A study of the Manipulability of the PHANToM™ OMNi™ Haptic Interface

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Abstract

In order to perform the assembly of a haptic device in the set of a simulator, we have to check that it operates in its optimal workspace. In general it will depend on the required functionality. One of the characteristics that define the performance of a haptic device is the manipulability. In this paper we are going to accomplish a kinematics study of the device PHANToM OMNi. This study will produce as result the calculation of a map, in which to each of the points of the workspace, we assign a value of manipulability. In this way it is obtained a drawing that identifies the best zones of functioning of the device and therefore those in which we wish that our manipulator preferably works as a design criteria.

Categories and Subject Descriptors (according to ACM CCS): I.3.4 [Graphics Utilities]: Virtual Device Interfaces. H.5.2 [User Interfaces]: Haptic I/O, Input Devices and Strategies.

1. Introduction

At the moment of evaluating the performance of a haptic device, one of the elements to consider is the capability to reach, and move around, a position inside its workspace that will need a certain configuration of the different joints of the system. Depending on the requirements, the set of the constructive characteristics and skills of the device will allow to transmit movement and force with major or minor difficulty up to the end of the kinematics chain. These characteristics are associated with the definition of manipulability of the device that this work will introduce in detail in section 5.

In order to accomplish the study of manipulability we need to make an analysis of the kinematics behavior of the haptic device, in our case, PHANToM OMNi device of the company SensAble Technologies. This analysis is parallel to the works about the PHANToM haptic device version 1.5 already presented by different authors [CFT02] [CF01] [RB05].

The first step would be the study of the constructive characteristics of the OMNi device. It is necessary the resolution of inverse and forward kinematics problems of the device.

As result of the kinematics study we obtain the Jacobian matrix associated to the device. Then, from the Jacobian, we can calculate the index of manipulability $\mu$. If we determine the value of this index for all the points that we can reach with the end of the manipulator we can achieve a map of manipulability of the workspace with curves of iso-manipulability, that is, curves that join the points with equal values of $\mu$. This will allow us to determine a map of functionality zones of the device.

2. Structural Analysis.

Firstly we will do the simplification of the object of study reducing it to those components of relevancy in kinematics. Then we will define these components of the OMNi examining its constructive characteristics and identifying each of its elements.

2.1 Defining the Object of Study.

In order to circumscribe the results that mathematically could be obtained to the functioning zone of the device we must take in account three definitions relative to the device working area, (a) Nominal Workspace that in case of the OMNi comes defined by a rectangular prism of dimensions 160 W x 120 H x 70 D mm [Sen04]. That is the area in which the manufacturer guarantees force feedback with a certain stability and precision. (b) Real Workspace understanding it as the subset of the total space that we can reach with the end of the manipulator, it means the solid angle described by the end of the mechanism that constitutes the device. (c) Effective Workspace, as the spatial environment in which it will actuate according to the task of simulation entrusted the OMNi. For instance the working space used by a surgeon inside of a knee in a simulation of Minimally Invasive Surgery.

In order to simplify the structural study of the kinematics of the OMNi, the study extends only to the final point where force feedback is transmitted, eliminating the three gimbal elements (D, E and F in the figure 1). These gimbal angles are just used to indicate an orientation of a possible tool. Kinematics study will simplify the end of the device to a point, a small virtual sphere located in D without orientation. Here in after we will name this point End Effector.
We have used the following names in order to identify the different Components of the OMNi device (fig. 1): Element A (Head) turns around Y axis (yaw) including Force Feedback. Element B (Crank) turns around X axis (pitch) including Force Feedback. Element C (Connecting Rod) turns around X' relative axis (pitch) including Force Feedback. Elements D (Wrist), E (Fork) and F (Stylus) turns around orthogonal axes located at the End Effector and are the Gimbal angles.

Finally the base or pedestal on which the system is supported and include turn axis for element A, has the function of giving stability to the system avoiding overturns. It lodges the control system and the engine that allows applying force feedback to the element A. This is a fixed component and no relevant in the kinematics study.

### 2.2 Degrees Of Freedom.

As we have said above, we analyze only the degrees of freedom (DOF) corresponding to the nodes of the mechanism where force feedback will be applied. We define three angles \( \theta_1 \), \( \theta_2 \) and \( \theta_3 \) so the three gimbal angles that determine only the orientation of stylus are obviated. First is the turn of A element (Head) around Y axis. Angle \( \theta_1 \) (fig. 2). Note that there is kinematics symmetry in the plane defined by the set ‘Crank’-‘Connecting Rod’ respect to this turn. Values of \( \theta_1 \) range from -50º to 55º.

The second DOF to consider is the turn of B element (Crank) around X axis. Angle \( \theta_2 \) (fig. 3a-3b). Values of \( \theta_2 \) range from 0 to 105º.

Finally the third DOF will be the turn of C element around X relative axis (Connecting Rod). This movement corresponds with angle \( \theta_3 \) (fig. 4).

The angles \( \theta_2 \) and \( \theta_3 \) are determined from the pieces ‘connecting-rod’ and ‘crank’. Angle \( \theta_2 \) indicates the inclination of the piece ‘crank’ with the horizontal plane as the axis Z, and \( \theta_3 \) the inclination of the piece ‘connecting-rod’ respect of the vertical one, the axis Y. Minimum and maximum values of \( \theta_3 \) are not constant and depend on value of \( \theta_2 \) and the angle inter-arms (l1-l2). So the table in fig. 5 shows the correspondence between these two angles.
### 3. Kinematics.

There is going to be completed the kinematics study defined in previous works [WLH*05]. The references in the whole analysis are going to be done with regard to the center of the element ‘Head’ that we will name Coordinate System Origin (CSO). In previous works about PHANToM 1.5 version [CFT02] [CF01], they carries out as CSO from the rest-calibration position similar to the inkwell of calibration of the OMNi. This election is based on the fact of this position coincides with a value of the angles $\theta_2$ and $\theta_3$ equal to zero. However the situation of the stylus of the OMNi inside the inkwell, which is the position of calibration, has not this advantage so we have decided to place the CSO in the center of the ‘Head’ element because it produces important simplification in the system equations.

The study of the kinematics of the device is going to include three steps. The first one is to solve the forward kinematics problem; it involves determining position of End Effector of the haptic in relation to the CSO reference, known the angles of the joints and the geometric parameters of the elements of the device. The second is to solve the kinematics inverse problem, determining the configuration that must adopt the haptic device for a known position of the End Effector. The third step is to solve the differential model (Jacobian Matrix) establishing the relations between angular velocities of the joints and those of the End Effector of the device.

#### 3.1 Forward Kinematics.

The aim is to calculate the matrix that defines the behavior of the device depending on each one of angles of the joints $\theta_i$. We want to define the translation matrix $T_{04}(\theta)$ of the device from CSO to End Effector (fig. 7). Partial derivative of this $T_{04}(\theta)$ respect to $\theta_{1,2,3}$ will produce the Jacobian matrix.

The fig. 7 represents a set of CS placed in every joint. Our calculation will consist of performing a transformation sequence from CSO to End Effector following the kinematics chain path CSO-CS1-CS2-CS3-End Effector. CS1 is the Coordinate System associated to the ‘Head’ element; CS2 is the Coordinate System associated to the ‘Crank’ element; CS3 is the Coordinate System associated to the ‘Connecting Rod’ element.

### Table 1: Relative values of $\theta_3$ depending on $\theta_2$ (value of the angles in degrees).

<table>
<thead>
<tr>
<th>$\theta_2$</th>
<th>$\theta_3$ minimum</th>
<th>$\theta_3$ maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-20</td>
<td>65</td>
</tr>
<tr>
<td>15</td>
<td>-15</td>
<td>90</td>
</tr>
<tr>
<td>30</td>
<td>-9</td>
<td>105</td>
</tr>
<tr>
<td>40</td>
<td>0</td>
<td>110</td>
</tr>
<tr>
<td>50</td>
<td>10</td>
<td>112</td>
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<tr>
<td>60</td>
<td>20</td>
<td>113</td>
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<td>80</td>
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<td>90</td>
<td>50</td>
<td>114</td>
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<tr>
<td>105</td>
<td>60</td>
<td>110</td>
</tr>
</tbody>
</table>

**Figure 5:** Relative values of $\theta_3$ depending on $\theta_2$ (value of the angles in degrees).

In the fig. 6 appear both configurations in which a manipulator might reach the end of the OMNi, simplified to the extreme of the End Effector where force feedback is applied. An observer who looks in the direction of the arrow will be able to see the possible OMNi configuration as concave whereas the convex configuration in bold lines is impossible due to constructive structure of the device, similar to the human elbow.

**Figure 6:** Force feedback point and non-possible configuration in bold lines.

The transformation matrix from CSO to CS2 system excluding turn $\theta_2$ is:

\[
T_{02} = \begin{pmatrix}
\cos(\theta_1) & 0 & \sin(\theta_1) & 0 \\
0 & 1 & 0 & 0 \\
-\sin(\theta_1) & 0 & \cos(\theta_1) & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

And then including $\theta_2$ we obtain transformation matrix between CS2 to CS3

\[
T_{23} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & \cos(\theta_2) & \sin(\theta_2) & l_1 \sin(\theta_2) \\
0 & -\sin(\theta_2) & \cos(\theta_2) & l_1 \cos(\theta_2) \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

Therefore transformation matrix from CSO to CS3:

\[
\begin{pmatrix}
\cos(\theta_1) & -\sin(\theta_1) \sin(\theta_2) & \cos(\theta_1) \sin(\theta_2) & l_1 \cos(\theta_2) \sin(\theta_1)
\\
0 & \cos(\theta_2) & \sin(\theta_2) & l_2 + l_1 \sin(\theta_2)
\\
-\sin(\theta_1) & -\cos(\theta_1) \sin(\theta_2) & \cos(\theta_1) \cos(\theta_2) & -l_1 + l_1 \cos(\theta_2) \cos(\theta_1)
\\
0 & 0 & 0 & 1
\end{pmatrix}
\]

**Figure 7:** Coordinate Systems CSO, CS1, CS2 and CS3 defined in the study of the OMNi.
Translation from joint CS3 to End Effector:

\[
T_{34} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & -l_2 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

Finally transformation matrix \( T_{04} \) from End Effector regard to the CSO:

\[
\begin{bmatrix}
\cos(\theta_1) & -\sin(\theta_1) \sin(\theta_3) & \cos(\theta_3) \sin(\theta_1) & 0 \\
\sin(\theta_1) & \cos(\theta_1) \sin(\theta_3) & -\cos(\theta_3) \sin(\theta_1) & 0 \\
0 & -\cos(\theta_3) \sin(\theta_1) & \cos(\theta_3) \cos(\theta_1) & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

R04 system rotation matrix:

\[
\begin{bmatrix}
\cos(\theta_1) & -\sin(\theta_1) \sin(\theta_3) & \cos(\theta_3) \sin(\theta_1) & 0 \\
\sin(\theta_1) & \cos(\theta_1) \sin(\theta_3) & -\cos(\theta_3) \sin(\theta_1) & 0 \\
0 & -\cos(\theta_3) \sin(\theta_1) & \cos(\theta_3) \cos(\theta_1) & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

Coordinates of the force feedback point End Effector of the manipulator referred to CSO are last column of the \( T_{04} \) transformation matrix.

\[
x = (l_1 \cos \theta_1 + l_2 \sin \theta_1) \sin \theta_1 \\
y = l_1 \sin \theta_1 - l_2 \cos \theta_1 \\
z = (l_1 \cos \theta_1 + l_2 \sin \theta_1) \cos \theta_1
\]

3.2 Inverse Kinematics.

This problem consists of the expression for the angle \( \theta_i \) of each joint depending on the geometry of the device. So from the workspace (\( P_x, P_y, P_z \)) through the resolution of the inverse kinematics problem until Joint space (\( \theta_1, \theta_2, \theta_3 \)).

According to cosine theorem calculations:

\[
\theta_1 = -\text{atan} \left( \frac{x}{z} \right)
\]

\[
\theta_2 = \alpha_1 + \alpha_2;
\]

Therefore

\[
\alpha_1 = \text{atan} \left( \frac{y}{H} \right);
\]

And we obtain \( H \) from \( H^2 = x^2 + z^2 \). According to cosine theorem again:

\[
\alpha_2 = \arccos \left( \frac{L^2 + l_1^2 - l_2^2}{2l_1 L} \right)
\]

Finally

\[
\theta_3 = \text{atan} \left( \frac{H - l_1 \cos \theta_1}{l_1 \sin \theta_1 - y} \right);
\]

4. Jacobian Definition.

A Jacobian is the matrix of the first partial derivatives of a function, \( F : \mathbb{R}^n \rightarrow \mathbb{R}^m \) with \( m \) components \( y_1 \) to \( y_m \) each of them with \( n \) independent variables \( x_1 \) to \( x_n \). We achieve the Jacobian matrix as result of each of the partial derivatives of \( y_i \) respect of each one of the \( x_i \), a row of the Jacobian.

\[
J(f,g) = \begin{bmatrix}
\frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\
\frac{\partial g}{\partial x} & \frac{\partial g}{\partial y}
\end{bmatrix}
\]

Generalizing

\[
J = \begin{bmatrix}
\frac{\partial y_1}{\partial x_1} & \cdots & \frac{\partial y_1}{\partial x_n} \\
\vdots & \ddots & \vdots \\
\frac{\partial y_m}{\partial x_1} & \cdots & \frac{\partial y_m}{\partial x_n}
\end{bmatrix}
\]

We can also say that from Jacobian we obtain the best linear approximation of the value of a function \( F \) in the surroundings of a point \( P \)

\[
F(x) \approx F(p) + J_p \cdot (x-p)
\]

The Jacobian is a matrix constructed in two parts. The upper half represents the relation that exists between the linear velocities of the end of the manipulator with the angular velocity of each one of the joints simplified in the equation:

\[
V = J_u \cdot \dot{\theta}/dt
\]

The matrix lower half is not used in the calculation of the manipulability, but represents the relation that exists between the angular velocity of the end of the manipulator with the angular velocity of each one of the joints simplified in the equation:

\[
\omega = J_l \cdot \dot{\theta}/dt
\]

Jacobian matrix is of great help in the analysis and control of the movement of a manipulator for example in the determination of singular configurations, in which for a small change of position a very high torque is needed, or configurations from which some directions are
unreachable, therefore zones we are not interested device works in.

The calculation of the manipulability from the Jacobian will determine that the real workspace of the device does not present wells of singular points in its habitual zone of functioning.. A singular point drives to an unrealizable solution of the kinematics inverse problem. Near to the singular points, the actions of the manipulator are undetermined, and for this combination of \( \theta \) values the Jacobian determinant is zero.

In formula (1) from \( d\theta/dt \) as a known value, applied by the engines in the joints and determined by the control system by the encoders, and calculated \( J_u \), we would be able to obtain a measure of the linear velocity \( V \) of the end of the manipulator.

In the case of PHANToM OMNi, upper half of the Jacobian \( J_u \) has the form \( J(3 \times n) \) being \( n \) the number of degrees of freedom, and depending on \( \theta \).

The calculations realized with the tool Mathematica in addition with the packages Screws and RobotLinks [Mur92] containing some of the defined functions used in the program, results in the following Jacobian matrix:

\[
J = \begin{bmatrix}
\dot{l}_1 \cos(\theta_2) + l_2 \sin(\theta_2) & 0 & 0 \\
0 & \dot{l}_1 \cos(\theta_2 - \theta_3) & 0 \\
0 & -\dot{l}_1 \sin(\theta_2 - \theta_3) & l_2 \\
0 & 0 & \cos(\theta_3) \\
\sin(\theta_3) & 0 & 0
\end{bmatrix}
\]

5. Manipulability.

One of several definitions that allows studying the performance of a manipulator in general and for extension a haptic device is the manipulability. A definition might be the skill for transmit movement and to apply forces in arbitrary directions [PK98]. We can also say that the manipulability of a device indicates its ability of moving freely in all the directions in the workspace [MLS94].

Another definition is that manipulability is the efficiency with which a manipulator transmits force and velocity to its end effector [SBS02].

The manipulability of a device was conceptually defined by Craig and Salisbury (1982) and the first formulation that allowed a mathematical simple quantification was brought up by Yoshikawa [Yos85]. Its study needs of the calculation of the Jacobian. And we will complete the representation of manipulability with information from the inverse kinematics.

5.1 Calculation of manipulability map

A widely used definition of manipulability is from Yoshikawa (it implies the Jacobian and its transposed one).

\[
\mu = \sqrt{\det(J_u \ast J_u')},
\]

Nevertheless other approximations use different formulation of the manipulability value [CFT02] [TPM04] defined like:

\[
\mu = \sigma_{\text{min}}(J_u)/\sigma_{\text{max}}(J_u)
\]

Where \( \sigma_{\text{min}} \) and \( \sigma_{\text{max}} \) are the minimum and maximum singular values of the matrix \( J_u \). In this case we should obtain the singular values \( \sigma \) of the \( J_u \) matrix by means of the decomposition in singular values, of the form:

\[
J_u = U \Sigma V'
\]

\( J_u \) is the matrix which decomposition we are looking for, \( S \) the diagonal matrix corresponding the values of its diagonal with the singular values of the matrix \( J_u \) and \( U \) and \( V \) orthogonal matrices. Once accomplished these singular values we get the maximum and the minimum of them and in consequence the definition of manipulability.

Therefore it is a design option to avoid singular configurations in order to maximize measurement of manipulability. Examining the singular values of \( J_u \) in a point we know how closely we are of a singular configuration.

5.2 Isomanipulability curves map.

The first problem to be solved is to represent for a value of \( x=0/\theta_1=0 \), the corresponding map of the different points that belong to this plane, each of them with its manipulability value. In this map we join the points with an equal value of \( \mu \) and create the curves of isomanipulability. Nevertheless the obtained formulation of the Jacobian does not depend on \( \theta_1 \), so the manipulability would be equal for any track of the space of a constant value of \( \theta_1 \). Symmetry exists therefore in the behavior of the manipulator with regard to the turn around axis Y that defines \( \theta_1 \).

The second consideration is in defining that values of manipulability have only sense for these points inscribed
inside the real workspace of the manipulator. On the map with the different numerical values of manipulability we will have to project the curve that describes the path of the End Effector as for its maximum scope. Defining the value of manipulability as formula (2) we obtain graphics (Z/Y) as fig. 9 illustrates for $\theta_1=0$.

From this calculus it is determined that workspace of the OMNi is singularity-free with a uniform distribution of the value of manipulability, reflecting wide zones of high values.

5.3 Evaluation of real workspace

Nevertheless we can only include in the analysis the values that are inside the real workspace. So we must obtain the section of the map of manipulability that corresponds with this space. The measured angles values are:

\[
\begin{align*}
\theta_1 &= \text{min. } -50^\circ / \text{max. } 55^\circ \\
\theta_2 &= \text{min. } 0^\circ / \text{max. } 105^\circ
\end{align*}
\]

For $\theta_1$ it is observed that it depends on the value of $\theta_2$ in agreement to the table defined in the as shown in the fig. 5 above.

We are going to describe the maximum area that End Effector can reach with the device in the plane YZ (fig. 10). A circumference of radius $l_1$ is created. Inside of it we draw radiuses simulating different positions of 'Crank' element. The first one is for the value of $\theta_2=0$ and going on with the values defined in the fig. 5. From the ends of each one of these radiuses, we draw different arches of radius $l_2$ with the measure of the arch for correspondent range of $\theta_3$ according to table of fig. 5.

Once joined the ends of maximum movement range of each one of the selected configurations and by means of interpolation, the corresponding arches are drawn in order to join the ends of all the defined surfaces. We have created the surrounding curve that determines, for this plane all the points of the space the End Effector can reach. The real workspace is obtained as the interior of this curve.

The projection of the surrounding curve on the map of manipulability is performed as fig. 11 illustrates. Then it must be done the extraction of the portion of the map that is going to interest us where the End Effector can really reach.

Figure 11: Projection of the real workspace on the manipulability map.

Of the whole space defined by the map of manipulability we are only going to consider a portion. In this case we will observe, as fig. 12 shows, that the chosen area is the optimum one, with the best values of manipulability, in order to coincide with the real working area.

Figure 12: Subspace of manipulability defined for the real workspace.
6. Conclusion

Our study has remarked the importance of the manipulability in determining the performance of a device. The increase in the knowledge of a haptic device provided by the map of manipulability will allow us to recognize the optimum configuration for an application that, in this case, involves the PHANToM OMNi.

We have found an optimum manipulability zone of the OMNi device at inter-arms $l_1$ and $l_2$ angle values near to 90º, coinciding with the central area of curves in fig. 12, with an optimum value in the upper zone of the map.

It is desirable that the Effective Workspace overlaps with the optimum manipulability zone of the OMNi as design criteria of an application. Even it could be possible to develop a methodology in order to use these criteria on every haptic system.

By selecting the location of the OMNi properly we will improve the performance of the manipulator, increasing efficiency of its transmission of velocity and torque to the considered force feedback End Effector.

7. Bibliography


[WLH*05] DAVID WANG, GEORGE LEE. MEHRDAD HOSEINIZADEH. JAMES KUNZ, Handshake proSENSE™ Lab Series. Handshake VR Inc., LM-101205. 2001-2005