite sequence of finite horizon problems, reintroducing into macroeconomics the double infinity (Shell [9]) feature that in the traditional overlapping genetational models with no intergenerational altruism resulted in the nonoptimality of competitive equilibria, in a role for the social contrivance of money, and caused indeterminacy and sunspot equilibria.

Since this model is very simple, it justifies the expectation of interesting results from more complicated frameworks: incomplete participation (Cass and Shell [5]) may matter in modern macroeconomics and may explain inefficiencies and endogenous self-fulfilling fluctuations.

REFERENCES