Unmixing the Directional Reflectances of AVHRR Sub-Pixel Landcovers
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Abstract—Recent progress in canopy bidirectional reflectance distribution function (BRDF) model inversions has allowed accurate estimates of vegetation biophysical characteristics from remotely sensed multi-angle optical data. Since most current BRDF inversion methods utilize one-dimensional (1-D) models, surface homogeneity within an image pixel is implied. The Advanced Very High Resolution Radiometer (AVHRR) is one of the few spaceborne sensors capable of acquiring radiometric data over the range of view angles required for BRDF inversions. However, its relatively coarse spatial resolution often results in measurements of mixed landcovers, and thus the data may not be ideal for BRDF inversions. We present a three-step spectral unmixing method for retrieving AVHRR sub-pixel directional reflectances in regions of high spatial heterogeneity. The reflectances of individual vegetation types are deconvolved using co-located Landsat TM and AVHRR data. The three major steps in the model include: 1) unmixing of vegetation endmember concentrations in TM imagery; 2) correction of dissimilar shadow fractions between TM and AVHRR data; and 3) unmixing of AVHRR sub-pixel reflectances of vegetation types for any sub-sensor geometry. We tested the method using simulated TM and AVHRR data. A savanna landscape simulation, comprised of a canopy radiative transfer model and a crown geometric-optical model, was used to create images containing mixed pixels of tree, grass, and shade endmembers. TM and AVHRR spectral response functions, viewing geometries, and off-nadir pixel shape calculations were incorporated into the simulations. Following the successful testing of the unmixing method on error-free simulations, random noise representing atmospheric perturbations and co-registration inaccuracies was added to the data. The method is stable when errors resulting from either the first unmixing step or image co-registration inaccuracies are introduced. Potential errors in the AVHRR data may result in inaccurately retrieved reflectances if the image scene contains a spatially homogeneous mix of landcovers. A method for detecting and mitigating this problem is presented.

I. INTRODUCTION

The spectral and angular dependence of reflected photons from a surface is described by the bidirectional reflectance distribution function (BRDF). In the specific case of vegetation canopies, this dependence is primarily a function of leaf-level biochemical characteristics [1], canopy-level bio-

physical traits [2], [3], soil surface attributes [4], illumination conditions, and viewing geometry [2], [5]. Plant canopy BRDF models simulate the angular distribution of scattered photons based on these leaf- to canopy-level characteristics [5]–[7].

Canopy BRDF models can be numerically inverted by adjusting the input parameters until the modeled reflectances best fit an actual sampling of the BRDF acquired from remotely sensed data. The model parameters producing this optimal fit are retrieved as estimates of the biophysical characteristics of the vegetation. Recent progress in one-dimensional (1-D) canopy BRDF inversions has allowed for accurate estimates of biophysical information from multi-angle remotely sensed data for a variety of vegetation types [8]–[10]. The success of these inversions largely depends on the spatial, spectral, angular, and temporal resolution of the data [11]. Specifically, BRDF inversions require an angularly diverse dataset gathered over a temporally invariant landcover to maximize the likelihood of successful parameter retrieval. Since most current BRDF inversion methods must utilize 1-D models due to data and computational limitations, surface homogeneity within an image pixel is also implied.

The NOAA Advanced Very High Resolution Radiometer (AVHRR) is one of the few spaceborne sensors currently capable of acquiring radiometric data over the range of view angles required for BRDF inversions [12]. However, the relatively coarse spatial resolution of the AVHRR (1.1–4.3 km) most often results in measurements of mixed landcovers, and thus the data may not be ideal for BRDF inversions. If the land-cover components that comprise a mixed pixel can be separated, the resulting reflectance data for each vegetation type (endmember) could be used for BRDF inversions.

Savannas and shrublands cover more than one-fourth of the global vegetated land surface [18]. They are mainly comprised of a continuous layer of grasses with scattered woody (e.g., tree) species that vary in spatial density across the landscape. Most of these regions are currently subject to significant anthropogenic pressures such as increased fire frequencies and grazing intensities [19], [20], and these pressures greatly affect the relative cover fraction of woody and herbaceous (e.g., grass) vegetation. Consequently, shifts in tree and grass cover substantially influence biogeochemical cycles and biosphere-atmosphere interactions over extensive geographic regions [21]. Woody and herbaceous vegetation types contribute differently to biogeochemical processes such as nutrient and fire cycles, trace gas emissions, and water fluxes. To evaluate and monitor changes in savanna and shrubland function at regional scales, estimates of the extent and biophysical properties (e.g.,
leaf area index) of tree and grass covers are needed. The 1-D canopy BRDF inversion is a promising tool for acquiring quantitative measures of vegetation attributes; however, sub-pixel mixing of vegetation types is a significant barrier in applying these techniques to spatially heterogeneous biomes.

Spectral mixture analysis is a process by which the spectrum of a mixed cover is deconvolved into component endmember spectra. These spectra can be used for sub-pixel quantification of the endmember abundances within a remotely sensed image [13]. However, spectral unmixing of AVHRR data is tenuous due to its spectral limitations (e.g., two bands in the optical wavelengths) and coarse spatial resolution (which potentially results in more land covers per pixel than can be feasibly separated). Several recent studies successfully addressed this problem by spectrally unmixing AVHRR imagery using co-registered high spatial resolution data from classified Landsat TM images [14]–[17]. These studies assumed that covers did not cause shadows, were homogeneous at TM (30 m) scales, and were Lambertian reflectors. Unfortunately, these assumptions are often violated in regions containing a heterogeneous mix of cover types.

Our goal is to resolve the inconsistency between the 1-D canopy BRDF models and remote sensing data by deconvolving the angular AVHRR reflectance data into single landcover reflectances. To achieve this goal, we have developed a method that combines two spectral unmixing steps with a geometric-optical shadow correction technique. This method is specifically designed for ecosystems of high spatial heterogeneity such as savannas and shrublands. The retrieved angular endmember reflectances could then be used to estimate biophysical parameters of each vegetation type via BRDF model inversions (not demonstrated here). In this paper, we present the three-step spectral unmixing model which provides access to the AVHRR sub-pixel reflectances, an unmixing demonstration using simulated Landsat TM and AVHRR imagery, and the results of a sensitivity study designed to assess each component of the method.

II. DESCRIPTION OF MODEL

A. Spectral Unmixing of High-Resolution Imagery

Our model is based on a three-step methodology (Fig. 1). First, a high spatial/spectral resolution (HR) image is used to generate component land-cover fractions via a recently developed spectral unmixing technique [22]. This method employs a multidimensional visualization procedure to allow interactive searching for a set of spectral signatures (one for each covertype) within the eigenvector space of an image principal components analysis. This step captures fine-scale heterogeneity (e.g., Landsat TM: sub-30 m), and has proven successful in unmixing spatially heterogeneous tallgrass, open woodland, and savanna regions [23], [24]. Further description of the HR unmixing step will not be discussed here as it has been described in detail elsewhere [22], [23].

Although any type of multispectral imagery can be employed in the first spectral unmixing step, we chose to simulate Landsat TM data. Note that other linear spectral unmixing methods could be used in the HR unmixing step [25], [26]. Our choice of this high-resolution unmixing method is based on its proven accuracy using actual data in semi-arid regions, especially in highly aggregated and spatially complex landscapes which do not provide the homogeneous image pixels of individual landcover endmembers that may be required by other spectral unmixing techniques.

B. Geometric-Optical Model Inversion for Shadow Correction

In savanna ecosystems, typical cover types obtained from the above HR spectral unmixing step include tree, grass, soil, and shade fractions. For convenience in describing the model, we will only consider three savanna endmembers (tree, grass, and shade). Because the cover fractions of endmembers viewed by a satellite change with illumination and view angle, we cannot use the TM-retrieved cover fractions with AVHRR data. We must transform the cover fraction values associated with the TM sun-view geometry to new fractions based on the AVHRR sun-view geometry. To address this problem, we employ a geometric-optical (G–O) canopy shadow model for vegetation covers consisting of discrete plant crowns [27], [28]. The model simulates each crown as an ellipse, and the version of the model used here does not allow for crown self-shadowing. Therefore, we assume that the landscapes are comprised of a relatively sparse tree cover (e.g., 30% of total cover with little crown overlap) which is appropriate for most semi-arid savanna and shrubland biomes. G–O model input parameters include the areal density of tree stems (λ), the ratio of the crown vertical to horizontal radius (b/c/r), the ratio of the tree height (ground to crown center) to vertical crown radius (h/b), and view, solar zenith and azimuth angles. The G–O model computes areal fractions of sunlit overstory (tree), sunlit grass, and shaded grass based on these parameters.

In our procedure, the G–O model is numerically inverted to retrieve the input parameters (b/c/r, h/b, λ) from the sunlit and shaded endmember fractions (generated in Step 1) and sun-sensor geometry for a set of TM pixels (Fig. 1, Step 2). In savanna ecosystems, the dimension ratios of trees vary less than their spatial density (G. Asner, unpub. data, S. Archer, pers. comm.), and these ratios are relatively constant amongst dominant savanna tree species (Fig. 2). Thus we assume that canopy dimension ratios (b/c/r and h/b) are nearly constant over a 4 TM pixel grid (60 m x 60 m), but that λ varies between individual TM pixels. This assumption is necessary since a single TM pixel provides only two independent cover fractions (e.g., sunlit tree and shaded grass). The third endmember is merely the remaining fraction that sums all three components to 1. Since two known values per pixel are not sufficient for solving for three unknowns (b/c/r, h/b, λ), we must use more than one pixel to retrieve the parameters.

A four-pixel TM grid provides eight known parameters (one sunlit tree and one shaded grass fraction for each pixel in the grid) to solve for six unknown parameters including 4 λs (one per TM pixel) and a mean b/c/r and h/b (one per four-pixel grid) values. Inversion of the G–O model for each grid is accomplished using an optimization routine (E04JAF, Numerical Algorithms Group) which minimizes the merit
Fig. 1. Schematic representation of the AVHRR unmixing model which uses high (e.g., Landsat TM) and low (AVHRR) spatial/spectral resolution data. The main components of the model are: 1) Spectral unmixing of high resolution imagery into component land-cover fractions (e.g., overstory, sunlit understory, shaded understory); 2) numerical inversion of a canopy geometric-optical model to retrieve overstory canopy dimensions; 3) forward geometric-optical modeling to correct shadow fraction differences between images; and 4) spectral unmixing of low resolution data.

The merit function:

\[ \varepsilon^2 = \sum [(c_j - c^*_j)^2 + (b_j - b^*_j)^2] \]  

where \( c_j \) and \( b_j \) are the actual sunlit canopy and shade background fractions obtained from the four-pixel TM grid, and \( c^*_j \) and \( b^*_j \) are the modeled sunlit tree and shaded grass fractions. The G–O model is iteratively run and the input parameters \( (b/r, h/b, \lambda) \) are adjusted by the optimization routine to minimize the merit function. When the merit function has been minimized to a satisfactory tolerance \( (\varepsilon^2 < 1.0 \times 10^{-6}) \), the input parameters producing the successful minimization are stored, and the next grid containing 4 TM pixels is inverted. When all pixels within a TM image are processed, the retrieved canopy parameters are used in a forward G–O model simulation, but with the sun-sensor geometry of the co-registered AVHRR imagery, again for each four-pixel grid, to produce corrected high resolution sunlit and shaded grass fractions (Fig. 1, Step 3). These cover fractions are stored as individual images.

The sunlit tree fractions should have the same value in the original TM and shadow-corrected (AVHRR geometry) images since only the background shade fraction is intended to be adjusted. This check is used to ensure that the inverse-to-forward modeling steps are numerically stable, viz. to confirm that the overstory tree canopy fractions do not change during the shadow correction process.
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C. Low Resolution Spectral Unmixing

After high resolution spectral unmixing and image shadow-correction, each HR endmember fraction image (sunlit tree, sunlit grass, shaded grass) is convolved to the AVHRR spatial resolution (1.1–4.3 km) using a modeled AVHRR point spread function (PSF) [17]. The actual AVHRR PSF is not known [29], but can be adequately approximated with a 2-D Gaussian model [17]. The AVHRR PSF is highly dependent on the viewing angle [29], thus we scale the Gaussian filter independently in the along- and across-track directions based on sensor view angle. In addition, we account for earth curvature effects which arise when imaging the surface off-nadir with the AVHRR [29].

Upon convolving endmember images to the AVHRR scale, a system of equations is constructed for each 3 × 3 pixel sub-scene in the AVHRR image:

\[
\begin{bmatrix}
C_{11} & C_{12} & \cdots & C_{1j} \\
C_{21} & C_{22} & \cdots & C_{2j} \\
\vdots & \vdots & \ddots & \vdots \\
C_{k1} & C_{k2} & \cdots & C_{kJ}
\end{bmatrix}
\begin{bmatrix}
\text{ENDMEMBER}_1 \\
\text{ENDMEMBER}_2 \\
\vdots \\
\text{ENDMEMBER}_J
\end{bmatrix}
= \begin{bmatrix}
\text{AVHRR}_1 \\
\text{AVHRR}_2 \\
\vdots \\
\text{AVHRR}_J
\end{bmatrix}
\]

where \( \rho_k \) are the AVHRR pixel reflectances \((k = 1 \cdots 9)\), \( C_{kj} \) are the PSF-convolved endmember cover fractions \((j = 3 \text{ endmembers: sunlit tree, sunlit grass, shaded grass})\), and \( \rho_j \) are the AVHRR sub-pixel endmember reflectances. This system of equations is solved using singular value decomposition to retrieve endmember reflectances \( \rho_j \) [30]. The mean reflectances for tree and grass canopy are thus obtained for a nine-pixel AVHRR sub-scene (Fig. 1, Step 4). This procedure is repeated throughout the AVHRR image until all sub-scenes are processed, resulting in directional reflectance maps for each land-cover type. This method of solving for many AVHRR sub-scenes allows for variation in endmember reflectances across the landscape. Moreover, because a single AVHRR image is used in the matrix inversion, and since we assume that the sky acts as a diffuse radiation source, the endmember reflectances are actually hemispherical-directional reflectances.

Past studies have employed other regression methods for solving this system of equations [14]–[17]; however, these methods are prone to matrix ill-conditioning when the AVHRR image contains relatively homogeneous pixel reflectances. This situation occurs when attempting to employ these methods on uniformly mixed landcovers (e.g., savannas and shrublands). Solution of the matrix equation by singular value decomposition largely remedies this situation [30], but the problem is further minimized by operating the AVHRR unmixing inversion on a 3 × 3 pixel sub-scene. We have found that when fewer than nine pixels are used in the inversion, the potential for singularity in the system of equations resulting from even mixing of landcovers (e.g., equations are not linearly independent) is greatly increased. This singularity leads to sub-pixel endmember reflectance results that are highly sensitive to small errors in the AVHRR mixed-pixel reflectances or the convolved endmember fraction images. In our model, the size of the sub-scene used for inversion can be chosen based on the pixel-to-pixel reflectance heterogeneity of the area to be inverted; however, we have found that nine pixels provide satisfactory stability in most cases. Further comments regarding this sensitivity will be provided below.

III. SAVANNA LANDSCAPE SIMULATION

In an effort to test the unmixing model on a large number of scenes, a landscape simulation was developed to produce realistic TM and AVHRR images representative of a savanna region located in Northern Texas (Fig. 3). We chose to employ simulated satellite imagery for two reasons: 1) It allowed us to assess the numerical accuracy and efficiency of the method on a controlled dataset and 2) We could systematically add noise to the dataset to assess the sensitivity of the method to errors that will likely be encountered with actual data. The simulations were parameterized using field radiometric and biophysical measurements obtained at our Texas savanna research site. The site is roughly 200000 ha in size and is comprised of a scattered mesquite (Prosopis glandulosa) overstory canopy with a continuous layer of understory grasses.

A 1-D turbid medium BRDF model (DISORD) based on the discrete ordinates solution to the radiative transport equation was used to simulate individual tree and grass canopy
Fig. 3. Black and white aerial photograph of a typical savanna landscape in North Texas. Total area of the scene is 90 × 90 m. Sunlit tree, sunlit grass, and shaded grass endmembers are present.

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<td>AVHRR</td>
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<td>0-60</td>
<td>15 incr.</td>
</tr>
</tbody>
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reflectances [31]. DISORD accounts for all known leaf and canopy scattering mechanisms and includes a hot spot formulation. The model is based on several field-measurable biophysical properties including leaf area index (LAI), leaf angle distribution (LAD), leaf reflectance and transmittance, stem and soil reflectance, and the diffuse fraction of incoming solar radiation. Soil reflectance is anisotropic [4], and diffuse sky irradiance is assumed to be isotropic. Scattering of photons at the leaf level is comprised of a diffuse bi-Lambertian distribution and a specular component [32]. The model has been validated against reflectance data for several different crop [33] and grassland canopies [34].

The parameter values for DISORD are listed in Table I. Each biophysical parameter was chosen based on the canopy characteristics measured at the Texas savanna site (G. P. Asner, unpublished). Tree and grass leaf hemispherical reflectance and transmittance properties were obtained using a spectroradiometer (ASD Inc., Boulder, CO) and an integrating sphere (Li-cor, Lincoln, NE), combined with a method for obtaining spectra of narrow leaves [35] (Fig. 4). The leaf spectra were weighted by the solar spectrum and band sensitivity functions for AVHRR optical channels 1 (visible) and 2 (near-IR). Tree stem and soil reflectance data were also acquired at the savanna site under clear sky conditions. Tree and grass canopy angular reflectance distributions were generated with DISORD using input parameters based on these field measurements.

The canopy G–O model was used to compute sunlit tree, sunlit grass, and shaded grass fractions for a wide range of sun-sensor geometries [27], [28] (Table I). From pixel to pixel within an image, the G–O model parameters \((b/r, h/b, \lambda)\) were permitted to randomly vary within a realistic range for common savanna tree species based on a field survey at the Texas savanna site (Fig. 2). Each resulting sunlit tree and sunlit grass fraction was multiplied by the modeled tree and grass canopy reflectances for the range of AVHRR sun and view
angles listed in Table I. The shadow endmember was simulated using the relationship:

$$\rho_{\text{shade}} = W_{\text{shade}} \cdot \rho_{sb} \cdot e^{(-1/A\Delta I/cos\theta)} \tag{3}$$

where $W$ is the shade fraction generated with the G–O model, $\rho_{sb}$ is the full sunlight grass canopy reflectance (from DISORD), and the tree canopy transmission function is dependent on the randomly chosen tree LAI used in DISORD for each pixel as well as the prescribed AVHRR solar zenith angle ($\theta$).

The three endmember reflectance images were linearly summed to produce mixed-pixel reflectances for a 400 $\times$ 400 pixel image. AVHRR solar zenith angles (SZA) were permitted to range from 0–60$^\circ$ in 15$^\circ$ increments, while view zenith angles (VZA) ranged from 0–50$^\circ$ in 5$^\circ$ increments (Table I). In total, 105 AVHRR images were created with the same BRDF and G–O model canopy parameters, but with the appropriate AVHRR sun-view angles (5 SZA’s, 21 VZA’s along the principal plane) and scaled AVHRR PSF convolution kernels. The TM endmember concentration images (tree, sunlit grass, and shaded grass) were constructed using the same randomly chosen G–O parameters, but with a nadir view angle (0$^\circ$) and SZA = 30$^\circ$, and were linearly mixed to produce a single TM scene. For model sensitivity analyzes, all simulated satellite views were constrained to the solar principal plane.

IV. EVALUATION OF MODEL

A. Numerical Stability of the Model

The computational accuracy and stability of Steps 2 and 3 of our technique were tested on the series of simulated savanna landscapes. Perfect co-registration between TM and AVHRR images was assumed for this initial exercise. The simulated AVHRR images were spectrally unmixed with the method described above. Average tree and grass reflectances for all 3 $\times$ 3 AVHRR sub-scenes were retrieved for each image. The “true” mean AVHRR endmember reflectance of each 3 $\times$ 3 pixel grid was also calculated ($\rho_{true}$). The relative error ($\varepsilon$) between retrieved and actual tree and grass canopy reflectances was then calculated:

$$\varepsilon = \frac{\rho_{true} - \rho_{retrieved}}{\rho_{true}} \tag{4}$$

where $\rho_{retrieved}$ is the retrieved average reflectance of an endmember within an AVHRR nine-pixel sub-scene.

The mean true and retrieved principal-plane reflectance distributions in the AVHRR visible channel (1) for tree and sunlit grass endmembers are shown in Fig. 5. Relative errors of 0–2% indicate that the unmixing model provides satisfactory estimates of endmember reflectances at any viewing or solar zenith angle. Changes in endmember canopy reflectance factors in the backward and forward scattering directions, as
well as in the retrosolar direction (hot spot), are accurately retrieved. This initial analysis indicates that the G–O shadow correction technique provides an accurate solution to the image shadow fraction problem. Furthermore, the original tree cover fractions are maintained following the shadow correction step (Fig. 6). From this analysis, it appears that the method, which uses two image sources, spectral unmixing, and a geometrical-optical model inversion and forward simulation, is numerically accurate.

### B. Effects of HR Cover Fraction and Image Co-Registration Errors

Errors in the retrieved AVHRR endmember reflectances can result from a number of sources including inaccuracies in the first HR spectral unmixing step or co-registration errors between the TM and AVHRR images [15]. To assess the unmixing model sensitivity to these potential problems, random Gaussian mean-zero noise was added in 5, 10, and 15% variance increments to the simulated TM cover fractions. The simulated AVHRR scenes were spectrally unmixed, and the relative error between retrieved and actual endmember reflectances was calculated (Table II). This exercise was repeated for both the visible and near-IR spectral regions. As shown in the previous section, errors in the no-noise case are very low (<2.0%) for all solar and view geometries, cover types, and wavebands. Errors increase with increasing TM endmember noise and AVHRR SZA. These errors are relatively small for all scenarios except for overstory tree cover when the SZA = 60°; however, it is unlikely that the AVHRR will image sub-tropical or tropical savanna regions at this extreme illumination angle.

### C. Effects of Shadow on Spectral Unmixing Results

To illustrate the shadow difference effects that arise when coordinating data from multiple satellite instruments over savanna regions, the AVHRR images were unmixed using the TM cover fractions but without the G–O model shadow correction technique. Fig. 7 shows the true and retrieved endmember reflectances in AVHRR channel 1 and the relative error in the results. In general, without the G–O model inversion correction, tree canopy reflectances are underestimated at all VZA and SZA. These errors increase as VZA increases, but are dependent on the overall fractional cover of an endmember in the sub-scene. In these simulated savanna landscapes, tree reflectances can be underestimated by as much as 20%, while the grass reflectances may be incorrect by a maximum of only 13%. Fig. 8 indicates the relative error along the principal plane in the specific case when AVHRR SZA = TM SZA = 30°, that is, when errors are only due to view geometry differences. The errors can be significant for tree and grass components, but decrease at nadir (AVHRR sun-sensor geometry coincident with TM sun-sensor geometry), in the extreme forward- and back-scattered directions, and at the
Fig. 7. Simulated and retrieved tree and grass angular reflectance distributions when images are not shadow-corrected. Results represent AVHRR channel 1 along the solar principal plane. Relative errors increase away from the hot spot as retrieved reflectances are increasingly underestimated.

hotspot (VZA = $-30^\circ$). Tree canopy reflectances are greatly underestimated at $|\text{VZA}| < 35^\circ$ (excluding nadir) since the projected area of sunlit tree canopy and shaded background in the AVHRR direction is incorrect, resulting in reflectances that are too high for shade and too low for tree cover. Again, the grass endmember is less sensitive to shadow fraction errors since a much greater proportion of any pixel contains this cover type in our savanna simulations, thus dampening small errors in fractional shade cover. Errors decrease at large VZA since the projected tree area increases while the projected shade area decreases. Solar zenith angle differences between images produce even larger errors in retrieved endmember reflectances when sun-sensor geometries, and thus shadow fraction differences, are not taken into account (Fig. 7).

D. Effects of Spatial Homogeneity

In matrix notation, (2) can be written as

$$\overrightarrow{\mathbf{A}} \mathbf{x} = \overrightarrow{\mathbf{b}},$$

where $\overrightarrow{\mathbf{A}}$ is the convolved endmember concentration matrix, $\overrightarrow{\mathbf{b}}$ is the AVHRR pixel reflectance vector, and $\mathbf{x}$ is the unknown endmember reflectance vector. The relationship is ill-conditioned when small perturbations to $\overrightarrow{\mathbf{A}}$ or $\overrightarrow{\mathbf{b}}$ produce large changes in the solved endmember reflectances, $\mathbf{x}$. These relationships can be described in matrix norm notation

$$\frac{||\delta\mathbf{x}||}{||\mathbf{x}||} \leq \kappa(\overrightarrow{\mathbf{A}}) \frac{||\delta\overrightarrow{\mathbf{A}}||}{||\overrightarrow{\mathbf{A}}||},$$

and

$$\frac{||\delta\mathbf{x}||}{||\mathbf{x}||} \leq \kappa(\overrightarrow{\mathbf{A}}) \frac{||\delta\overrightarrow{\mathbf{b}}||}{||\overrightarrow{\mathbf{b}}||}$$

where $\delta\overrightarrow{\mathbf{A}}$ and $\delta\overrightarrow{\mathbf{b}}$ represent the perturbations introduced to the convolved endmember fraction or AVHRR pixel reflectance matrix, respectively, and $\delta\mathbf{x}$ is the resulting error in the retrieved endmember reflectance values. The condition number $\kappa(\overrightarrow{\mathbf{A}})$ of the convolved endmember concentration matrix is defined as the ratio of the largest to smallest eigenvalues of a principal components analysis of $\overrightarrow{\mathbf{A}}$. In matrix norm notation, the condition number is denoted

$$\kappa(\overrightarrow{\mathbf{A}}) = ||\overrightarrow{\mathbf{A}}|| \cdot ||\overrightarrow{\mathbf{A}}^{-1}||.$$

A large condition number leads to a large error in the retrieved endmember reflectances ($\mathbf{x}$) when a small perturbation is introduced to either the convolved concentrations ($\overrightarrow{\mathbf{A}}$) or AVHRR reflectances ($\overrightarrow{\mathbf{b}}$). Thus a check for this potential problem can be made when combining information from multi-resolution datasets for spectral unmixing, that is, before solving (2).

As mentioned earlier, the problem of ill-conditioning can occur when unmixing an AVHRR sub-scene containing very similar pixel-to-pixel land-cover endmember concentration values, viz. when the fractions of sunlit tree and grass are nearly the same across neighboring pixels within a sub-scene. There is no set dividing line between “well conditioned”
and "ill conditioned" in a system of equations, and it is partially dependent on computer precision. Therefore, the relationship between the condition number and the sensitivity of the retrieved endmember reflectances to perturbations to $\mathbf{A}$ or $\mathbf{b}$ must be determined empirically. The effects of a perturbation to $\mathbf{A}$ (convolved endmember concentrations) were previously discussed and shown to have minimal effects mainly because there are so many TM pixels within an AVHRR pixel. However, an error in $\mathbf{b}$ could result from errors in atmospheric correction, instrument noise, and other factors. Errors in $\mathbf{b}$, when combined with a convolved concentration matrix ($\mathbf{\bar{A}}$) having a large condition number, result in erroneous values for unmixed land-cover reflectances.

To obtain a relationship between the condition number of $\mathbf{A}$ and the potential error in retrieved reflectances following unmixing, we used the savanna simulation model to construct images of varying spatial heterogeneity. The heterogeneity of an image was adjusted by increasing or decreasing the tree cover in each sub-scene. For a well-mixed image, the amount of tree cover was randomly chosen in each pixel. For an aggregated (clumped) image, sub-scenes of varying size were constructed within an image, and some of those sub-scenes received 100% tree cover (using DISORD to simulate complete crown coverage without canopy gaps). The condition number of $\mathbf{\bar{A}}$ was computed for each simulation (6), and 20% mean-zero Gaussian noise was added to the AVHRR samples ($\mathbf{\bar{b}}$). The resulting error in the retrieved landcover reflectances, averaged for tree and grass endmembers, was then computed using (5).

The retrieved reflectance errors ($\mathbf{e}$) are very low (<8% relative) for condition numbers less than ~50; however, they increase quickly as the condition number increases beyond that level (Fig. 9). For a completely random mixing of the three endmembers in each pixel of a simulated image (e.g., the average concentration of an endmember in the entire image is 33.3%), the condition number is roughly 200, which leads to a 600% relative error in retrieved endmember reflectances following a 20% perturbation to the AVHRR samples. This is the worst case scenario for (5) since $\mathbf{\bar{A}}$ approaches the condition of singularity (e.g., the system of equations becomes linearly dependent).

Our experience has been that most savanna landscapes contain sufficient clumping or variation in landcover types to avoid this potential problem. As an example, three savanna image sub-scenes at the 30 × 30 m TM spatial resolution are shown in Fig. 10. The first image is a simulated TM scene consisting of a random mix of the three endmembers. This scene produces an unsatisfactorily high condition number (~180), and thus, it is prone to spectral unmixing errors. The second and third images are actual Landsat TM data from two Texas savanna research sites. After image convolution, the condition numbers (31 and 2, respectively) for these scenes are sufficiently small to allow for stable spectral unmixing model inversions.

In practice, we will apply this empirical relationship to the high resolution imagery to ensure that the condition number is satisfactorily low prior to spectral unmixing. If the nine-pixel AVHRR sub-scenes are found to produce ill-conditioned systems of equations, the size of the sub-scene will be adjusted until the landscape heterogeneity of tree and grass cover types increases sufficiently [$_{\mathbf{\bar{A}}}$ decreases adequately] for reliable endmember reflectance retrievals.

V. CONCLUSION

While 1-D BRDF model inversions have been successfully employed to retrieve canopy biophysical characteristics of horizontally homogeneous land covers [8]–[10], these methods cannot be used in regions containing a mix of covers if the canopy parameters of individual vegetation types are of interest. Many ecosystems including savannas, shrublands, and woodlands are comprised of mixed vegetation types, and many others (e.g., boreal and deciduous forests) contain significant
gaps which make the 1-D assumption difficult to apply. To our knowledge, the invertibility of a general three-dimensional (3-D) canopy radiative transfer model has not been established, nor does it seem likely that such an inversion could be successful without significant ancillary data.

Spectral unmixing of remotely sensed imagery provides one way to rectify the problem in applying 1-D BRDF inversions to heterogeneous landscapes. Unfortunately, the extremely low spectral resolution of the AVHRR precludes the use of more conventional unmixing methods that rely on image or library endmembers. As a solution to this problem, we presented a three-step spectral unmixing method for retrieving AVHRR sub-pixel land-cover reflectances in regions of high spatial heterogeneity. The reflectances of individual vegetation types are deconvolved using co-located Landsat TM and AVHRR data. The three major steps in the model include: 1) retrieval of vegetation endmember concentrations from high resolution (TM) imagery; 2) correction of dissimilar shadow fractions between TM and AVHRR data; and 3) retrieval of AVHRR sub-pixel reflectances of vegetation types for any sun-sensor geometry.

Step 1 (TM spectral unmixing) has been explained in detail elsewhere [22], [23]. In this paper, Step 1 was only explored with respect to its effect on other components of the overall method. Step 2 (shadow correction) is critical to the success of our method when applied to arid and semi-arid environments (e.g., savannas, shrublands, deserts) which tend to have landscape reflectance patterns that contain significant shadow contamination. Without the shadow correction, land-cover reflectances are underestimated. The degree of error, however, depends on the spatial extent of the covertypes. In other geographic regions or at coarser spatial scales, the shadows cast by the vegetation may be relatively small compared to the extent of other covertypes, and thus the shadow correction may be unnecessary.

Step 3 performs the unmixing of the AVHRR samples into component endmember reflectances. We show that the method is robust and stable when noise resulting from either the first unmixing step or from image co-registration errors is introduced. However, the unmixed reflectances are sensitive to an ill-conditioned endmember concentration matrix resulting from spatial homogeneity in cover fractions. Therefore, we devised a simple method based on the matrix condition number for determining the number of AVHRR pixels required for a stable solution prior to unmixing.

Based on these tests using simulated TM and AVHRR data, our technique appears useful for application in savanna regions to estimate individual reflectance of tree and grass covers. This method will be used in future work to compile directional reflectance datasets from actual AVHRR images for 1-D BRDF model inversions. A progress report on these aspects of our research is forthcoming.

Although other instruments are currently available for sampling the BRDF of vegetation canopies [12], none of them has the relatively fast repeat time or ease of data acquisition like the AVHRR. Thus we believe that a combination of sensors is necessary for unmixing AVHRR data. Some of the difficulties in spectrally unmixing AVHRR data may be alleviated when the first era of EOS instruments become available. Both MODIS and MISR will provide greatly increased spectral, spatial, and angular resolution over the AVHRR, and these gains will allow for further improvement of unmixing methods.

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