Observer-Based Control Techniques for the LBT Adaptive Optics under Telescope Vibrations

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This paper addresses the application of observer-based control techniques for the adaptive optics system of the LBT telescope. In such a context, attention is focused on the use of Kalman and $H_\infty$ filters to estimate the temporal evolution of phase perturbations due to the atmospheric turbulence and the telescope vibrations acting on tip/tilt modes. We shall present preliminary laboratory experiments carried out at the Osservatorio Astrofisico di Arcetri using the Kalman filter.

Keywords: Adaptive optics, modal control, tip/tilt control, atmospheric turbulence, telescope vibrations

1. Introduction

The Large Binocular Telescope (LBT, for short) is an optical/infrared ground-based telescope using two 8.4 m diameter primary mirrors. Having both the primary mirrors on the same mechanical mount, LBTs can achieve the diffraction-limited image sharpness of a 22.8 m diameter aperture. As in any large ground-based telescope, the diffraction limit can be obtained only by means of Adaptive Optics (AO) techniques, viz. those techniques apt to reduce, thanks to the use of deformable mirrors, the effects of wavefront distortion caused by the atmospheric-turbulence [19].

The LBT will be equipped soon with two AO systems1, one for each arm of the telescope. Specifically, each AO unit (Fig. 1) comprises a pyramid wavefront sensor (WFS), an adaptive secondary mirror (ASM), and a real-time computer (RTC). The pyramid wavefront sensor delivers a signal that is proportional, by a first-order approximation, to the first derivative of the incoming wavefront, sampled with a maximum of $30 \times 30$ sub-apertures [21]. Conversely, the ASM consists of a deformable mirror2 with 672 voice-coil (electro-magnetic force) actuators, distributed in concentric rings, and whose task is to change the shape of the 1.6 mm-thick and 911 mm-diameter Zerodur shell [17].

As discussed in [2], in addition to atmospheric turbulence, large telescopes (including the LBT) also suffer from structure vibrations which can reduce AO performance. Vibrations may arise from many different situations, e.g., telescope orientation, telescope tracking errors, and wind shaking. In particular, since vibrations cause displacements of the image (but do not deform it), they have a major impact on noise amplifications in the so-called tip/tilt modes, viz. the modal coefficients which quantify image displacements in the two orthogonal directions. In the literature, experimental studies of control techniques apt to enhance AO performance in the presence of

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vibrations only recently appeared [13, 14]. In this paper, we consider possible mixed-control strategies (see also [5]) for the LBT, which, by jointly combining observer-based and classical control approaches, can drastically reduce the impact of telescope vibrations, while retaining a moderate RTC computational burden.

In this respect, the LBT employs a set of accelerometers, placed on the structure supporting the ASM, which characterize, in terms of frequency and amplitude, the vibrations acting on the telescope. On that basis, the vibration parameters have been calibrated for the design of two observer-based controllers (deriving from Kalman and $H_{\infty}$ filters [8]), both specifically aimed at tip/tilt control. A simple integrator-based control approach [7] has been instead considered for all the other modes. As will be shown, such a mixed-control approach yields an effective trade-off between RTC constraints and AO performance. It is finally worth noting that AO is an active research area, spanning several diversified techniques for estimation, prediction and control design. The interested reader is referred, e.g., to [4, 5, 9] (and the references therein) for a comprehensive literature review.

The remainder of this paper is as follow. Section 2 introduces the general control setting for the LBT-AO system. Sections 3 and 4 describe the AO models as well as the mixed-control approaches. In Section 5, such schemes are comparatively analyzed, through numerical simulations, from both performance and robustness viewpoints. Finally, preliminary experimental results comparing the integrator-based control with the mixed-control approach based on the Kalman filter, are presented in Section 6.

2. General Control Setting

The AO control system architecture for the LBT is depicted in Fig. 2. The AO controller receives the WFS measurements $y(k)$ and computes the commands vector $u(k)$ so as to drive the actuators of the ASM. It is to be pointed out that ASM has non-negligible dynamics and, in order to compensate for wavefront distortions, it must take the desired shape within a short settling time. Because of this, the ASM has an appropriate control loop. Such a control loop relies on the position feedback provided by a set of capacitive sensors, placed at the back of the mirror shell. The design of the ASM controller is based on a proportional-derivative (PD) position feedback plus a feedforward signal which is proportional to the desired
position [18]. Summing up, there are two control systems involved:

- A global AO control system (working @ 1 kHz), whose task is to determine the commands to the ASM in order to correct for the residual wavefront distortion;
- A local ASM control system (working @72 kHz), which is responsible for shaping the mirror within one AO loop step (< 1 ms).

2.1. Global AO Control System – RTC

Constraints

Hereafter, attention will be focused on the global AO control loop. To this end, one shall consider the so-called modal control approach [19]. Typically, the controllers employed in the current generation of AO systems rely on classical (modal) integrators [7]. It is well known that such controllers, though yielding acceptable performance in terms of atmospheric turbulence correction, need not satisfactory compensate for telescope structure vibrations [14]. Structure vibrations are in fact typically greater than the closed loop transfer function bandwidth. This is also the case for the LBT under consideration, wherein the swinging arm supporting the ASM has resonant peaks, caused by vibrations, in the frequency-band [15, 30] Hz [6]. As a result, by using classical controllers, the closed loop transfer function bandwidth turns out to be proportional to the integrator gain, but such a gain cannot be arbitrarily increased due to stability constraints 3.

In the RTC firmware (see Section 6.2) one can only implement a time-invariant digital controller as in (21). Nonetheless, as previously noted, the vibrations affect mostly the tip/tilt modes, which yield more than 80% of the overall atmospheric turbulence variance [5]. For this reason, an observer-based approach has been considered for tip/tilt control. In practice, the goal is to estimate, via Kalman or \( H_\infty \) filters [8], the temporal evolution of phase perturbations due to the atmospheric turbulence and the telescope vibrations that affect such modes 4. One shall describe in Section 3 all the AO system models as well as the relevant exogenous signals involved in the observer-based control design.

As a result, the global AO control system considered here is a mixed type controller – see also [1, 5]: as depicted in Fig. 2, tip/tilt modes are controlled by resorting to Kalman or \( H_\infty \) filters, while all the remaining modes by means of a simple integrator-based controller. The modal basis chosen was created from Karhunen-Loève modes [19] defined in the LBT pupil, projected onto the ASM influence functions and then re-orthonormalized (the so-called Modes-to-Commands matrix). A total of 672 modes (corresponding to the total number of ASM actuators, and, hence, to the total number of degrees-of-freedom) were computed in such a way. Finally, tip/tilt modes were decoupled from all the other modes for the mixed-control design.

3. AO System Model

3.1. WFS and ASM Models

The pyramid WFS model is described by

\[
y(k) = D\Phi_{\text{res}}(k - 1) + v_n(k)
\]

where \( y(k) \) is the process measurement, while \( v_n(k) \) a measurement noise – \( y, w \in \mathbb{R}^q \) where \( q \) is the number of measurements; \( \Phi_{\text{res}}(k) \) stands for the residual phase after ASM correction, computed as \( \Phi_{\text{res}}(k) = \Phi_{\text{cor}}(k) - \Phi_{\text{vib}}(k) \), where \( \Phi_{\text{cor}}(k) \) is the phase correction applied by the ASM, while \( \Phi_{\text{tot}}(k) = \Phi_{\text{tur}}(k) + \Phi_{\text{vib}}(k) \), i.e., the sum of the phase distortions introduced by the turbulence \( \Phi_{\text{tur}}(k) \) and the telescope vibrations \( \Phi_{\text{vib}}(k) \). Finally, \( D \) is the WFS response matrix, describing the geometric relationship between the phase and the measurement 5. Notice that all phase variables represent modal coefficient vectors on \( \mathbb{R}^m \), where \( n \) denotes the number of coefficients.

The ASM model can be expressed by:

\[
\Phi_{\text{cor}}(k) = \mathcal{N}u(k - 1)
\]

where \( u(k) \) is the command vector, and \( \mathcal{N} \) is the so-called Commands-to-Modes matrix which describes the geometric relationship between the command vector, \( u(k) - u \in \mathbb{R}^m \), where \( m \) is the number of actuators – and the modal phase 6. In turns, as shown in Fig. 2, the AO command vector \( u(k) \) is the input to the ASM control loop and such a loop guarantees that the ASM takes the desired shape.

3.2. Turbulence and Vibration Models

Turbulence and vibrations need to be represented by two different models because they arise from two different physical phenomena. More precisely, while the

3 The AO system can be considered as a system with a two-step delay, so that the integrator gain is limited, for stability, between 0 and 1.
4 Of course, all modes of interest could be, in principle, controlled by using the same approach. However, this is practically impossible in the current RTC due to the size of the controller state-vector (limited up to a maximum of 672 entries), and the fact that such a controller requires more than one state element for each mode to be controlled.
5 Such a matrix defines the sensitivity of the WFS to ASM modal deformations. It is measured experimentally during the AO system calibration.
6 In fact \( u(k) \) is expressed in the command space, which depends on ASM influence functions; instead, the desired correction phase is expressed in the modal space, which depends on the selected modal basis.
same turbulence temporal statistics can be shared by large groups of modes [3]—e.g., modes with the same radial degree [11], vibrations dynamics change for each mode. The atmospheric turbulence evolution can be described by

\[ \Phi^{\text{tar}}(k + 1) = A_f \Phi^{\text{tar}}(k) + v_t(k) \]  

(3)

where \( v_t(k) \) is a white-noise process. In this respect, a convenient way to approximate Eq. (3) is via an Auto-Regressive (AR) first-order model [10] (see Fig. 3):

\[ \Phi^{\text{tar}}(k + 1) = A_f \Phi^{\text{tar}}(k) + v_t(k) \]  

(4)

where \( A_f \) is a stability matrix in diagonal form, calculated as in [20], whose entries are given by \( e^{-2\pi n/3f/\phi} \) (\( \eta \) radial order, \( V \) wind speed, \( f \) sampling frequency), and where \( v_t(k) \) is a white-noise process whose statistics can be computed from the Noll matrix [11]. Conversely, the vibration model can be computed from the damped oscillator expression [13]

\[ \ddot{\omega}(t) + 2\eta_0 \dot{\omega}(t) + \omega_0^2 \omega(t) = f(t) \]  

(5)

where \( \omega \) is the position, \( \omega_0 \) is the natural frequency, \( \eta_0 \) is the damping coefficient, and \( f(t) \) the external forcing function which excites the vibration. Hence, the transfer function from \( f(t) \) to \( \omega(t) \) in the Laplace variable, is given by

\[ \frac{\omega(s)}{f(s)} = \frac{1}{s^2 + 2\eta_0 s + \omega_0^2} \]  

\[ = \frac{1}{s - \eta_0 [1 + \eta_0 \mu]} \]  

(6)

where \( \mu := \sqrt{1 - \eta^2} \). An approximation of the discretized version of (6) can be computed by replacing the poles \( \frac{1}{s - \eta_0} \) by \( \frac{1}{z^{-\eta / T}} \), \( T \) denoting the sampling time,

\[ \frac{\omega(z)}{f(z)} = \frac{z^2 T^2}{z^2 - 2e^{-\eta_0 T} \cos(\omega_0 T \eta_0) z + e^{-2\eta_0 T}} \]  

(7)

Hence, if one considers the forcing function \( f(t) \) as a white noise, the vibration model can be expressed as

\[ \Phi^{\text{vib}}(k + 1) = A_1 \Phi^{\text{vib}}(k) - A_2 \Phi^{\text{vib}}(k - 1) + v_v(k) \]  

(8)

where \( A_1 \) and \( A_2 \) are stability matrices in diagonal form, whose entries depend upon the vibration frequency \( \omega_0 \) and the damping constant \( \eta \). Further, \( v_v(k) \) is a white-noise process, whose variance depends on the forcing function power and the sampling time (see (7)). Notice that \( \Phi^{\text{vib}}(k) \) and \( v_v(k) \) take on nonzero value only in \( p \) entries corresponding to the modes affected by vibrations. Accordingly, from now on, \( \Phi^{\text{vib}} \) in (8) should be understood as a vector on \( \mathbb{R}^p \).

Equation (8) describes a model for a single damped vibration. However, by extending the model, more than one vibration for each mode can be easily taken into account.

### 4. AO Control System

This section describes the AO control system to be implemented in the RTC. Let \( u_I \) and \( u_O \) denote the command vector related to the integrator, and, respectively, the observer-based controller. Likewise, let \( u_{M_I} \) and \( u_{M_O} \) be the corresponding variables in the modal space, related by

\[ \Phi^{\text{cor}}(k + 1) = N^T \begin{bmatrix} u_{O}(k) \\ u_{I}(k) \end{bmatrix} = N^T \mathcal{M} \begin{bmatrix} u_{M_O}(k) \\ u_{M_I}(k) \end{bmatrix} \]  

(11)

where \( \mathcal{M} \) is the Modes-to-Commands (or projection) matrix, and coincides with \( N^T := (N^T N)^{-1} N^T \). Let \( u_{M_O} \in \mathbb{R}^p \), and, accordingly, \( u_{M_I} \in \mathbb{R}^{r-p} \). Further, denote the first \( p \) entries of \( \Phi \) by \( \Phi_p \). We note that the matrices \( N \) and \( \mathcal{M} \) were calibrated so (see last paragraph of Section 2.1) as to yield

\[ \Phi^{\text{cor}}(k + 1) = u_{M_O}(k) \]  

(12)

#### 4.1. Integrame-Based Control Design

The objective of the global AO control system is to regulate the phase \( \Phi^{\text{ex}} \) about the value 0. As depicted in Fig. 4, the diagonal entries, say \( A_j(i,j), j = 1, 2 \), can be computed from (7)

\[ A_1(i,j) = 2e^{-\eta_0 T} \cos(\omega_0 T \eta_1) \]  

(9)

\[ A_2(i,j) = -2 \eta_0 T \]  

(10)

---

**Fig. 3.** PSDs of the AR1 model and of the turbulence (Taylor).
the classical approach consists of equipping the controller with an integral action, which is updated from closed-loop WFS data by least-square reconstruction [12],

\[ u_{MI}(k) = u_{MI}(k - 1) + g \mathcal{R}_I y(k) \] (13)

where \( g \) is the integrator gain (equal for all modes). Further,

\[ \mathcal{R} := \mathcal{D}' = (\mathcal{D}' \mathcal{D})^{-1} \mathcal{D}' \]

is the so-called Reconstruction matrix. \( \mathcal{R}_I \in \mathbf{R}^{q \times (n-p)} \) is the \( n-p \) lines of \( \mathcal{R} \) which correspond to the modes controlled by the integrator. The matrix \( \mathcal{R} \) yields the geometric relationship between the measurements of the WFS and the deformations of the ASM – see also [15, 19], and it is selected in such a way that \( u_{MO} \) has no effect on (13).

It is to be pointed out that the gain \( g \) was in fact selected by trial and error, but it is considered as a future improvement.

4.2. Observer-Based Control Design

In view of (12), a simple but effective approach for tip/tilt control consists of selecting \( u_{MI} \) so as to provide an optimal estimate, in a sense to be defined, of \( \Phi_p^{tot}(k + 1) \). In particular, for tip/tilt control, \( p = 2 \).

Consider the following state vector

\[ x(k) = \begin{bmatrix} \Phi_v^{tib}(k + 1)' & \Phi_v^{tib}(k)' & \Phi_p^{tot}(k + 1)' & \Phi_p^{tot}(k - 1)' & u_{MO}(k - 1)' & u_{MO}(k - 2)' \end{bmatrix}' \] (14)

In accordance with Section 3, let

\[ x(t + 1) = \begin{bmatrix} A_1 & -A_2 & 0 & 0 & 0 & 0 \\ I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & A_1 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I \end{bmatrix} x(k) \]

\[ + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} u_{MO}(k) + \begin{bmatrix} I & 0 & 0 & 0 & 0 & 0 \end{bmatrix} w(k) \]

(15)

where \( w := [v_x', v_y', v_z'] \). Furthermore, the measurement equation can be written as \((\mathcal{D}_O \in \mathbf{R}^{q \times p})\)

\[ y(k) = \mathcal{D}_O \begin{bmatrix} 0 & 0 & 0 & I & 0 & -I \end{bmatrix} x(k) \]

\[ + \begin{bmatrix} 0 & 0 & I \end{bmatrix} w(k). \] (16)

Notice that \( A \) is a stability matrix. In (16), we made use of the following: (i) \( y(k) = \mathcal{D}_O \left[ \Phi_p^{tot}(k - 1) - \Phi_p^{cor}(k - 1) \right] + \mathcal{D}_I \left[ \Phi_p^{tot}(n-p)'(k - 1) - \Phi_p^{cor}(n-p)'(k - 1) \right] + v_n(k) \), where we denoted the last \( n-p \) entries of \( \Phi \) by \( \Phi_p^{(n-p)'} \); (ii) \( u_{MI} \) is independent from \( u_{MO} \). Accordingly, the contribution given by the term \( \mathcal{D}_I \left[ \Phi_p^{tot}(n-p)'(k - 1) - \Phi_p^{cor}(n-p)'(k - 1) \right] \) can be treated as a form of disturbance, and, hence, incorporated into \( v_n \); (iii) by (12), \( \Phi_p^{cor}(k - 1) = u_{MO}(k - 2)^8 \).

Hence, the variable to be minimized turns out to be

\[ \epsilon(k) := u_{MO}(k) - K x(k) \] (17)

with \( K := \begin{bmatrix} I & 0 & I & 0 & 0 & 0 \end{bmatrix} \).

4.3. Central Filters

Given the system embedded in (15) and (16), let us consider the map \( G : w \mapsto \epsilon, \epsilon \) as in (17). As anticipated, attention is here focused on \( H_{\infty} \) and Kalman filters, and,

\[ \text{It is to be pointed out that the validity of (12) crucially depends on the selected modal basis and } p. \]
where:

\[
\begin{align*}
\text{map, viz. to find a } & \gamma \geq 0 \text{ such that } \|G\|_\infty < \gamma, \text{ where } \\
\|G\|_\infty & := \sup_{w \neq 0, \ell_2(-\infty, \infty)} \frac{\|Gw\|_2}{\|w\|_2}
\end{align*}
\]

while \(\ell_2(-\infty, \infty) := \{s : \|s\|_2 < \infty\}\) denotes the linear space of all sequences with bounded energy. Accordingly, the \(H_\infty\) central filter – see also (22) – has the form [8] (see Fig. 5)

\[
\begin{align*}
\xi(k+1) &= A \xi(k) + Bu_{MO}(k) + AL \left[y(k) - C \xi(k)\right] \\
u_{MO}(k) &= K \xi(k) + KL \left[y(k) - C \xi(k)\right]
\end{align*}
\]

(18)

where \(K\) is as in (17), while the gain matrix \(L\) is given by \(L := QC'(QC' + DD')^{-1}\), \(Q \geq 0\) being the symmetric and nonnegative definite matrix, solution of

1. \(Q = AA' + EE' - \begin{bmatrix} CQA' \\ -KQA' \end{bmatrix}^* \\
\begin{bmatrix} CQC' + DD' & -CQK' \\ -KQC' & KKK' - \gamma^2 I \end{bmatrix}^{-1} \begin{bmatrix} CQA' \\ -KQA' \end{bmatrix}
\]

subject to

2. \(\gamma^2 I - KQK' + QCK' (QC'C + DD')^{-1} QCK' > 0\)

3. \(A - AQ \begin{bmatrix} C \\ -K \end{bmatrix} W \begin{bmatrix} C \\ -K \end{bmatrix} \text{ stable.}\)

It is to be pointed out that stability of the overall system deriving from (18) follows from stability of \(A\) in (15).

In practice, it is clear that stability also depends on the accuracy of the models related to (15). In this respect, explicitly accounting for plant model uncertainties, e.g., as in [22], may prove relevant. Some comments are in order.

- As exemplified in Section 5, although none of the filters was designed here so as to explicitly address modelling errors (this lies outside the scope of this paper), the \(H_\infty\) filter proves to be less sensitive to uncertainties in the vibration frequency than the Kalman filter.
- In connection with (19), when the signal \(w\) driving the system is a unit intensity white-noise process, one obtains, as \(\gamma \to \infty\), the \(H_2\) central filter which minimize the average RMS power of the estimation error

\[
\|G\|_2 = \lim_{N \to \infty} \mathcal{E} \left\{ \frac{1}{N+1} \sum_{k=0}^{N} \epsilon(k)' \epsilon(k) \right\}^{1/2}
\]

where \(\mathcal{E}\) denotes expectation. In particular, as \(\gamma \to \infty\), the feasibility conditions specified by (1)–(3) reduce to the existence of a symmetric and nonnegative definite matrix \(Q_*,\) solution of the Riccati algebraic equation

\[
Q_* = AA' + EE' - (CQA')^* (CQC + DD')^{-1} CQA'
\]

(20)

Eq. (20) is recognized to be the solution of the Kalman filtering problem when \(w\) is a unit intensity white noise. In particular, the related filter is as in (18) with the gain matrix \(L = Q_* C'(CQC' + DD')^{-1} + Q_*\) solution of (20). Nonetheless, here, the solution yielding the Kalman filter equations has been obtained based upon prior assumptions on the statistical properties of the exogenous signals, viz. by replacing \(EE'\) and \(DD'\) so as to take into account of the disturbance covariance matrices, as described in [1].
Table 1. Summary of simulation parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Telescope</td>
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</tr>
<tr>
<td>Effective diameter (D)</td>
<td>8.22 m</td>
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<tr>
<td>Central obstruction</td>
<td>0.11 D</td>
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<tr>
<td>Pyramid WFS</td>
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<td>Sensing wavelength (λ)</td>
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<td>Tilt modulation radius</td>
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<tr>
<td>Number of sub-apertures</td>
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<tr>
<td>Number of photons per integration time per sub-aperture</td>
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<tr>
<td>Number of electrons per pixel of readout noise</td>
<td>8</td>
</tr>
<tr>
<td>ASM</td>
<td></td>
</tr>
<tr>
<td>Number of modes</td>
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</tr>
<tr>
<td>Turbulence</td>
<td></td>
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<tr>
<td>Seeing</td>
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<tr>
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</tr>
<tr>
<td>Wind speed</td>
<td>20 m/s</td>
</tr>
<tr>
<td>Loop parameters</td>
<td></td>
</tr>
<tr>
<td>Sampling frequency</td>
<td>800 Hz</td>
</tr>
<tr>
<td>Total delay</td>
<td>2 frames</td>
</tr>
</tbody>
</table>

5. Numerical Simulations Studies

5.1. Preliminary Considerations

Prior to implementation of the mixed-control approach on the RTC, several studies based on numerical simulations have been carried out. All simulations reported hereafter rely on an End-to-End Simulator of the LBT-AO system. Table 1 presents a summary of the simulation parameters – additional details on the simulator characteristics can be found in [1]. As a first step, in Section 5.2, we compare the performance of the three controllers (say integrator, mixed-Kalman and mixed-$H_{\infty}$) when only atmospheric turbulence is present. Then, in Section 5.3, one also considers telescope vibrations affecting tip/tilt modes. Finally, a robustness analysis, with respect to changes in the vibration frequency, is provided in Section 5.4. We point out that, in all simulations considered hereafter, the integrator gain in (13) has been set to $g = 0.7$. Further, the observer-based controllers were designed corresponding to a nominal vibration frequency of 20 Hz. In this respect, for the $H_{\infty}$ filter, one obtains $\gamma \approx 7$.

In order to measure the performance of the AO system we use the Strehl Ratio (SR). It is the ratio of the observed peak intensity at the detection plane compared to the theoretical maximum peak intensity of a diffraction-limited Point Spread Function (PSF) image (see Fig. 6).

5.2. Performance Under Turbulence

One first consider the case where no vibrations are present. In such a case, mixed-Kalman controller gives a SR of 80.7%, the mixed-$H_{\infty}$ controller a SR of 80.4%, and the integrator ($g = 0.7$) a SR of 84.1% (Table 2). Note that the performance of the mixed-controllers is slightly lower than the performance achieved by the integrator, since the AR1 dynamic model of the turbulence is significantly different from the actual turbulence (see Fig. 3): it is in fact just a first-order approximation of the Taylor’s hypothesis model of the turbulence temporal evolution [20].

5.3. Performance Under Both Turbulence and Vibrations

We now consider the case where, in addition to turbulence, a vibration affects tip/tilt modes with an amplitude of 80 milliarcseconds (mas) at a frequency of 20 Hz. Under such conditions, the mixed-Kalman controller provides a SR of 80.4%, while the mixed-$H_{\infty}$ one a SR of 80.2%, hence they show performance very similar to the one previously obtained. On the contrary, the integrator performance deteriorates up to a 30.9% SR (Table 2). It is to be pointed out that this latter result was obtained by increasing the integrator gain up to $g = 0.9$ in order to achieve better vibration attenuation. Of course, the gain cannot be arbitrarily increased due to stability constraints (see Section 2). In other terms, the performance achievable with the integrator are invariably limited under telescope vibrations.

Vibration amplitudes and frequencies considered here are the ones typically experienced during the standard LBT operating conditions.

By letting $g = 0.7$ the SR is in fact 27.3%
Observer-Based Control Techniques for the LBT AO

5.4. Robustness

In order to test robustness of the mixed-controllers against modelling errors, we introduced an error in the vibration frequency of the state model, whereas the actual vibration frequency was left equal to 20 Hz. From Fig. 7 and Table 3 one sees that the two controllers exhibit similar performance up to a magnitude of the error on the frequency less than 0.5 Hz. When such a magnitude is greater than 1 Hz the performance of the mixed-$H_\infty$ controller is ≈10% in SR better than the mixed-Kalman one. Note that, under the same conditions, the SR obtained with the integrator is significantly lower for almost every value considered.

The simulation results can be intuitively explained by considering the corresponding sensitivity functions. Fig. 8 depicts the maximum singular values of the transfer functions from the disturbance to the estimation error11. From Fig. 8, one can observe that the sensitivity functions corresponding to the Kalman filter peak around the vibration frequency, indicating that disturbances and modelling errors around this frequency has significant impact on the estimate, and, invariably, on the overall closed-loop behavior.

6. Laboratory Experimental Results

6.1. Tests Facility

Laboratory experimental tests have been carried out using the Solar Tower at the Osservatorio Astrofisico di Arcetri (Fig. 9). The Solar Tower is a concrete frame with a dome on the top and a room at the bottom. An isothermal optical bench – the LBT optical bench – was installed between the tower pillars and in the room. The bench includes the ASM and the WFS. The Solar Tower Optical Bench does not comprise a rotating phase screen emulating the evolution of the atmospheric turbulence in time. Therefore, all disturbances were introduced in the optical path by the ASM with a disturbance command vector, which is added, at each loop iteration, to the final position command.

During the laboratory tests, the infrared test camera was not yet installed. However, using the PSF images acquired with the acquisition camera (a CCD47 camera with an optical filter at 900 nm, 10 nm bandwidth), we obtained some preliminary estimations of the SR at 900 nm, scaled to 2.2 μm12.

6.2. Implementation

The implementation of the mixed-control approach (i.e., observer-based control for tip/tilt modes and integrator for all the other modes) is described hereafter. The RTC relies on a particular fixed structure, which is as follows

$$\begin{align*}
\zeta(k) &= \Lambda \zeta(k-1) + \Delta \sum y(k) \\
u(k) &= \Sigma \zeta(k)
\end{align*}$$

(21)

where $\zeta \in \mathbb{R}^{672}$ is the state vector, $y \in \mathbb{R}^{1600}$ is the measurement vector, and $u \in \mathbb{R}^{672}$ is the command vector. The number of elements in the state vector used by the observer-based control is denoted by $\chi (\chi = 14)$. Since the size of the state vector is limited to 672 elements, there

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11 For the sake of clarity, only the sensitivity functions corresponding to tip/tilt modes are depicted.

12 $SR@\lambda_2 = e^{\left(\sigma_1/\sigma_2\right)^2 \ln(SR@\lambda_1)}$
Fig. 8. Maximum singular values of the disturbance-to-estimation error transfer functions corresponding to tip/tilt modes (the vertical line denotes the Nyquist frequency).

are 672 − χ state elements remaining for the integrator-based control applied to the remaining modes. Hence, the mixed-controller can correct up to 672 − (χ − 2) modes, where χ is the filter state dimension. The RTC matrices are as follows

• Λ ∈ R^{672×672} (state update matrix);
• Ξ ∈ R^{672×1600};
• Δ ∈ R^{672×672} (gain matrix) is a diagonal matrix;
• Σ ∈ R^{672×672}.

As can be checked [16], in view of (21), the central filters defined in (18) can be rewritten as

\[
\begin{align*}
\dot{\vec{\zeta}}(k+1) &= \Lambda \vec{\zeta}(k) + B u_{MO}(k) + L \left[ y(k+1) - C \left( \Lambda \vec{\zeta}(k) + B u_{MO}(k) \right) \right] \\
u_{MO}(k) &= K \vec{\zeta}(k)
\end{align*}
\]

(22)

where \( \vec{\zeta} \in R^{\chi} \). Accordingly, by letting \( \vec{\zeta} \in R^{672−\chi} \) denote the state of the integrator, one has \( \vec{\zeta} = [\vec{\xi} \ \vec{\zeta}']' \), initialized from \( \vec{\zeta}(-1) = 0_{672} \). Likewise, if \( u_I \in R^{672−2} \) denotes the integrator output, one has \( u = [u_O \ u_I]' \), initialized from \( u(-1) = 0_{672} \). Note that \( \vec{\xi}(-1) = 0 \) yields \( u_{MO}(0) = KL y(0) \), which corresponds to the output of (18) with \( \xi(0) = 0 \). Hence, one concludes that

\[
\begin{align*}
\Lambda &:= \begin{bmatrix} (I - LC)(A + BK) & 0 \\ 0 & I_{672-\chi} \end{bmatrix} \\
\Xi &:= \begin{bmatrix} L \\ R_I \end{bmatrix} \\
\Delta &:= \begin{bmatrix} I_{\chi} & 0 \\ 0 & g I_{672-\chi} \end{bmatrix}
\end{align*}
\]

(23) (24) (25)

Fig. 9. Solar tower.
where $I$ is the identity matrix; $g$ is the integrator gain; $K \in \mathbb{R}^{2 \times \chi}$ is the observer-based output matrix; $M \in \mathbb{R}^{672 \times 672}$ is the modes-to-commands matrix. The $\Sigma$ matrix shows that the mixed-controller can correct up to $672 - (\chi - 2)$ modes, since $\chi$ of the $672$ total states are used by the filters to determine tip/tilt estimates.

### 6.3. Experimental Results

Hereafter, experimental results are given, showing the performance improvements achieved by mixed-Kalman controller over the integrator in case of both atmospheric turbulence (seeing $= 0.8''$, $v = 15\text{ m/s}$) and vibrations. The parameters of the experimental tests are summarized in Table 4, while Figs 10 and 11 compare three PSFs – the open-loop one, the one obtained with the simple integrator controller, and the one with the mixed-Kalman – when the vibration amplitude is equal to 10 mas, and, respectively, 100 mas. The advantage of mixed-Kalman controller is evident under larger values of the vibration amplitude. Table 5 summarizes the comparison between the mixed and the simple integrator controllers under the presence of the above-mentioned vibration with amplitudes varying from 0 to 150 mas. Note that the integrator performance is slightly better in the absence of vibrations (cf. Section 5.2). Nonetheless, for vibration amplitudes greater than $\sim 10$ mas, the integrator performance constantly deteriorate with the vibration amplitude up to $\sim 37\%$. This is not the case for the mixed-Kalman controller, as the $SR$ remains greater than $69\%$ regardless of the vibration amplitudes considered.

### 7. Conclusions and Further Work

In this paper, attention has been focused on mixed-control approaches, combining classical and observer-based techniques for the LBT-AO system. It was shown that the mixed-controller is able to effectively reduce the effects of telescope structure vibrations on the global AO performance. Both $H_{\infty}$ and Kalman observer-based controllers were investigated in numerical simulations, while only the latter one was experimentally tested, confirming the dramatically improvements achievable over classical control techniques.

It is finally worth noting that, in order to achieve more effective vibration compensation, it is crucial to characterize accurately the vibration parameters, in particular the vibration frequency value. Indeed, several robustness studies carried out via numerical simulations indicate that an absolute loss of $10\%$ of $SR$ at $2.2\mu\text{m}$ is expected in the presence of a frequency error of $\pm 1.2\text{ Hz}$ and $\pm 0.9\text{ Hz}$ in the vibration model for the $H_{\infty}$ and the Kalman filter, respectively.

![Fig. 10. PSF images taken with the acquisition camera (Filter 900 nm–10 nm). (Left) Open-loop PSF with disturbance introduced: atmospheric turbulence (seeing $= 0.8''$, $v = 15\text{ m/s}$) plus a narrow vibration at 21 Hz with 10 mas amplitude on the tip mode ($x$ axis). (Center) Closed-loop PSF obtained with the simple integrator controller. (Right) Closed-loop PSF obtained with the mixed controller.](image-url)
**Fig. 11.** PSF images taken with the acquisition camera (Filter 900 nm–10 nm). (Left) Open-loop PSF with disturbance introduced: atmospheric turbulence (seeing = 0.8′′, $v = 15$ m/s) plus a narrow vibration at 21 Hz with 100 mas amplitude on the tip mode ($x$ axis). (Center) Closed-loop PSF obtained with the simple integrator controller. (Right) Closed-loop PSF obtained with the mixed controller.

**Table 5.** Performance for vibrations of different amplitudes.

<table>
<thead>
<tr>
<th>Amplitude [mas]</th>
<th>%SR @ 2.2 μm</th>
</tr>
</thead>
<tbody>
<tr>
<td>vibration @21 Hz</td>
<td>Integrator</td>
</tr>
<tr>
<td>0</td>
<td>83.8</td>
</tr>
<tr>
<td>10</td>
<td>76.4</td>
</tr>
<tr>
<td>20</td>
<td>61.4</td>
</tr>
<tr>
<td>50</td>
<td>51.7</td>
</tr>
<tr>
<td>100</td>
<td>45.8</td>
</tr>
<tr>
<td>150</td>
<td>36.9</td>
</tr>
</tbody>
</table>

*In addition to experimental tests with the $H_{\infty}$ filters, further studies will be therefore concerned with more elaborate vibration patterns (combination of narrow and broad vibrations) as well as possible specific design procedure apt to explicitly accounting for modelling errors.*

**References**