A Spectrum Trading Model with Strict Transmission Power Control

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Abstract—The underutilization of spectrum coupled with developments in network technologies has prompted a number of proposals for managing spectrum. Dynamic spectrum access radio technology, which is based on cognitive radio technology, promises to increase spectrum sharing and thus overcome the lack of available spectrum for new communication services. In this paper, the pricing and the transmission power control processes are investigated in a cognitive radio network. The considered network consists of multiple primary service providers which have some unutilized bandwidth; and multiple secondary users that require spectrum bands. In this multiple-seller and multiple-buyer environment, the proposed framework aims at determining the optimum price values for unit spectrum bands that maximizes PSPs profits while protecting the social welfare of their network. Furthermore, the framework considers the power control, especially the effect of transmission power on the profit of PSPs. Modeling the competitive relationships among network elements as games ensures analyzing all elements’ behaviors and actions in a formalized way. The existence of various network elements that want to maximize its own profits makes the problem very complex, with usually conflicting objective functions. We have used a model based on game theory for PSPs’ pricing problem to provide with a well defined equilibrium. The simulation results show that the proposed framework allow PSPs to make up to 45%-86% of additional profit while preserving the social welfare of the network.

Keywords-component; radio networks, power management, pricing, spectrum trading

I. INTRODUCTION

The traditional way of spectrum management is to let government agencies statically allocate communication frequencies to different wireless service providers for large geographical areas and for long periods of time. This approach is referred as the command and control model. The government opens a radio frequency band for bidding and specifies a certain type of wireless technology/application for this particular radio frequency band (e.g. TV broadcast, cellular services, etc.). Any interested user/company submits the bid and the government determines the winning user/company, which is generally the user/company offering highest bid. Although the government-issued licenses intend to enhance the efficiency of spectrum usage by specifying user rights and obligations, they can cause inefficient spectrum usage. Apart from the spectrum scarcity, the limitations on the wireless technology put by regulatory requirements are the causes of spectrum usage inefficiency. The limitations may prevent an authorized user from changing its wireless transmission techniques and services according to market demand.

A wireless service provider buys spectrum with a certain price from the telecommunication regulatory authority, and then sells the spectrum to its end users in the form of services (bandwidth) [1]. The wireless service providers that are the long-term spectrum owners are referred as the primary service providers (PSPs). The regular customers of the PSPs are referred as the primary users (PUs). The PSPs have the opportunity to sell the spectrum opportunities to secondary service providers or secondary users (SUs), when their allocated spectrum is not fully utilized. This is called as spectrum trading or secondary trading of spectrum [2]. Price is the fundamental component of the economic model, since it indicates the value of the spectrum to both seller and buyer. The spectrum price should be set based on the demand of the buyers (SUs) and the supply of the seller. Also, competition among buyers/sellers has an impact on price setting. In this regard, it is important to investigate the economic issues that arise due to the presence of multiple service providers. Another important component in a spectrum management system is the power control. Power control serves as a means for both battery savings at the mobile equipment, and interference management. When power emission is not controlled, the intended quality of service (QoS) levels can be achieved at arbitrary high power levels. This leads to interference on PUs, as well as on the other SUs.

In this paper, we aim at establishing a pricing model in a secondary market in the presence of multiple competing PSPs. The proposed secondary market consists of SUs who seek for spectrum bands, the PSPs who have available spectrum bands to sell and a telecommunications coordinating authority (TCA) which assures the satisfaction of all the players in the market. The TCA aims at managing the transmission powers of SUs so as to maximize the QoS both SUs and PSPs’ regular customers get. By considering these optimum power values, PSPs determine which SUs are more profitable for them to serve. Then, as the second step, PSPs decide on the unit price of their spectrum bands. The pricing model of spectrum bands is based on a non cooperative game, where the PSPs are the players. Solving the game, the mutual best response strategies that determine the equilibrium point(s) are studied. With the optimum prices, the PSPs calculate their actual bandwidth demands and maximum possible profits.
II. PROBLEM DESCRIPTION AND FORMULATION

In related literature, there are several studies that look at the impact of pricing on power control [3] [4] [5]; however to the best of our knowledge, no one has considered the impact of power control on profit maximization and on price setting of PSPs. Thus, with our proposed framework, the main question that we have focused to answer is: “What kind of impact transmission power has on the profit maximization, as well as on the unit spectrum prices?”

A. Framework Information Flow

The framework consists of a cognitive radio system with multiple PSPs and multiple SUs who are co-located in the same geographical region R. SUs are randomly distributed over the geographical area. Each PSP has its own frequency band and its own regular costumers (i.e. PUs) in region R. PSPs intend to sell portions of their unutilized spectrum to SUs in order to obtain additional revenue. Also, this spectrum trading process enables SUs to satisfy their spectrum requirements. For analytical purposes, a single cell wireless CDMA data network is considered where the PUs and SUs of a PSP transmit simultaneously in a common frequency band. The model concentrates on the power control for the uplink communication in the given carrier frequency. Thus, the base station (BS) of a PSP receives independent signals transmitted from multiple SUs. The proposed framework can be extended to other types of wireless networks with slight modifications. Each BS is assumed to have perfect knowledge of channel information from SUs to BS and PUs to BS. In practice, certain cooperation is needed between secondary and primary network in order to have such perfect information. This might be in terms of parameter feedback which is carried out directly by the primary network or indirectly through a spectrum manager or spectrum broker that mediates between the two networks [6] [7].

The information flow of the proposed framework can be summarized as: 1. SUs broadcast their minimum bandwidth requirements to all the PSPs in the same region. 2. PSPs send the channel information and minimum desired bandwidths of the SUs to the TCA. 3. The TCA computes the optimum transmission power levels of SUs, for each PSP individually. Using an initial price vector and the optimum transmission power levels of SUs, the TCA determines the transmission power values, PSPs play the pricing game. The TCA is not an element of the pricing game. The pricing game takes place. This will be determined by the TCA taking transmission power levels of SUs into consideration for each combination of selection. These power levels maximize the SINR levels for both SUs and PUs; in other words, they are the smallest transmission powers to achieve the best transmission quality for SUs. The TCA directs PSPs keeping their PUs’ satisfaction by balancing the tradeoff between transmission quality and resulting interference. Finding a good balance between these two conflicting objectives is the primary focus of an efficient power control component. By the time PSPs declare their unit spectrum price, we do not know how SU and PSP matching takes place. This will be determined by the TCA taking transmission power levels of SUs into consideration for each combination of selection. These power levels maximize the SINR levels for both SUs and PUs; in other words, they are the smallest transmission powers to achieve the best transmission quality for SUs. The TCS can use the results to choose which SU(s) to offer the spectrum. By investing in the TCA, PSPs seek to minimize their churning rate and maximize their profits in long-term by keeping their primary customer base. One way to reach the optimum points in a network is to satisfy data rates of both PUs and SUs in the system. Shannon’s channel capacity establishes the bound of the maximum amount of error-free digital data that can be transmitted with a specified bandwidth in the presence of the noise [9]:

\[ C = B \log_2(1 + SINR) \]  

where \( B \) is the bandwidth of the channel in Hertz and \( C \) is the channel capacity in bits/second. For spread spectrum transmission, the SINR of SU\(_j\) at the base station of PSP\(_i\) can be represented as:

\[ SINR_{SU_j}^{PSP_i} = \frac{L_s L_b |h_{SU_j^{PSP_i}}|^2}{\sum_{k=1}^{M} L_s L_b |h_{SU_j^{PSP_i}}|^2 + \sum_{k=1}^{M} L_s L_b |h_{PU_k^{PSP_i}}|^2 + \sigma^2} \]  

The SINR of the \( k^{th} \) primary user of PSP\(_i\) (\( PU_k^{PSP_i} \)) at the base station of PSP\(_i\) can be represented as:

\[ SINR_{PU_k}^{PSP_i} = \frac{L_s L_b |h_{PU_k^{PSP_i}}|^2}{\sum_{j=1}^{S} L_s L_b |h_{SU_j^{PSP_i}}|^2 + \sigma^2} \]  

where \( L \) is the spreading gain, \( s_l^l \) is the transmission power of SU\(_j\), \( s_k^l \) is the transmission power of PU\(_k\) in the PSP\(_i\)'s network, \( h_{SU_j^{PSP_i}} \) is the channel gain between the BS of PSP\(_i\) and the SU\(_j\), \( h_{PU_k^{PSP_i}} \) is the channel gain between the BS of PSP\(_i\) and the PU\(_k\), \( n_{PU_k} \) is the background noise power, \( \eta_{PU_k} \) is the number of PUs in the PSP\(_i\)'s network and \( M \) is the number of SU\(_j\) in region \( R \). The background noise power is assumed to be the same for all users. The channel gain between the BS of PSP\(_i\) and the SU\(_j\) is modeled as [10]:

\[ h_{i \rightarrow j} = \frac{1}{d_{ij}} \]  

Since the same frequency bands are reused by multiple PUs and SUs; links operating over the same frequency interfere with each other. SINR value is a quality measure for such networks that represents the overall ratio of the wanted signal to any other unwanted power at the receiver side [8].
where \( d_{ij} \) is the distance between the BS of \( PSp_i \) and the \( SU_j \) and \( \alpha \) is the path loss exponent. Using the above functions, the TCA aims at adopting the transmission powers of SUs in order to maximize their cognitive capacity, while maximizing their SINR and minimizing the interference in the environment. We do not propose adjusting the power levels of PUs. They are taken from a lookup table which involves the average transmission power values with respect to distance. It is always possible to let the transmission powers of PUs be variable and optimize them; however in this research, we want to examine the impact of our proposed approach on the secondary market separately.

For a PSP with \( \bar{n}_{PU} \), PUs and \( M \) SUs in the region, the objective function of the TCA is expressed below. The TCA uses this objective function for each PSP individually and calculates the optimum transmission powers of SUs:

\[
\max \sum_{j=1}^{M} b_j^{min} \log_2 \left( 1 + SINR_{SU_j}^i \right) + \sum_{l=1}^{n_{PU}} B_l^i \log_2 \left( 1 + SINR_{PU_l}^i \right)
\]

s.t. \( l_i < SINR < l_u, \quad \forall i, i \quad (6) \)

\[
0 < s_j^U < 2, \quad \forall j \quad (7)
\]

where \( b_j^{min} \) is the minimum desired bandwidth of \( SU_j \). \( B_l^i \) is the bandwidth requirement of \( PU_l \) of \( PSp_i \). \( l_i \) is the lower bound for \( SINR \) and \( SINR \) values and \( l_u \) is the upper bound for \( SINR \) and \( SINR \) values. The transmission powers of SUs are bounded by a lower limit of 0 and upper limit of 2 Watts. Similarly, the SINR values are bounded with a lower and upper limit, depending on the SU’s application type.

2) Primary Service Providers (PSPs): The regular customers of the PSPs are referred to as the primary users. A part of the revenue of a PSP is obtained from what the PUs pay for the services. We assume that the \( PSp_i \) receives a fixed amount of fee \( (\bar{p}_i) \) from each PU, in return for a guarantee of a minimum bandwidth \( B_l^i \) and a minimum quality of service level in terms of SINR, call dropping probability and call blocking probability. In this framework, we have considered the SINR value as an indicator of the quality of service level. The number of PUs in region \( R (\bar{n}_{PU}) \) can vary over time due to random arrivals and departures; however during the spectrum trading process, it is assumed to be constant. Moreover, we assume that the prices are calculated fast enough to assume the PUs and SUs have not moved too much during the trading process. Otherwise, the corresponding channel gains cannot remain constant during the convergence of transmission powers. When their allocated spectrum of size \( W_i \) is not fully utilized by PUs, the PSPs (spectrum owners) have the opportunity to lease/sell these spectrum opportunities to SUs in order to obtain additional revenue. Each spectrum owner sets its own price \( (p_i) \) per unit of spectrum. The unit of spectrum is measured by a bandwidth measurement unit, Hertz. The PSPs compete with each other to sell extra spectrum to SUs by setting the optimal price that maximizes their utilities. In this regard, the price should be set properly by considering both the demand of SUs and the price strategies of other PSPs. Moreover, in this paper, we aim at proposing an alternative management model by allowing PSPs to sell more spectrum than they have. If the PSP, sells too much spectrum to SUs, this will cause degradation on \( B_l^i \). Therefore, PSP, has to be penalized with a cost of quality degradation. This profit diminution can be considered as a discount which is offered to PUs. A similar approach is envisioned by Niyato and Hossain [11], but they introduce a cost for each spectrum band which is shared by SUs. The difference of our model is that in our model, a PSP does not experience any cost, if the minimum quality requirements of its PUs are met. Apart from excessive spectrum sell, the interference which arises from undue transmission power also degrades the service quality of PUs. Even if the power emission is controlled by the TCA, a PSP should be penalized because of the power emission of SUs \((s_j^U)\) in its network.

A PSP can sell spectrum bands to more than one SU; however a SU is assumed to buy its required spectrum band from only one PSP. The set \( M_i < \{SU_1, SU_2, ..., SU_M\} \) represents the set of the SUs that are selected to serve by \( PSp_i \). \( M_i = M_k \), where \( M_k \) represents the set of the SUs that are selected by other PSPs, \( \forall k \neq i, k \in \{1,..,N\} \). The profit function of \( PSp_i \) can be expressed as:

\[
\pi_i = \bar{p}_i \sum_{j=1}^{M_k} a_i b_j - \left( \sum_{j=1}^{M_k} b_j - \alpha_i \right) \max \left( \sum_{j=1}^{M_k} b_j, -\left( W_i - B_{\alpha_i} \right)_{SU_i} \right), 0 \right) - d_i \sum_{j=1}^{M_k} s_j^U \]

s.t \( p_i \geq 0, \quad \forall i \quad (8) \)

\[
 b_j \geq b_j^{min}, \quad \forall j \quad (9) \]

\[
 \sum_{j=1}^{M_k} b_j \leq W_i, \quad \forall i \quad (10) \]

where \( \bar{p}_i \) is the fixed fee that \( PU_l^i \) pays, \( p_i \) is the unit price offered to SUs by \( PSp_i \), \( b_j \) is the size of the spectrum that \( SU_j \) buys (actual demanded bandwidth), \( W_i \) is the total size of spectrum of \( PSp_i \), \( a_i \) is the weight coefficient of cost due to the quality degradation and \( d_i \) is the weight coefficient of cost due to the interference created by SUs. The factors \( a_i \) and \( d_i \) are the weight coefficients, as well as the factors that convert utility units to currency. The higher the \( a_i \), the more the PSP is penalized for quality degradation. The lower bound of the offered bandwidth determines the upper bound of the offered price, and it is set by (9). The \( b_j^{min} \) is the minimum desired bandwidth of \( SU_j \) that is broadcasted to SUs in the same region. By setting an initial average price vector, \( PSp_i \) calculates its initial profit for each case by using the power levels \((s_j^U)\) given by the TCA. \( PSp_i \) chooses the case that brings it the highest positive profit. In other words, \( PSp_i \) selects the SUs that it is going to offer its available spectrum band. Then PSPs play a game among each other for unit price setting.

3) Secondary Users (SUs): Each SU is assumed to have a minimum desired bandwidth \((b_j^{min})\) at a point in time. The SU broadcasts its requirement to all the \( PSp_i \)s in the same region. They then receive price offers from the \( PSp_i \)s. The spectrum offered by a \( PSp_i \) should meet the minimum desired bandwidth requirement of a SU. The \( PSp_i \)s’ price offers also include the transmission power level information, which is set
by the TCA. A SU has to obey this power limitation of the PSP, if this PSP is chosen as the seller. A SU chooses the offer that provides the best utility in terms of received quality and price. SUs are free to choose and buy spectrum opportunities. They are interested in getting high quality service at a low cost and they pay for the spectrum they accessed. We assume that the demand coming from a SU shows a linearly decreasing trend when the price offered by PSP increases. Besides, it shows a linearly increasing trend when other PSPs’ prices increase. Therefore, demand is a function of PSPs’ price decisions as well as a function of competitors’ price decisions:

\[
D_j = \delta_j - \beta_j p_i + \sum_{k=1, k \neq i}^N \gamma_{jk} p_k \tag{11}
\]

where \(\delta_j\) is the base demand of \(SU_j\), \(\beta_j\) is the positive constant that represents to what extent the \(SU_j\) is influenced from price offered by \(PSP_j\), \(\gamma_{jk}\) is the positive constants that represent to what extent the \(SU_j\) is influenced from prices offered by the competitors of \(PSP_j\). We assume that the influence of a \(PSP_j\)’s price on \(SU_j\)’s demand is greater than the influence of the competitors’ prices:

\[
\beta_j > \frac{1}{N} \sum_{k=1, k \neq i}^N \gamma_{jk} \tag{12}
\]

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III. GAME MODEL

When determining the unit spectrum prices, we want to analyze the competition and interactions among PSPs. Game theory provides us to deal with the interactive nature of the price setting process. The elements of the proposed pricing game are: Players: Primary service providers, Strategy of players: The unit price of the offered bandwidth, and Payoff of players: Profit function of PSPs. Each PSP tries to maximize its own payoff (profit) function without considering the effect of its actions on other users. The one shot game is a part of our proposed framework. It begins by the optimum transmission power levels coming from the TCA to PSPs, and the game ends by the declaration of unit prices of PSPs. The optimum unit price of a PSP depends on the unit prices (strategies) that other PSPs determine. Solving such a game means predicting the strategy of each PSP. One can see that if the strategies from the players are mutual best responses to each other, no player would have a reason to deviate from the given strategies and the game would reach a steady state. Such a point is called the Nash equilibrium (NE) point of the game [12]. The PSPs use the demand information to adjust their price strategies. The shared bandwidth \((b_j)\) in the profit function of \(PSP_j\) (7) is replaced with the demand function of \(SU_j\) (11):

\[
\pi_j = \bar{p}_j n_{PU_i} + p_i \sum_{j=1}^N \left( \delta_j - \beta_j p_i + \sum_{k=1, k \neq i}^N \gamma_{jk} p_k \right) - a_j \max \left\{ \sum_{j=1}^N \left( \delta_j - \beta_j p_i + \sum_{k=1, k \neq i}^N \gamma_{jk} p_k \right) - \left( W_i - B_i n_{PU_i} \right), 0 \right\} - d_i \sum_{j=1}^N s_{ij}^* \tag{12}
\]

The vector \(p^* = [p_1^*, p_2^*, \ldots, p_i^*, \ldots, p_N^*]\) denotes the solution (the NE) of this game for:

\[
p_i^* = BR_i (p_j^*, \text{for } j \neq i) \tag{12}
\]

where \(p_j^*\) represents the vector of best responses for player \(j\) for \(j \neq i\). PSPs negotiate with each other to determine their optimum unit price value. In each round, a PSP declares its price, given the others’ prices \((p_j)\). The negotiation process iterates until the steady state (Nash equilibrium) is reached. The NE is the point that solves the set of equations: \(\frac{\partial \pi_j}{\partial p_i} = 0\) for all \(i\). The set of equations is given as:

\[
\frac{\partial \pi_j}{\partial p_i} = \begin{cases} 
\sum_{j=1}^M \delta_j - 2\beta_j p_i + \sum_{k=1, k \neq i}^N \gamma_{jk} p_k + a_i \sum_{j=1}^M \beta_j, & \text{if } \sum_{j=1}^M b_j > W_i - B_i n_{PU_i}, \\
\sum_{j=1}^M \delta_j - 2\beta_j p_i + \sum_{k=1, k \neq i}^N \gamma_{jk} p_k, & \text{otherwise}
\end{cases}
\]

IV. NUMERICAL ANALYSIS

This section attempts to reveal the performance of the proposed power control and pricing mechanism. The framework is simulated numerically using MATLAB. As the parameters are scaled for a less-congested network, the scenario can be considered to reflect a real-life situation. As a competitive demonstrative example, we have assumed that the network consists of two PSPs (\(PSP_1\) and \(PSP_2\)), one PU for each one (\(PU_1^1\) and \(PU_1^2\)) and two SUs (\(SU_1\) and \(SU_2\)) (Figure 1). The simulation parameters are chosen as in Table 1. The PSPs are differentiated as price sensitive and quality sensitive, according to their attitudes toward quality degradation of PUs. \(PSP_1\) is the quality sensitive one with a higher \(a_1\) value, \(a_1 = 3.5\), while \(PSP_2\) is the price sensitive one with \(a_2 = 2.75\). In the same way, the SUs are differentiated as price sensitive and quality sensitive, with the minimum desired bandwidths \(b_{min} = 0.34\) MHz, and \(b_{min}^2 = 0.46\) MHz, respectively. They are differentiated according to their sensitivities on offered unit prices. \(SU_1\) is the price sensitive one with a higher \(\gamma_1\) value, while \(SU_2\) is the quality sensitive one.

In this example with two SUs, PSPs have four choices. The TCA calculates optimum transmission powers of \(SU_1\) and \(SU_2\) in each one of these cases for each PSP. Taking these power values and an initial price vector, PSPs calculate their initial profits (Table 2). The results reveal that, \(PSP_1\) has higher profit when serving only \(SU_1\), but serving both \(SU_1\) and \(SU_2\) is more profitable for \(PSP_2\). After determining which SUs to serve, each PSP decides on its unit price as a result of the pricing game (Table 3). In order to determine the performance

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of our proposed framework, especially to separate the effect of the TCA to the profit, we compare it with a model without any power control. As the benchmark model, we consider the same model with same formulations; however the TCA does not optimize the transmission powers of SUs. Instead, transmission powers are randomly selected from the interval [0.03, 2] Watts.

Figure 1. The demonstrative example with two PSPs, two PUs and two SUs

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<tr>
<th>TABLE I</th>
<th>PARAMETERS OF THE DEMONSTRATIVE EXAMPLE</th>
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</thead>
<tbody>
<tr>
<td>Spread gain ($L_i$)</td>
<td>128</td>
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<tr>
<td>Background noise ($\sigma$)</td>
<td>0</td>
</tr>
<tr>
<td>Min. bandwidth requirement of PU ($B_{i1} - B_{i2}$)</td>
<td>0.46 MHz</td>
</tr>
<tr>
<td>Path loss exponent ($\alpha$)</td>
<td>4</td>
</tr>
<tr>
<td>Total size of the spectrum ($W_i = W_j$)</td>
<td>1.5 MHz</td>
</tr>
<tr>
<td>Flat price ($p_1 = p_2$)</td>
<td>4</td>
</tr>
<tr>
<td>Transmission powers of PUs ($S_{i1}^j$)</td>
<td>0.5 Watt</td>
</tr>
<tr>
<td>Initial price ($p_i = p_2$)</td>
<td>4</td>
</tr>
<tr>
<td>Coefficient of cost due to the quality degradation of $PSP_1$ and $PSP_2$ ($\alpha$ and $\alpha_2$)</td>
<td>3.5 and 2.75</td>
</tr>
<tr>
<td>Coefficient of cost due to the interference created by SUs of $PSP_1$ and $PSP_2$ ($d_{i1} = d_{i2}$)</td>
<td>2</td>
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<tr>
<td>Base demand of SU ($d_{i1} = d_{i2}$)</td>
<td>1.8 MHz</td>
</tr>
<tr>
<td>Coefficients representing to what extent the $SU_1$ and $SU_2$ are influenced from $PSP_i$’s price ($\beta_i$, $\beta_2$)</td>
<td>0.4 and 0.25</td>
</tr>
<tr>
<td>Coefficients representing to what extent the $SU_1$ and $SU_2$ are influenced from other $PSP_i$’s prices ($\gamma_i$, $\gamma_2$)</td>
<td>0.15 and 0.03</td>
</tr>
</tbody>
</table>

Note that, the power values have a direct effect on the profit of PSPs: The profit of PSP decreases by the increase of the transmission power levels of SUs in its network. The power values do not have a direct effect on unit prices. However, PSPs determine which SUs to serve by considering their profits, and accordingly, they set their unit prices. $PSP_1$ will not serve $SU_1$, hence it only offers a unit price to $SU_2$, which is 3.8809. $PSP_1$ expects that the demand from $SU_2$ will be 0.97 with this price. $SU_2$ has another spectrum offer with the unit price of 4.6816. $SU_1$’s actual demand to $PSP_1$ is 0.50 MHz and $SU_2$’s actual demand to $PSP_1$ is 0.74 MHz, with this price. The results reveal that the proposed spectrum trading framework with power control allows $PSP_1$ to make 45% and $PSP_2$ to make 86% of additional profit without violating the social welfare in the networks. These values correspond to the maximum possible profits that the PSPs can make. Our proposed framework is also more profitable when compared to the benchmark model without power control. The PSPs can make up to 34% of additional profit by controlling the transmission powers.

<table>
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<tr>
<th>TABLE II</th>
<th>PSPS’ INITIAL PROFITS IN EACH CASE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case</td>
<td>Offer no band</td>
</tr>
<tr>
<td>Initial profit of $PSP_1$</td>
<td>4.000</td>
</tr>
<tr>
<td>Initial profit of $PSP_2$</td>
<td>4.000</td>
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<table>
<thead>
<tr>
<th>TABLE III</th>
<th>PARAMETERS OF THE DEMONSTRATIVE EXAMPLE</th>
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<tr>
<td>$SU_1$</td>
<td>$SU_2$</td>
</tr>
<tr>
<td>$SU_1$</td>
<td>$SU_1$</td>
</tr>
<tr>
<td>Offered price</td>
<td>3.8809</td>
</tr>
<tr>
<td>Demanded bandwidth</td>
<td>-</td>
</tr>
<tr>
<td>Max. possible profit</td>
<td>5.7936</td>
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<tr>
<td>Social welfare</td>
<td>4.4712</td>
</tr>
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V. ACKNOWLEDGEMENT

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REFERENCES