Enhancing potential field data using filters based on the local phase

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Abstract

Measures of the local phase of potential fields can be a useful aid to their interpretation. There are several variations in use, such as the tilt angle, tilt derivative and the Theta map. This paper compares the results of these filters, and introduces some new phase-based filters which show improved performance as edge detectors in different ways. The filters are demonstrated on synthetic gravity data and on magnetic data from Australia. Source code in Matlab format is available from the server at: www.iamg.org.

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1. Introduction

Images of the magnetic and gravity field of the Earth are used worldwide as part of exploration programs for mineral resources. When the data quality permits, a range of highpass filters, such as downward continuation or vertical derivatives, can be applied to bring out fine detail. Local phase filters provide an alternative approach but conventional phase functions need to be unwrapped to remove phase ambiguity (Fitzgerald et al., 1997). In noise-free data, the phase can be unwrapped by simply adding 2\(\pi\) to the phase whenever a sudden change from \(\pi\) to \(-\pi\) occurs but the presence of noise and aliasing can make the procedure very difficult. An alternative approach to the conventional phase filter is the tilt angle (Miller and Singh, 1994) which is defined as

\[
T = \tan^{-1}\left(\frac{\partial f/\partial z}{\sqrt{((\partial f/\partial x)^2 + (\partial f/\partial y)^2)}}\right),
\]

where \(f\) is the magnetic or gravity field. The gradient tilt angle has some interesting properties. As a dimensionless ratio it responds equally well to shallow and deep sources and to a large dynamic range of amplitudes for sources at the same level. The tilt angle is positive when over the source, passes through zero when over, or near, the edge where the vertical derivative is zero and the

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horizontal derivative is a maximum and is negative outside the source region. The tilt angle has a range of \(-90^\circ\) to \(+90^\circ\) and is much simpler to interpret than the analytic signal phase angle.

Verduzco et al. (2004) suggested using the total horizontal derivative of the tilt angle (which they termed the THDR) as an edge detector, i.e.

\[
THDR = \sqrt{\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2}
\]  

(2)

Pilkington and Keating (2004) show that the THDR is closely related to the local (instantaneous) frequency as defined by Thurston and Smith (1997). Pilkington and Keating (2004) compare five edge detectors including tilt angles and conclude that no single method has all of the desirable properties for a reliable contact mapper. The familiar trade-off between resolution and stability applies with techniques having the greatest resolution suffering from noise amplification. They conclude that using all of the five methods in concert is the best approach with co-located solutions from different methods providing increased confidence in the reliability of a given contact location and lessening the adverse effects of source magnetisation, dip, interference and depth extent.

Recently Wijns et al. (2005) introduced the Theta map which is based upon the analytic signal

\[
\text{Theta} = \cos^{-1}\left(\frac{\sqrt{\left((\partial f / \partial x)^2 + (\partial f / \partial y)^2\right)}}{\sqrt{\left((\partial f / \partial x)^2 + (\partial f / \partial y)^2 + (\partial f / \partial z)^2\right)}}\right).
\]  

(3)

Fig. 1 shows the gravity response from a simple model, and the tilt angle, THDR, Theta map, and the total horizontal derivative for comparison. The tilt angle (Fig. 1c) is positive over the model, but the response is blurred due to the model depth. The THDR (Fig. 1d) locates some of the model edges well, but others less well. In addition it is very susceptible to noise problems, being a derivative of a derivative-based function. The negative contour of the Theta map (Fig. 1e) also locates the model edges, but is again blurred due to the model depth. Note that edges located in magnetic data will only be correctly located relative to the geology if the data has previously been reduced to the pole.

2. Phase-angle-based filters with improved spatial resolution

It was found that use of the real part of the hyperbolic tangent function in the tilt angle
calculation achieved better delineation of the edges of the body than the filters discussed above, i.e.

\[ HTA = \text{arctanh} \left( \frac{\partial f / \partial z}{\sqrt{((\partial f / \partial x)^2 + (\partial f / \partial y)^2)}} \right) \].

(4)

The hyperbolic tangent function is defined as (Spiegel, 1972, p. 36)

\[ z = \tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}} \]  

and so

\[ x = \text{tanh}^{-1} z = 0.5 \ln \frac{1 + z}{1 - z}. \]  

(6)

The maximum value of the hyperbolic tilt angle (HTA) gives improved location of the body edges, as is shown in Fig. 1f. It is also less sensitive to noise than the THDR since it uses derivatives of lower order. Note however the negative contour of the HTA that surrounds the body—this is unwanted and only serves to complicate interpretation. Overlying only the values of the HTA above some user-defined threshold on the original data proved useful in solving this problem.

3. The second-order tilt angle

As mentioned above, the tilt angle filter did not serve as a very accurate method of locating deep sources, giving a diffuse response in that case. A much improved result came from using the second-order tilt angle, i.e. the tilt angle of the tilt angle. A comparison of Figs. 1c and g shows the tighter alignment of the filter response to the body outline. In theory the calculation of the second-order tilt angle is not physically valid, because Eq. (1) uses the vertical derivative of the potential field, and the tilt angle is not a potential field. For that reason the vertical derivative of the tilt angle was also computed from the difference between the tilt angle derived from the potential field data at two different heights. Fig. 1h shows the second-order tilt angle computed in this way. In practise, although not physically valid, the direct calculation of the second-order tilt angle (Fig. 1g) gave better results than the theoretically valid approach (Fig. 1h) on all the datasets on which the algorithms were compared. Regardless of how it was calculated, the second-order tilt angle was sensitive to noise, and improved results were achieved when the first-order tilt angle

data was smoothed (using a simple Gaussian convolution kernel) prior to computation of the second-order tilt angle. Alternatively, for noisy data, the tilt angle of the vertical integral or pseudogravity of the data can be calculated.

4. Directional tilt angles

The standard tilt angle (Eq. (1)) can be considered as an amplitude normalised vertical derivative. It is possible to design two other tilt angles which act as normalised horizontal derivatives:

\[ T_x = \tan^{-1} \left( \frac{\partial f / \partial x}{\sqrt{((\partial f / \partial z)^2 + (\partial f / \partial y)^2)}} \right) \]  

and

\[ T_y = \tan^{-1} \left( \frac{\partial f / \partial y}{\sqrt{((\partial f / \partial z)^2 + (\partial f / \partial x)^2)}} \right). \]  

(8)

\[ T_x \] and \[ T_y \] produce Tilt datasets with features enhanced in the \( x \) and \( y \) directions, respectively. Fig. 2a shows a synthetic gravity dataset consisting of the response from two rectangular sources at different depths. Fig. 2b is the EW gradient of the gravity data, while Fig. 2c is the EW horizontal tilt angle. The conventional EW gradient has defined the NS edges of the shallow body well, but the response from the smaller amplitude anomaly from the deeper body is comparatively faint. By comparison \( T_x \) is more diffuse than the conventional gradient across the shallower body, but it gives a much clearer response over the deeper body, due to the amplitude normalisation process. Directional tilt functions at any angle can be computed by using horizontal derivatives in the chosen direction and at 90° to it.

Sunshading is a commonly used image processing filter. The data is considered as a topographic surface which is illuminated with light from a ‘sun’ which has a particular azimuth and elevation. The surface reflectance is calculated, and the result enhances linear features with an orientation of 90° to the sun azimuth, and diminishes those that lie parallel to the sun azimuth (Horn, 1982). The data in Fig. 2a is shown sunshaded from the west (with a sun elevation of 30°) in Fig. 2d, which may be compared with Figs. 2b and c.

The maximum horizontal gradient amplitude (sometimes called the total horizontal derivative)
enhances edges of any orientation, and is given by

\[ f_{x_{\text{tot}}} = \sqrt{\left( \frac{\partial f}{\partial x} \right)^2 + \left( \frac{\partial f}{\partial y} \right)^2}. \]  

(9)

A normalised version of this filter is

\[ TDX = \tan^{-1} \left( \frac{\sqrt{\left( \frac{\partial f}{\partial x} \right)^2 + \left( \frac{\partial f}{\partial y} \right)^2}}{|\frac{\partial f}{\partial z}|} \right). \]  

(10)

In Eq. (1) the amplitude of the vertical derivative was normalised using the horizontal gradient amplitude, whereas in Eq. (10) the situation is reversed. Fig. 2e shows the total horizontal derivative of the data in Fig. 2a, which may be compared with the plot of \( TDX \) in Fig. 2f. Again the conventional filter performs well on the data from the shallow source, but the response from the deeper source is relatively subdued. \( TDX \) however performs almost equally well with both sources.

Cooper (2003) introduced a radial sunshading filter which was useful in delineating the edges of near-circular bodies of any radius, providing an estimate of the centre of the feature could be given. This idea can be extended to the tilt angle also, i.e. \( T_0 \) is used, where \( \theta \) is not constant across the whole image but is the angle between the chosen feature centre and each pixel in the image. Rotating \( \theta \) by 90° produces a filter that removes circular tilt features and enhances radial tilt features.

Note that the result of the application of the directional filters described in this section is not the same as can be achieved by the application of the standard version of the filters to the standard tilt angle \( T \) (i.e. \( T_{xz} \) is not the same as the horizontal derivative of \( T \)), because the latter involves calculating higher derivatives of the data and thus is more sensitive to noise.

5. Application to aeromagnetic data from Australia

The various tilt angle-based filters have been applied to semi-regional aeromagnetic data provided by Primary Industries and Resources of South Australia (PIRSA) to highlight magnetic signatures of interest in part of the North Flinders Ranges. The aeromagnetic data were acquired along east-west flight lines, 400 m apart at a nominal terrain clearance of 80 m. The North Flinders Ranges forms part of the Adelaide Fold Belt in South Australia. Palaeoproterozoic to Mesoproterozoic basement consisting of granites, gneiss and metasediments which outcrops in the Mt Painter Inlier is overlain by variable thickness of Neoproterozoic to Cambrian sedimentary rocks of the Adelaide Geosyncline. Both the Palaeoproterozoic–Mesoproterozoic cratonic basement and the Neoproterozoic–Cambrian cover have been affected by deformation and metamorphism during the Cambro-Ordovician Delamerian orogeny (Paul et al., 1999). Interpretation of filtered magnetic images has been used to map important magnetic elements and lineaments and provide objective structural and lithological information on the geological setting of mineralisation. The focus was on mapping favourable structures in Neoproterozoic Adelaidean and
Cambrian metasediments that may host mineralisation including diapiric structures. The area to the north contains two small copper deposits at Blinman and Sliding Rock (Dentith and Stuart, 2003).

Interpretation of enhanced images provides objective information on magnetic markers within the Adelaidean sequence and the magnetic signature of major folds corresponds closely to mapped structures. However, the magnetic data provides additional information on subtle changes in anomaly trends and amplitude along strike, which are interpreted as fold hinges, faults, and fractures, and which may be favourable sites for copper and gold mineralisation.

Fig. 3a is a total magnetic intensity greyscale image covering an area of 48 x 48 km. The image is dominated by long wavelength magnetic anomalies due to crystalline basement as most of the Adelaidean fold-thrust belt has subdued magnetic response, fortunately with clear traces of slightly higher amplitude stratigraphic markers visible. The low amplitude linear and curvilinear magnetic anomaly trends are due to iron-rich facies of tillites and sandstones within the Adelaidean. Higher amplitude, local anomalies with a rugose magnetic texture are due to the presence of mafic igneous material in diapirs. Derivatives of potential fields enhance the field component associated with shallow features and de-emphasise the field from deeper sources. Filtering has sharpened the response of both the higher amplitude diapir magnetic anomalies as well as dramatically improving the resolution of the weakly magnetic markers in the Adelaidean metasediments. A complex pattern of dome and basin folds and faults is clear in the image. The higher amplitude diapir magnetic anomalies are mainly located in domes. Tilt angle filters provide an alternative approach to enhancing shallow magnetic anomalies while preserving information on basement sources. Fig. 3b shows the conventional tilt angle. The image highlights the complex folding and faulting within the Adelaidean fold belt without attenuating the basement magnetic signal. Fig. 3c is the second-order tilt angle, which provides the best resolution of the magnetic markers in the Adelaidean metasediments compared with Fig. 3b. Fig. 3d is the HTA which probably provides the best resolution of the edges of basement magnetic sources. Fig. 3e shows the HTA response above a user-selected threshold overlain on the data itself, showing how it maps the edges of features, while Fig. 3f is a superposition of the second-order tilt angle on the data itself, using a transparency of 50%. The transparent overlay was used instead of the more conventional approach of using the high-pass filtered data (in this case the second-order tilt angle) as the image intensity layer, because the degree of transparency can be adjusted as required.

Fig. 4a shows the EW gradient of the data, computed in the space domain, whereas Fig. 4b shows the amplitude stabilised EW gradient $T_x^*$. The standard gradient is dominated by the large amplitude responses over the high-amplitude features, whereas $T_x^*$ shows significant detail even in the areas where the field gradients are very small. Figs. 4c and d compare the total horizontal gradient (Eq. (9)) with $TDX$ (Eq. (10)). Fig. 4c is noisy and
the image is dominated by a few large amplitude responses. TDX is also noisy, but shows the smaller features with greater clarity than does the conventional filter. Fig. 4e is the data in Fig. 3a sunshaded with the sun in the east (at an elevation of 30°), for comparison with Fig. 4b.

Finally, the tilt angle radial derivative is demonstrated on aeromagnetic data over the Yallalie possible impact structure from Australia. The Yallalie structure (Cowan and Cooper, 2005a, b), located at 30°27’S, 115°46’E in Mesozoic sediments of the Perth Basin, Western Australia was first identified from seismic reflection data. Aeromagnetic data were collected along north–south flight lines at 200 m line spacing. Fig. 5a shows the aeromagnetic data while Fig. 5b shows the conventional radial derivative (Cooper, 2003) centred on the middle of the structure. Fig. 5c is the radial derivative computed using tilt angle-based horizontal derivatives of data in (a) and (e) data in Fig. 3a sunshaded from east using an elevation of 30° from horizontal.

6. Conclusions

The tilt angle is a useful form of highpass filter for enhancing subtle detail in potential field data. The hyperbolic tilt angle is effective in delineating the edges of features, while the second-order tilt angle sharpens the filter response considerably. Amplitude balanced horizontal derivatives can be produced,
and these are useful in enhancing features with a given orientation. The tilt angle radial derivative performs well in enhancing near circular features. The filters were demonstrated on synthetic data and on aeromagnetic data from Australia.

References