How do Value Creation and Competition Determine Whether a Firm Appropriates Value?

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How do value creation and competition determine whether a firm appropriates value?*

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Abstract

We define value creation, competition, and value appropriation, and show that (i) there is a minimal level of value creation that is required if competition is to allow a firm to appropriate value; (ii) there is a higher level of value creation guaranteeing competition will result in value appropriation; (iii) there is a measure of scarcity, which we call minimum value, having the feature that competition ensures a firm may appropriate value if and only if the firm’s minimum value is positive; and (iv) if an agent is to appropriate value, a particular structure of competition is required.

Our results are relevant to the theoretical foundations of strategy. For example, we show that ownership of assets that are non-imitable and productivity-enhancing does not guarantee value appropriation.

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1 Introduction

Strategy is about the creation and appropriation of value. Value creation is a process that is inherently difficult to analyze. But value appropriation is arguably less mysterious and, so, has been studied extensively. A familiar idea is that a firm’s ability to appropriate some or all of the value it creates is determined by features of the value creation process somehow interacting with competition among firms; see, e.g., Saloner et al. (2001, chap. 3). In particular, if a firm can create value in a manner that is not easily imitated, this may limit competition and allow the firm to appropriate some or all of the value created.¹

In this paper we define value creation, competition and value appropriation, and then show that (i) there is a minimal level of value an agent must create if competition is to allow the agent to appropriate value; (ii) there is a higher level of value creation that guarantees competition will result in value appropriation; (iii) there is a measure of scarcity, which we call minimum value, having the feature that competition ensures a firm may appropriate value if and only if the firm’s minimum value is positive; and finally, (iv) if an agent is to appropriate value, a particular structure of competition is required. We also relate our results to the existing literature on the foundations of Strategy. For example, we show that ownership of assets that are non-imitable and productivity-enhancing does not guarantee value appropriation.

Our propositions follow from answering a very specific question: *If a firm creates value, what determines whether it must receive at least part of what is created?* Thus, we are concerned with whether the value creation process and the nature of the competition among firms invariably results in the firm appropriating value; equivalently, we ask whether the minimum the firm might obtain, given the value creation process and nature of competition, is more than what the firm could earn simply

¹According to Barney (1991, p. 102): “... a competitive advantage is sustained only if it continues to exist after efforts to duplicate it have ceased.” Similarly, Porter (1996, p. 62) says “[a] company can outperform rivals only if it can establish a difference that it can preserve.”
by not participating in this value creation process at all (and pursuing its next best alternative).

Why do we explore a lower bound on what the firm appropriates under competition instead of asking exactly how much the firm appropriates? The primary reason is that competition alone may be insufficient to determine exactly what the firm appropriates. As we demonstrate, the forces of competition imply an upper and lower bound on the firm’s appropriation opportunities, with the exact level of appropriation being determined by some sort of bargaining process. Thus, analyzing precisely what the firm earns requires a specification of how bargaining works. Numerous well-articulated theories of bargaining exist; see Osborne and Rubinstein (1990). However, predictions about the outcome of bargaining are generally sensitive to what is assumed about how bargaining operates – and there is little consensus on what constitutes reasonable assumptions (this especially so if bargaining involves multiple agents). By focusing on whether the firm must appropriate at least part of what is created, our results are robust in the sense that they do not depend upon any assumed structure of bargaining. Or, to put it slightly differently, we identify exactly those aspects of value creation and competition that guarantee, on their own, that the firm appropriates value.²

Our analysis is based on “cooperative” game theory, on which there is also an extensive literature; e.g., Osborne and Rubinstein (1994, Section IV) and Moulin (1988). Furthermore, we are not the first to suggest that thinking along these lines may be a fruitful way to study Strategy. For example, Brandenburger and Stuart (1996, 2000) make convincing arguments, and Brandenburger and Nalebuff (1996) draw on this framework. The novelty in what we do is simply to focus upon an individual firm

²This is not to suggest that the issue of how value creation and competition interact with aspects of bargaining is unimportant or uninteresting. Indeed, methods like those we utilize below can be applied to these questions, as well as to related ideas such as: what is the most the firm can hope to appropriate given value creation and the nature of competition.
and to characterize the competitive conditions under which it appropriates value; the general analysis appears in MacDonald and Ryall (2001).

We forecast that our framework and results will provide both structure and new insights when applied to familiar Strategy frameworks, e.g., the Five Forces (Porter, 1980), the Governance Perspective (e.g., Williamson, 1999), the Resource-Based View (e.g., Barney, 1991) and Coopetition (Brandenburger and Nalebuff, 1996). We illustrate this by providing a simple formalization of the governance perspective, and showing that a firm having non-imitable, productivity-enhancing, governance structure is no guarantee of value appropriation. Agents turn out to be scarce in unexpected ways and, when this is the case, they may appropriate any or all value created irrespective of the cleverness with which their incentives are structured or their jobs designed.

Our approach is relatively formal. One advantage of proceeding this way is as follows. Within the Strategy literature, several conditions are thought to influence a firm’s ability to “sustain competitive advantage;” Peteraf (1993) surveys the early literature. Collis and Montgomery (1998, p. 30-31) say that, “the value of a firm’s resources lies in the complex interplay between the firm and its competitive environment along the dimensions of demand, scarcity, and appropriability.” Seemingly, controlling resources with these qualities is both necessary and sufficient for value appropriation. While these informal discussions are both interesting and intuitively appealing, they can also be problematic in the sense that without precise definitions, assumptions and analysis, it is difficult to determine any of: what the theory assumes, whether it is internally consistent, how it might be applied, whether it is consistent with data, and so on. By making our primitives and assumptions explicit, the reader

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3 Many trace their intellectual roots to Penrose (1959), who argued that firms should be viewed as unique bundles of productive resources. Wernerfelt (1984) was the first direct application of these ideas to issues in business strategy. Barney (1991) provided a comprehensive argument linking resource heterogeneity and imperfect resource mobility to competitive advantage.
will know exactly what underlies our results, and can, therefore, both decide whether what we have done makes sense, and modify, augment, or challenge our model.

Our approach is also abstract and general. We proceed this way in part because it allows a particularly simple presentation. More importantly, this generality enables us to identify broadly-applicable principles (i.e., (i)-(iv) mentioned earlier) about the forces determining value appropriation.

In Sections 2 and 3 we present the model and main results. Section 4 contains our application to the governance perspective.4

2 Model

2.1 Value creation

Consider a strategic interaction among some agents, e.g., firms, suppliers, consumers, employees, government, etc. The set \( N \equiv \{1, 2, \ldots, n\} \) corresponds to all the agents in the interaction, i.e., agent 1, agent 2,\ldots, agent \( n \). Any subset of \( N \) (a “group”) is denoted \( G \), and for any group \( G \) including \( i \), the group obtained by excluding \( i \) is \( G_{-i} \); specifically, \( N_{-i} \) is the group consisting of all agents except \( i \). Likewise, for any group \( G \) not including \( i \), the group obtained by including \( i \) will be written \( G_{+i} \).

We have nothing to add concerning the creative process leading to value creation opportunities. Thus, we treat agents’ ability to create value as a primitive, and leave this as general as we can. Specifically, for any group of agents, \( G \), \( v(G) \) is the total value \( G \) can create without interacting with the agents outside \( G \). That is, \( v(G) \) is the value group \( G \) can create on its own.5

4Throughout, we illustrate with simple examples involving a small number of players. In other work-in-progress (notes available from the authors), we show how these same ideas can be employed in more complex settings, e.g., markets, and how the additional structure often assumed in such settings, e.g., fee entry, yields further results on value appropriation.

5To simplify notation, we assume no agent can create value alone: \( v(\{i\}) = 0 \). This is a harmless
In what follows, we will refer to the following simple example. Suppose a firm, $F$, has discovered a mine worth $1 million to one high valuation buyer, $B_h$, $.5$ million to a low valuation buyer, $B_l$, and $0$ to everyone else, including $F$. In this example, value creation possibilities are

<table>
<thead>
<tr>
<th>$G$</th>
<th>$v(G)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>${F, B_h, B_l}$</td>
<td>1</td>
</tr>
<tr>
<td>${F, B_h}$</td>
<td>1</td>
</tr>
<tr>
<td>${F, B_l}$</td>
<td>.5</td>
</tr>
<tr>
<td>${B_h, B_l}$</td>
<td>0</td>
</tr>
</tbody>
</table>

The rest of the paper is simplified by restricting value creation possibilities somewhat. Specifically, we assume that for any disjoint groups $G$ and $G'$,

$$v(G) + v(G') \leq v(G \cup G').$$

That is, combining disjoint groups of agents never reduces their aggregate ability to generate value. This is a kind of “no negative externalities” assumption. Assuming that (1) holds is not to say that we assume away negative externalities. Instead, the notion is that if there are any such effects, there is also enough flexibility to ameliorate their impact. For example, if groups $G$ and $G'$ do not function well as a large group, we assume they can be trivially “combined” by having the groups create value separately and then pool the value created; in this “worst case”, $v(G \cup G') = v(G) + v(G')$. Thus, by assuming (1), we are ignoring those situations in which there are both significant negative externalities and too little flexibility to avoid their having a significant impact. Although slightly more complicated, our approach can be applied when (1) is not assumed.

Note that (1) tells us that including more agents in a group never reduces the normalization.

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6Formally, $v$ is “superadditive”.

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value that might be created, i.e., for any disjoint groups $G$ and $G'$,

$$v(G) \leq v(G \cup G') ;$$

in particular, for $G$ not including $i$, $v(G) \leq v(G_{i+1})$. Thus, whatever the value creation possibilities are, the group that creates the most value is the group including all agents, i.e., $N$, which has value $v(N)$. Since this value will arise often below, we let $V \equiv v(N)$.\(^7\)

The interaction among agents is completely described by the pair $(N, v)$. In any specific application, $(N, v)$ is derived from the primitives of the interaction (i.e., the details of how agents create value). In particular, $v(G)$ represents the maximum (expected present) value $G$ can produce, implicitly accounting for limitations implied by information and agency problems, configuration of productive resources, barriers to technology transfer, institutional structure, regulation, etc.

### 2.2 Competition

We assume the agents in a strategic interaction are interested in the value they will obtain by participating. So let $\pi_i$ denote the amount of value obtained by agent $i$ as a result of the interaction. We refer to $\pi = (\pi_1, \ldots, \pi_n)$ as a distribution of value among the agents.

Strategic interaction must ultimately result in some distribution of value. Uncovering the features of such distributions is our objective. Thus, we must describe how strategic interaction turns value created into a distribution of value.

There are three forces shaping the way value is distributed. One is simply feasibility, i.e., the aggregate value distributed cannot exceed aggregate value created. Thus, recalling that the most productive group (the one with all agents participating) creates value $V$, we require that

$$\sum_{i \in N} \pi_i \leq V .$$

\(^7\)Since each agent can create zero value on his own, (1) also implies $v(G) \geq 0$. 

7
In the mining example, no more than $1$ million can be distributed.

Agents’ alternatives make up the second force constraining how value can be distributed. To see how this works, consider some distribution of value, \( \pi \), such that for some group \( G \), \( \sum_{i \in G} \pi_i < \nu(G) \). In other words, given \( \pi \), the agents in \( G \) obtain less (in total) than they could create on their own. This means that agents in \( G \) will not rationally participate in generating \( V \), since they can obtain more through other actions they can take unilaterally; i.e., without help from any agent outside \( G \). Thus, if a distribution of value is to be a plausible candidate for the outcome of a strategic interaction, it must be stable in the sense that no group could do better creating value on its own. Thus we require that for all groups \( G \), the distribution of value satisfies

\[
\sum_{i \in G} \pi_i \geq \nu(G). \tag{3}
\]

In the mining example, the mine being purchased by \( B_h \) for $0.3$ million, leaving \( B_t \) to receive $0$, is not a stable distribution of value. \( F \) and \( B_t \) could improve by exchanging the mine for $0.4$ million; indeed, it will turn out that they must receive at least $0.5$ million between them.

Note that (3) requires \( \sum_{i \in N} \pi_i \geq V \). That is, altogether, at least \( V \) must be distributed among the agents. Thus, recalling (2), a feasible and stable distribution of value always distributes exactly \( V \); that is,

\[
\sum_{i \in N} \pi_i = V. \tag{4}
\]

There may be many distributions of value that are both feasible and stable. What determines which distribution ultimately occurs? Both feasibility and the impact of agents’ alternatives have already be taken into account. Thus, the third force determining how value can be distributed is the above-mentioned bargaining process, a catch-all for all the means – apart from the threat of exercising their strategic alternatives as embodied in (3) – that agents might employ to cajole one another into parting with value. As indicated earlier, instead of making particular assumptions
about how bargaining operates and focusing on the implied distribution(s) of value, we consider a distribution of value to be a plausible candidate for an outcome of agents’ interaction if it is simply feasible and stable. Any such distribution is consistent with value creation and competition, but is not reliant on any arbitrary bargaining procedure. If $\pi$ is a feasible and stable distribution of value, we will refer to it as an FSD.\(^8\)

The value agent $i$ obtains generally depends on which FSD one has in mind. From agent $i$’s perspective, if all FSDs result in his receiving positive value, then no matter how bargaining ultimately determines what $i$ actually receives, he prefers participating to not participating. Thus, we will say that agent $i$ appropriates value if $\pi_i > 0$ in every FSD.

### 2.3 Airline example

Three air carriers, EuroAir (E), AmeriAir (A) and ChinaAir (C), consider a strategic alliance to offer customers from Europe return service through the United States to Asia. This service is expected to create $200$ million in additional value for travelers, net of operating costs. Further, market research indicates that E and A can form an independent alliance that generates $100$ million in new value; C and A can form an independent alliance that creates $150$ million in value; and E and C cannot link up without A. This strategic interaction can be summarized by $(N, v)$, where (using letters instead of numbers) $N \equiv \{E, A, C\}$ and $v$, for groups of more than one agent, $\vdots$.

\(^8\)Conditions on $(N, v)$ that are necessary and sufficient for the existence of an FSD are well known; see Osborne and Rubinstein, Sec. 13.3. We assume these conditions are satisfied, which restricts $(N, v)$. When these conditions play a role in our analysis, we point this out.
is given by

<table>
<thead>
<tr>
<th>$G$</th>
<th>$v(G)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>${E, A, C}$</td>
<td>200</td>
</tr>
<tr>
<td>${E, A}$</td>
<td>100</td>
</tr>
<tr>
<td>${A, C}$</td>
<td>150</td>
</tr>
<tr>
<td>${E, C}$</td>
<td>0</td>
</tr>
</tbody>
</table>

Note that $v$ satisfies (1).

In this example, (2) says that any distribution of value must distribute at most $200$ million. The stability condition, (3), says that no distribution can leave any group with a total payoff smaller than that group could achieve on its own.

### 3 Value appropriation

Our objective is to determine circumstances in which agent $i$ appropriates value in the sense that in every FSD, $\pi_i > 0$. Consider the collection of all FSDs. The associated values of $\pi_i$ can be shown to form a closed, nonnegative interval in the real line, say $[\pi_i^{\min}, \pi_i^{\max}]$. That is, given value creation possibilities as described by $v$, there are FSDs in which $i$ receives exactly $\pi_i^{\min}$, exactly $\pi_i^{\max}$, and every amount in between. In the mining example, it turns out that $F$ must receive at least $.5$ million and no more than $1$ million; so, $\pi_F^{\min} = .5$ and $\pi_F^{\max} = 1$.

Since saying $i$ appropriates value is equivalent to saying $\pi_i^{\min} > 0$, we can identify conditions necessary and/or sufficient for $i$ to appropriate value by identifying the corresponding conditions for $\pi_i^{\min} > 0$. This is the approach we follow below.

#### 3.1 $i$’s Marginal product

A familiar notion in strategy is that whether an agent can appropriate value is related to how much value that agent creates. For example, Saloner et. al. (2001, p. 39) say, “A firm achieves superior performance only if it can provide products or services
that customers will pay more for than it costs the firm to provide them.” Our first two results are directly related to this idea.

In our model, \( v \) describes the possibilities for value creation. Given this, there are many ways we might measure the value created by agent \( i \). Specifically, for any group \( G \) including \( i \), the additional value created, in comparison to if \( i \) is not included, is

\[
v(G) - v(G_{-i}).
\]

We show that one of these measures of \( i \)'s contribution constrains \( i \)'s value appropriation possibilities. Define agent \( i \)'s marginal product, \( mp_i \), to be the difference between \( V \) and the value that could be created by all agents other than \( i \), \( v(N_{-i}) \), i.e.,

\[
mp_i \equiv V - v(N_{-i}).
\]

This brings us to our first, and simplest, result.

**Proposition 1** Given \((N, v)\), agent \( i \) cannot appropriate more than his marginal product:

\[
\pi_i^{\text{max}} \leq mp_i.
\]

Intuitively, if the distribution of value results in agent \( i \) receiving more than his marginal product, there is so little left to distribute among the other agents that they would find creating value on their own, i.e., abandoning \( i \), preferable. Thus, if \( \pi_i > \pi_i^{\text{max}} \), \( \pi \) is not a stable distribution of value.

Elementary as Proposition 1 is, it immediately implies that a necessary condition for \( i \) to appropriate value is that \( i \) have a positive marginal product.

**Corollary 1** Given \((N, v)\), positive marginal product is a necessary condition for value appropriation. That is,

1. \( \pi_i^{\text{min}} > 0 \Rightarrow mp_i > 0 \); and
2. \( mp_i = 0 \Rightarrow \pi_i^{\text{min}} = 0 \).
Corollary 1 says that for $i$ to appropriate value, value creation possibilities must allow $i$ to increase value when included in $N_{-i}$. Note that when this is not the case, i.e. $mp_i = 0$, $i$ cannot appropriate value even if he adds value to every other group. Therefore, for purposes of analyzing value appropriation, the most relevant definition of value creation is positive marginal product.\textsuperscript{9}

Why does $i$ having a positive marginal product not guarantee value appropriation? That is, why is it that $i$ can add value, yet not be assured of receiving any of it? The answer is that the ability to add value does not imply that one is any more or less scarce than the others with whom one interacts. To see this, consider the simplest possible case: a strategic interaction with $n = 2$. In this situation both agents, acting together, create value $V$, and either, on his own, creates nothing. Thus, $mp_i = V$ for both agents. This is a classic “pure bargaining” problem: with only $V$ to distribute, one or both agents must obtain less than their marginal product. Indeed, receiving as little as 0 or as much as $V$ is a possibility for either.\textsuperscript{10} Here, both agents are equally scarce in the sense that each needs the other as much as the other needs him.

More generally, having a positive marginal product means that there is value that $i$ might hope to receive. But it is the nature of $i$’s (and, as we will show, other agents’) strategic alternatives that determine whether he will succeed in doing so.

\textsuperscript{9}Ostrov (1980, 1984) and Makowski and Ostrov (1995) explore the connection between perfect competition and individual marginal products in general equilibrium settings.

\textsuperscript{10}To see this, note that when $n = 2$, (3) implies $\pi_1 \geq 0$, $\pi_2 \geq 0$, and

$$\pi_1 + \pi_2 = V.$$ 

Thus, every FSD is of the form $(\pi_1, V - \pi_1)$, where $0 \leq \pi_1 \leq V.$
**Airline example, continued.** In the airline example, marginal products are:

<table>
<thead>
<tr>
<th>Firm</th>
<th>$mp_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$V - v(N_{-A}) = 200$</td>
</tr>
<tr>
<td>E</td>
<td>$V - v(N_{-E}) = 50$</td>
</tr>
<tr>
<td>C</td>
<td>$V - v(N_{-C}) = 100$</td>
</tr>
</tbody>
</table>

It is easy to check that the bounds on what agent’s can receive in an FSD are:

<table>
<thead>
<tr>
<th>Firm</th>
<th>$\pi^\text{min}_i$</th>
<th>$\pi^\text{max}_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>50</td>
<td>200</td>
</tr>
<tr>
<td>E</td>
<td>0</td>
<td>50</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>100</td>
</tr>
</tbody>
</table>

In this example, all firms satisfy the necessary condition for value appropriation, but only firm A is assured of appropriating value.

### 3.2 Minimum residual

In the preceding section we showed that an agent’s being able to create value, in the sense of having a positive marginal product, is required for the agent to appropriate value, but is generally not sufficient. Here, we develop a simple condition that, while not necessary for an agent to appropriate value, does, if satisfied, guarantee $i$ will appropriate value. To see how this works, recall (4), i.e., that any FSD must distribute exactly $V$. Since no agent can appropriate more than his marginal product, if distributing $mp_j$ to each agent $j$ other than $i$ leaves part of $V$ undistributed, $i$ must receive what is left, i.e., $\pi^\text{min}_i$ is at least as large as this undistributed residual. More formally, define $i$’s *minimum residual* by

$$mr_i \equiv V - \sum_{j \in N_{-i}} mp_j;$$

observe that $mr_i$ might be negative.\(^{11}\)

---

\(^{11}\)If an FSD is to exist, it is necessary that $mp_i \geq mr_i$. Otherwise, by Proposition 1, it is not possible to distribute the residual to $i$. 

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Proposition 2 Given \((N, v)\), \(mr_i \leq \pi_i^{\text{min}}\).

Corollary 2 Given \((N, v)\), positive minimum residual is a sufficient condition for value appropriation. That is,

1. \((mr_i > 0) \Rightarrow (\pi_i^{\text{min}} > 0)\), and

2. \((\pi_i^{\text{min}} = 0) \Rightarrow (mr_i \leq 0)\).

Another way to interpret Corollary 2 follows from the fact that \(i\) has a positive residual if \(i\)’s marginal product is sufficiently large relative to others’ marginal products. Precisely, observe that positive minimum residual is equivalent to

\[
mp_i > \sum_{j \in N - i} mp_j - v(N - i).
\] (5)

The preceding results identify the relationship between value creation and value appropriation. Corollary 1 describes the precise sense in which an agent creates too little value to guarantee appropriation. Conversely, Corollary 2 clarifies what it means for an agent to add value large enough to make appropriation a certainty.

The argument supporting Proposition 2 reveals a subtle feature of competition. Because abandoning agent \(j \neq i\) is attractive to all players other than \(j\) if \(\pi_j > mp_j\), agent \(j\) can never receive more than his marginal product; this is just the argument behind Proposition 1. Therefore, there is no way to distribute less to \(i\) than his minimum residual without making abandoning some other agent attractive. Thus, \(i\)’s guarantee of value appropriation is due to a blend of his value creation possibilities and alternatives available to \(i\) and to the other agents.

Why is \(i\) having a positive minimum residual not required for \(i\) to be certain of value appropriation? There are various reasons, all related to the fact that the situation minimum residual describes can be a very incomplete picture of the competitive forces at work. First, the calculation of minimum residual supposes that each players other than \(i\) receives his marginal product. However, there may be no FSD with this
property. For example, it is possible that $\pi_j^{\text{max}} = mp_j$ for all $j$, but stability precludes $\pi_j = \pi_j^{\text{max}}$ simultaneously for all $j \neq i$. Thus, the least that is left over for $i$ may be larger than his minimum residual (which is why it is called \textit{minimum} residual), in which case $i$ might appropriate value even though $mr_i < 0$. Second, minimum residual is determined solely by the value created in groups of size $n$ and $n - 1$. But it may be that there are smaller groups, including $i$, which can create a great deal of value on their own. If such groups exist, stability may limit how little $i$ can receive.

### 3.2.1 Airline example, continued

In the airline example,

<table>
<thead>
<tr>
<th>Firm</th>
<th>$mr_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$V - (m_E + m_C) = 200 - (50 + 100) = 50$</td>
</tr>
<tr>
<td>E</td>
<td>$V - (m_A + m_C) = 0$</td>
</tr>
<tr>
<td>C</td>
<td>$V - (m_A + m_E) = 0$</td>
</tr>
</tbody>
</table>

By Proposition 2, firm A must appropriate at least 50 in value, but the other firms’ minimum residuals are too small to guarantee appropriation.

### 3.3 Minimum value

Propositions 1 and 2 provide some insight into the determinants of value appropriation, and sometimes provide a definitive answer about whether agent $i$ can appropriate value; “yes” if value creation is large enough ($mr_i > 0$) and “no” if value creation is too small ($mp_i = 0$). However, in many cases of interest value creation is not so extreme (i.e., $mr_i \leq 0$ and $mp_i > 0$). When this occurs, Propositions 1 and 2 do not settle the question of whether an agent appropriates value. We now provide a more elaborate, but complete, description of the features of the strategic interaction that are both necessary and sufficient for value appropriation.

Suppose that $\pi$ is an FSD in which $i$ does not appropriate value, i.e., $\pi_i = 0$. 

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Since $\pi_i = 0$, the feasibility and stability conditions $\pi$ must satisfy reduce to
\[
\sum_{j \in N_{-i}} \pi_j \leq V, \tag{6}
\]
and\footnote{Note that (1) implies that if $i$ is not included in $G$, $v(G_{+i}) \geq v(G)$. Thus, if the inequalities in (7) are satisfied, the other stability conditions,

$$
\text{for all } G \text{ not including } i, \sum_{j \in G} \pi_j \geq v(G),
$$

are automatically satisfied.}

for all $G$ not including $i$, \[
\sum_{j \in G} \pi_j \geq v(G_{+i}). \tag{7}
\]

Condition (6), together with (7) for $G = N_{-i}$, implies that value $V$ must be distributed solely to members of $N_{-i}$. The rest of the inequalities in (7) require that value be distributed among groups not including $i$ in a way that no such group finds creating value on its own, but adopting $i$, attractive despite $\pi_i = 0$. That is, since $\pi_i = 0$, should there be some group $G$, not including $i$, for which
\[
\sum_{i \in G} \pi_i < v(G_{+i}),
\]
every member of the group could be made better off by offering $i$ a portion of $v(G_{+i}) - \sum_{i \in G} \pi_i$, which $i$ would prefer to $\pi_i = 0$, and dividing the rest among the members of $G$.

If it is possible to distribute $V$ among agents in $N_{-i}$ and satisfy (6) and (7), then $\pi_{i_{\text{min}}} = 0$, and $i$ cannot appropriate value. To develop the result, define $i$’s minimum value by
\[
mv_i \equiv \min x
\]
subject to
\[
x \geq 0,
\]
and there exists $\pi$ such that both

$$\sum_{j \in N_{-i}} \pi_j + x \leq V;$$

(8)

and

for all $G$ not including $i$, $\sum_{j \in G} \pi_j + x \geq v(G_{+i})$.

(9)

The minimum value is the smallest payoff $i$ can receive without the stability of the distribution of value being upset by some group finding it attractive to adopt $i$ and produce on its own. Since we have assumed at least one stable distribution exists, $mv_i$ is well-defined and nonnegative.$^{13}$

**Proposition 3** Given $(N, v)$,

$$(mv_i = 0) \Leftrightarrow \left( \pi_i^{\text{min}} = 0 \right).$$

**Corollary 3** Positive minimum value is necessary and sufficient for value appropriation, i.e.,

$$(mv_i > 0) \Leftrightarrow \left( \pi_i^{\text{min}} > 0 \right).$$

Proposition 3 exposes the two opposing forces determining whether $i$ appropriates value. To see what these forces are, consider (8) and (9) assuming $i$ cannot appropriate value, i.e., when $mv_i = 0$. In this case, there is an FSD, $\pi$, satisfying both

$$\sum_{j \in N_{-i}} \pi_j \leq V;$$

and

for all $G$ not including $i$, $\sum_{j \in G} \pi_j \geq v(G_{+i})$.

$^{13}$This minimization problem is not intended to be descriptive of any behavior of agents in the strategic interaction. Instead, it is simply an analytical device facilitating our determining the least $i$ can obtain in an FSD. This is analogous to the familiar process of analyzing an efficient allocation of resources to discover features of a competitive equilibrium.
In order for these constraints to be satisfied, the available value, $V$, must be distributed among agents other than $i$ in a way that, despite $i$’s receiving 0 (and, thus, being willing to join any group for a trivial share of the value created), leaves agents in the group content not to adopt $i$ and create value independently. The more productive are groups including $i$, the more appealing adopting $i$ is for those groups to adopt $i$, and, therefore, the more value must they receive in order for a distribution in which $i$ appropriates nothing to be stable. On the other hand, the more value there is to distribute, the easier it is to meet the aforementioned constraints. Below, in Subsection 3.5, we show that if value creation possibilities have a particular structure, the latter effect is always strong enough to preclude value appropriation.\(^{14}\)

To see this tension more clearly, consider the following example: suppose

$$v(\{1, 2\}) = v(\{1, 3\}) = v(\{2, 3\}) = 2 \text{ and } V = 4.$$ 

Then, constraints characterizing an FSD, assuming $\pi_1 = 0$, are

$$0 + \pi_2 \geq 2,$$

$$0 + \pi_3 \geq 2,$$

$$\pi_2 + \pi_3 \geq 2,$$

and

$$0 + \pi_2 + \pi_3 = 4.$$

These constraints can be satisfied, i.e., $\pi_1^{\text{min}} = 0$. Indeed, there is exactly one way that all value can be distributed among the among the other players and accomplish this: $\pi_2 = \pi_3 = 2$. But if, instead, $V = 3$, there is not enough value to distribute to others in a way that leaves them content not to adopt $i$ if they can attract him for a small share of the value created. For example, if agent 3 obtains $\pi_3 = 1.5$, then the

\(^{14}\) The proof of Proposition 3 shows that minimum value is a lower bound on what $i$ can receive in any stable distribution of value, i.e., $\pi_i^{\text{min}} \geq mv_i$. In MacDonald and Ryall (2001) we show that when $mv_i > 0$, $\pi_i^{\text{min}} > mv_i$ may occur.
most agent 2 can receive is $\pi_2 = 1.5$. But then agent 2 could improve by adopting player 1, creating value 2, and retaining all but $\varepsilon$ of it; player 1 would prefer this too. In fact, when $V = 3$, the unique FSD is $\pi = (1, 1, 1)$, i.e., $\pi_1^{\text{min}} = \pi_1^{\text{max}} = 1$.

### 3.3.1 Airline example, continued

First, consider $mv_E$, defined by

$$mv_E \equiv \min x$$

subject to

$$\pi_A + \pi_C + x \leq 200,$$
$$\pi_A + x \geq 100,$$
$$\pi_C + x \geq 0,$$
and $x \geq 0$.

It is easy to check that see that $x = 0$ is a solution with $\pi_A \in [100, 200]$ and $\pi_C = 200 - \pi_A$. Thus, $mv_E = 0$. Similarly, $mv_C = 0$. Now, consider $mv_A$. The problem is

$$mv_A \equiv \min x$$

subject to

$$\pi_E + \pi_C + x \leq 200,$$
$$\pi_E + x \geq 100,$$
$$\pi_C + x \geq 150,$$
and $x \geq 0$.

It is straightforward to show that $mv_A = 50$.

Notice that, in this example, each firm’s minimum value is equal to its minimum residual. However, $mv_i > 0$, unlike $mr_i > 0$, is both necessary and sufficient for
appropriation. Therefore, $A$ appropriates with certainty, which followed from Proposition 2 and $mr_A > 50$. But $E$ and $C$ are in the position $mr_i \leq 0$ and $mp_i > 0$, so Propositions 1 and 2 do not settle whether they may appropriate. Proposition 3 and $mv_E = mv_C = 0$ imply $\pi^\text{min}_E = \pi^\text{min}_C = 0$, i.e., no appropriation for $E$ or $C$.

### 3.4 Effective competition

Proposition 3 gives a complete description of the features of value creation which, together with competition, result in $i$’s appropriating value. Our final result is a closely-related one, which, while less informative in the sense of simply being a collection of jointly necessary conditions for value appropriation, is helpful for understanding the competitive forces at work when $i$ appropriates value.

**Proposition 4** Given $(N, v)$, if $\pi^\text{min}_i > 0$, for any FSD, $\pi$, in which $\pi_i = \pi^\text{min}_i$, there is a collection of groups, $\mathbf{G}$, with all of the following features:

1. Agent $i$ is not a member of any group in $\mathbf{G}$;

2. Agent $i$ adds value to every group in $\mathbf{G}$, i.e.,

   $$v(G_{+i}) - v(G) > 0;$$

3. $G$ includes at least two groups other than $N_{-i}$;

4. For every group in $\mathbf{G}$,

   $$\sum_{j \in G} \pi_j + \pi^\text{min}_i = v(G_{+i});$$

   and

5. No agent is a member of every group in $\mathbf{G}$.

Proposition 4 identifies the kind of competition for $i$ that ensures $i$ appropriates value. Suppose $i$ appropriates value, i.e., $\pi^\text{min}_i > 0$, and consider some FSD in which $i$ obtains
the least consistent with competition, i.e., \( \pi_i = \pi_i^{\text{min}} \). According to Proposition 4, given this FSD, there must be a collection of groups, \( G \), having certain characteristics. First, the groups do not include \( i \); thus any of the groups may consider adopting \( i \). Second, adopting \( i \) is potentially worthwhile in that \( i \) does add value to each group. Third, there are at least two such groups. Fourth, if \( i \) were to receive any less than \( \pi_i^{\text{min}} \), adopting \( i \) would immediately be attractive to each of these groups. Finally, none of the other agents are present in all the groups.

These conditions describe the sort of competition for agent \( i \) that must operate when \( i \) appropriates value, but does so minimally. Were \( i \) to receive any less, some group would, by adopting \( i \), undermine the stability of the distribution of value. Moreover, there are at least two such groups, so that were \( i \) to receive less, “competition” would arise for the opportunity to adopt \( i \). Agent \( i \) is “central” in the sense that there is no other agent who also belongs to all the competing groups. This last feature requires some explanation. If agent \( j \) belongs to every group competing for \( i \), the situation is as if \( i \) and \( j \) are in a pure bargaining situation. Indeed, if there were such an agent \( j \), then the distribution of value in which (for small \( \varepsilon \) \( i \) receives \( \pi_i^{\text{min}} - \varepsilon \), \( j \) receives \( \pi_j + \varepsilon \), and the distribution to all other players is unchanged, is also an FSD. But, this implies that \( \pi_i^{\text{min}} \) is not actually the lowest possible level of appropriation for \( i \).\(^{15}\)

### 3.4.1 Airline example, continued

In the airline example, \( \pi_i^{\text{min}} = 50 \), and the unique distribution of value consistent with \( A \) receiving so little is \( (\pi_A, \pi_C, \pi_E) = (50, 100, 50) \). The feasibility and stability conditions, (2) and (3) can be written

\[
\frac{\pi_A}{50} + \frac{\pi_C}{100} + \frac{\pi_E}{50} = 200,
\]

\(^{15}\text{The proof of this result appears in MacDonald and Ryall, (2001). The argument shows that whenever any of conditions 1-5 fail, it is possible to construct an FSD in which } i \text{ receives less than } \pi_i^{\text{min}}, \text{ contradicting the fact that } \pi_i^{\text{min}} \text{ is the least value } i \text{ can receive in any FSD.}\)
\[
\begin{align*}
\pi_A^{50} + \pi_C^{100} &\geq 150, \\
\pi_A^{50} + \pi_E^{100} &\geq 100,
\end{align*}
\]
and
\[
\pi_C^{100} + \pi_E^{50} \geq 0.
\]
In this example, the competing groups \( \mathbf{G} \) referred to in Proposition 4 are simply \( \{C\} \) and \( \{E\} \). Observe that (i) Neither group includes \( A \); (ii) \( A \) adds value to both groups; (iii) There is more than one group; (iv) Since the first two inequalities hold as equalities, if \( A \) received less than 50 either group would prefer to adopt \( A \); and finally, (v) Neither \( C \) nor \( E \) is a member of both groups.

### 3.5 Abundant value limits value appropriation

Earlier we mentioned that when value creation opportunities are such that value is abundant, competition for agent \( i \) is tempered, making value appropriation more difficult for \( i \). Under certain conditions, this effect has a particularly strong impact, and in fact, eliminates the possibility of value appropriation.

To see this, consider any two groups, \( G \) and \( G' \), neither including \( i \), and with all agents in \( G \) also included in \( G' \). If
\[
v(G_{+i}) - v(G') \geq v(G_{+i}) - v(G),
\]
v is called **supermodular**; see Topkis (1998).

Condition (1) requires value creation to be weakly increasing in group size. Supermodularity is a much stronger condition. When \( v \) is supermodular, larger groups not only create more value, but each agent’s value creation possibility is increasing with group size. (If \( v \) were simply a function of one variable, say the number of agents in the group, (1) amounts to monotonicity, i.e., positive slope, and supermodularity to convexity, i.e., increasing slope.)

Strategic interactions displaying complementarities imply supermodular \( v \); e.g., strong network externalities. Also, if participation by every agent is required for
value creation, i.e., \( v(G) = 0 \) unless \( G = N \), \( v \) is supermodular; interactions of this kind are the general case of “pure bargaining”. Another example is the situation in which the only group to which \( i \) adds value is \( N_{-i} \), implying \( v(G) = 0 \) unless \( G = N_{-i} \).

When \( v \) is supermodular, there is a sense in which the value created by the all agents, \( V \), is especially large relative to the value created by smaller groups. The next result, a particular version of a well-known proposition due to Shapley (1971), formalizes this idea.

**Proposition 5** (Shapley) If \((N, v)\) is supermodular, \( \pi_i^{\min} = 0 \).

**Corollary 4** A necessary condition for value appropriation is that \( v \) not be supermodular.

## 4 Governance

Williamson (1999, p. 1087) describes the governance perspective as an analytical framework in which alternative modes of governance arise as firms’ optimal responses to variation in transaction costs. While he also suggests that his framework is useful for analyzing Strategy, he does not explain how governance influences whether/how value created results in value being appropriated. In this section, we employ a very simple application of our theory to provide some insight into the circumstances under which well-chosen organizational structure yields value appropriation.

The structure we impose on agents’ value creation possibilities describes a competitive interaction in which governance influences value creation. Suppose firms might employ two types of organizational practices: \((i)\) industry “best practices” – techniques that can be understood and implemented by any firm in the industry, and that yield the “normal” rate of return (normalized to 0); and \((ii)\) “proprietary practices” – non-imitable, value creating, methods known only to one firm. These methods might, for example, have been discovered serendipitously through the firm
finding itself in unfamiliar circumstances and making unusual choices with surprising effects.

To keep the analysis as simple as possible, suppose there are two firms, \( F_1 \) and \( F_2 \), paired with their own management teams, \( M_1 \) and \( M_2 \). Management operates like a public good within the firm, so \( M_1 \) and \( M_2 \) are more productive when managing more resources; see Rosen (1982). We will assume \( F_2 \) is larger than \( F_1 \), but this will not, until later, play any role.

Suppose \( F_1 \), but not \( F_2 \), has a proprietary organizational practice, allowing its management to deliver a supra-normal return on \( F_1 \)'s assets; i.e., \( F_1 \) has learned something specific about how \( M_1 \) and the firm’s business interact. A structure of value creation consistent with this is:

<table>
<thead>
<tr>
<th>( G )</th>
<th>( v(G) )</th>
<th>( G )</th>
<th>( v(G) )</th>
<th>( G )</th>
<th>( v(G) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>{( F_1, F_2, M_1, M_2 } )</td>
<td>1</td>
<td>{( F_2, M_1, M_2 } )</td>
<td>0</td>
<td>{( F_2, M_1 } )</td>
<td>0</td>
</tr>
<tr>
<td>{( F_1, F_2, M_1 ) }</td>
<td>1</td>
<td>{( F_1, F_2 } )</td>
<td>0</td>
<td>{( F_2, M_2 } )</td>
<td>0</td>
</tr>
<tr>
<td>{( F_1, F_2, M_2 } )</td>
<td>0</td>
<td>{( F_1, M_1 } )</td>
<td>1</td>
<td>{( M_1, M_2 } )</td>
<td>0</td>
</tr>
<tr>
<td>{( F_1, M_1, M_2 } )</td>
<td>1</td>
<td>{( F_1, M_2 } )</td>
<td>0</td>
<td>{( F_1 }, { F_2 }, { M_1 }, { M_2 } }</td>
<td>0</td>
</tr>
</tbody>
</table>

That is, extra value is created when, and only when, \( F_1 \)'s assets are managed by \( M_1 \). Adding more management or resources – e.g., including either \( M_2 \) or \( F_2 \) – merely adds the normal rate of return.

These value creation opportunities are supermodular. Thus, from Proposition 5, \( \pi^\text{min}_{F_1} = 0 \); specifically, \( (0, 0, 1, 0) \) is an FSD. Despite \( F_1 \)'s non-imitable and productivity-enhancing know-how, \( F_1 \) is unable to guarantee that it will receive a positive share of the value created by its unique governance ability. Another way to see this is to note that \( F_1 \)'s unique ability is how to motivate \( M_1 \) to manage \( F_1 \)'s resources in a superior manner, which is not valuable without \( M_1 \). Effectively, this puts \( F_1 \) in a pure bargaining position with \( M_1 \), thereby preventing \( F_1 \) from being assured a share of the supra-normal value it creates by employing \( M_1 \).

This suggests \( F_1 \) might appropriate value if its proprietary know-how, instead of
being specific to $M_1$ (i.e., those in charge of $F_1$’s assets when the discovery was made), is applicable to any management team managing $F_1$’s assets. A structure of value creation consistent with this setup is:

<table>
<thead>
<tr>
<th>$G$</th>
<th>$v(G)$</th>
<th>$G$</th>
<th>$v(G)$</th>
<th>$G$</th>
<th>$v(G)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>{F_1, F_2, M_1, M_2}</td>
<td>1</td>
<td>{F_2, M_1, M_2}</td>
<td>0</td>
<td>{F_2, M_1}</td>
<td>0</td>
</tr>
<tr>
<td>{F_1, F_2, M_1}</td>
<td>1</td>
<td>{F_1, F_2}</td>
<td>0</td>
<td>{F_2, M_2}</td>
<td>0</td>
</tr>
<tr>
<td>{F_1, F_2, M_2}</td>
<td>1</td>
<td>{F_1, M_1}</td>
<td>1</td>
<td>{M_1, M_2}</td>
<td>0</td>
</tr>
<tr>
<td>{F_1, M_1, M_2}</td>
<td>1</td>
<td>{F_1, M_2}</td>
<td>1</td>
<td>{F_1}, {F_2}, {M_1}, {M_2}</td>
<td>0</td>
</tr>
</tbody>
</table>

As expected, in this situation $M_2$’s ability to respond to $F_1$’s superior practices eliminates $M_1$’s ability to appropriate. In fact,

$$mp_{M_1} = mp_{M_2} = 0.$$  

Therefore, using Corollary 1 and the stability condition

$$\pi_{F_1} + \pi_{M_1} \geq 1,$$

the only FSD is $\pi = (1, 0, 0, 0)$; i.e., $\pi_{F_1}^{\text{min}} = 1$. Consistent with Proposition 4, $M_1$ and $M_2$ form a group competing to manage $F_1$’s assets.

Thus far, we have focused on how the cross-management-team generalizability of $F_1$’s unique and proprietary know-how impacts competition between $M_1$ and $M_2$ to manage $F_1$’s assets; $F_2$ has played a passive role. What happens if, for example, both firms have unique experiences leading each to its own proprietary know-how? We explore effectively the same two cases. In both, $F_1$’s know-how is generalizable across management teams. But in the first, $F_2$’s proprietary practice applies only to $M_2$; in the second, $F_2$’s know how also generalizes.

Value creation possibilities consistent with the first case are:
\begin{center}
\begin{tabular}{|c|c|c|c|c|}
\hline
$G$ & $v(G)$ & $G$ & $v(G)$ & $G$ & $v(G)$ \\
\hline
$\{F_1, F_2, M_1, M_2\}$ & 3 & $\{F_2, M_1, M_2\}$ & 2 & $\{F_2, M_1\}$ & 0 \\
$\{F_1, F_2, M_1\}$ & 1 & $\{F_1, F_2\}$ & 0 & $\{F_2, M_2\}$ & 2 \\
$\{F_1, F_2, M_2\}$ & 2 & $\{F_1, M_1\}$ & 1 & $\{M_1, M_2\}$ & 0 \\
$\{F_1, M_1, M_2\}$ & 1 & $\{F_1, M_2\}$ & 1 & $\{F_1, \{F_2\}, \{M_1\}, \{M_2\}\}$ & 0 \\
\hline
\end{tabular}
\end{center}

Note that owing to the public good nature of managing, $F_2$’s larger size now plays a role, e.g., $v(\{F_2, M_2\}) = 2 > 1 = v(\{F_1, M_1\})$.

In this case, $\pi^\text{min}_{F_1} = 0$. To see this, note that $\pi = (0, 1, 1, 1)$ meets all of the distribution constraints implied by stability. This may seem somewhat counter-intuitive since $F_2$’s know-how has no direct bearing on $F_1$’s operations, and $F_2$’s know-how does not make $M_1$’s services more productive. However, even though $F_2$’s know-how applies only to its own team, its existence has effectively removed the competition between $M_1$ and $M_2$ to manage $F_1$’s assets, ending $F_1$’s ability to appropriate. This occurs because $F_2$’s proprietary practice and larger size implies $M_2$ is much more valuable when employed by $F_2$.

Now suppose $F_2$’s know how also generalizes. Value creation possibilities consistent with this case are:

\begin{center}
\begin{tabular}{|c|c|c|c|c|c|}
\hline
$G$ & $v(G)$ & $G$ & $v(G)$ & $G$ & $v(G)$ \\
\hline
$\{F_1, F_2, M_1, M_2\}$ & 3 & $\{F_2, M_1, M_2\}$ & 2 & $\{F_2, M_1\}$ & 2 \\
$\{F_1, F_2, M_1\}$ & 2 & $\{F_1, F_2\}$ & 0 & $\{F_2, M_2\}$ & 2 \\
$\{F_1, F_2, M_2\}$ & 2 & $\{F_1, M_1\}$ & 1 & $\{M_1, M_2\}$ & 0 \\
$\{F_1, M_1, M_2\}$ & 1 & $\{F_1, M_2\}$ & 1 & $\{F_1, \{F_2\}, \{M_1\}, \{M_2\}\}$ & 0 \\
\hline
\end{tabular}
\end{center}

This interaction is not supermodular.\textsuperscript{16} Also, Propositions 1 and 2 do not provide a

\textsuperscript{16}To see this, observe that

\[ v(\{F_1, F_2, M_2\}) - v(\{F_1, M_2\}) = 1 \quad \text{and} \quad v(\{F_2, M_2\}) - v(\{M_2\}) = 2, \]

26
definitive answer about which agents, if any, appropriate. That is:

\[
\begin{array}{|c|c|c|}
\hline
 & mr_i & mp_i \\
\hline
F_1 & -1 & 1 \\
\hline
F_2 & 0 & 2 \\
\hline
M_1 & -1 & 1 \\
\hline
M_2 & -1 & 1 \\
\hline
\end{array}
\]

In this case the necessary condition for appropriation is met for all agents, but the sufficient condition is met for none. Minimum values, as per Proposition 3 are required to determine which, if any, agents appropriate. Carrying out the calculations:

\[
\begin{array}{|c|c|c|c|}
\hline
i & mv_i & i & mv_i \\
\hline
F_1 & 0 & M_1 & 0 \\
\hline
F_2 & 1 & M_2 & 0 \\
\hline
\end{array}
\]

Since \( F_2 \) is larger, and its know-how generalizable, competition between \( M_1 \) and \( M_2 \) to manage \( F_2 \)'s greater assets allows \( F_2 \) to appropriate at least 1 of the available 3 units of value. But \( F_2 \) is only assured a part of this value. While it is true that \( M_1 \) and \( M_2 \) compete for to generate value with \( F_2 \), the generalizability of \( F_1 \)'s know-how gives \( M_1 \) and \( M_2 \) a valuable, albeit less valuable, alternative.

Finally, what is the source of \( F_2 \)'s advantage? Is it \( F_2 \)'s proprietary practices? Or is it \( F_2 \)'s larger resource base combined with the public good nature of managing? The answer is neither: \( F_2 \) benefits from the subtle interaction among its governance practices, its greater assets, and competition, all of which are captured by the assumed structure of \( v \), the requirement that the distribution of value be feasible and stable, and the implications for minimum value.

Therefore,

\[ v(\{F_1, F_2, M_2\}) - v(\{F_1, M_2\}) < v(\{F_2, M_2\}) - v(\{M_2\}). \]
References


5 Proofs

5.1 Proposition 1

If $i$ receives $\pi_i > mp_i$, we have

$$\pi_i > V - v \left( N_{-i} \right).$$

Using (4), it follows that

$$\pi_i > \sum_{j \in N} \pi_j - v( N_{-i} ),$$

or

$$\sum_{j \in N_{-i}} \pi_j < v( N_{-i} ).$$

That is, group $N_{-i}$ could improve by abandoning $i$ and creating value on their own. Thus, if $\pi_i > mp_i$, the distribution of value must violate (3).

5.2 Proposition 2

Since the definition of a stable distribution implies $\pi_{i \min} \geq 0$, if

$$mr_i < 0,$$

the result follows trivially. Thus, suppose

$$mr_i > \pi_{i \min} \geq 0.$$

Then,

$$V > \sum_{j \in N_{-i}} \left( mp_j + \pi_{i \min} \right),$$

and using Proposition 1,

$$\sum_{j \in N_{-i}} \left( mp_j + \pi_{i \min} \right) \geq \sum_{j \in N_{-i}} \pi_{j \max} + \pi_{i \min}.$$ 

And since $\pi_{i \max} \geq \pi_i$ for every FSD,

$$\sum_{j \in N_{-i}} \pi_{j \max} + \pi_{i \min} \geq \sum_{j \in N_{-i}} \pi_{j} + \pi_{i \min}.$$
altogether

\[ V > \sum_{j \in N \setminus i} \pi_j + \pi_i^\text{min}. \]

But since every stable distribution must satisfy (4), no stable allocation can result in \(i\)'s receiving only \(\pi_i^\text{min}\), contradicting the definition of \(\pi_i^\text{min}\).

### 5.3 Proposition 3

Note that \(\pi_i^\text{min}\) can be defined in a manner similar to the definition of \(mv_i\). That is,

\[ \pi_i^\text{min} \equiv \min x \]

subject to

\[ x \geq 0, \]

and there exists \(\pi\) such that all of

\[ \sum_{j \in N \setminus i} \pi_j + x \leq V, \]

for all \(G\) not including \(i\), \(\sum_{j \in G} \pi_j \geq v(G)\),

and

for all \(G\) not including \(i\), \(\sum_{j \in G} \pi_j + x \geq v(G_{\setminus i})\).

Since \(\pi_i^\text{min}\) is the solution to the same minimization problem as \(mv_i\), but with additional constraints, it is immediate that \(\pi_i^\text{min} \geq mv_i\). Thus, since \(mv_i \geq 0\), \(\pi_i^\text{min} = 0\) implies \(mv_i = 0\). Next, suppose \(mv_i = 0\). Then there exists \(\pi\) satisfying

for all \(G\) not including \(i\), \(\sum_{j \in G} \pi_j \geq v(G_{\setminus i})\).

Using (1)

\[ \forall G \text{ not including } i, \sum_{j \in G} \pi_j \geq v(G), \]

i.e., the additional constraints in the minimization problem defining \(\pi_i^\text{min}\) are also satisfied by this same \(\pi\). Thus, \(mv_i = 0 = \pi_i^\text{min}\).