Peer Admission Control in a Real-Time P2P Video Distribution Platform: Definition and Performance Evaluation

Giuseppe Incarbone, Giovanni Schembra
Dipartimento di Ingegneria Informatica e delle Telecomunicazioni - University of Catania
V.le A. Doria, 6 - 95125 Catania - ITALY
E-mail: (giuseppe.incarbone, schembra)@diit.unict.it

Abstract—Performance analysis frameworks for P2P networks are in great demand by Internet Service Providers (ISP) and telecommunications companies in order to redesign their systems so as to support the new paradigm. The target of this paper is to propose an analytical model of a real-time video transmission system using P2P to achieve multipoint communication on the current Internet. The system considered is defined in such a way as to maximize the number of users, while guaranteeing a given quality of service level in terms of the maximum number of hops connecting them with the video source.

Keywords: P2P, video transmission, real-time, Markov models, performance evaluation

1. Introduction

Most of the multimedia applications today are being shown great interest by both Internet service providers and users. In the first generation of such systems the most widely used approach is unicast streaming. However, as unicast streaming requires a separate streaming bandwidth for each client, the multimedia source and network bandwidth resources consumed will inevitably grow linearly with the user population. This approach is therefore not scalable with the number of users.

A widely accepted solution to this problem is application of peer-to-peer (P2P). A P2P communication infrastructure is built upon an overlay network whose topology is independent of the underlying physical network. According to the P2P approach, peers that participate in the same group organize themselves into an application layer multicast tree. A peer in the application layer multicast tree receives data from its parent, then duplicates and forwards them to its children. However, although P2P technology gives novel opportunities to define an efficient multimedia streaming application, at the same time it involves a set of technical challenges and issues due to its dynamic and heterogeneous nature. Even though the problem has already been studied in the literature [1-3], work on P2P media streaming systems is still in the early stages. At least the following issues have to be considered in designing a P2P-based multimedia communications platform: 1) peers are more unreliable than routers mainly because they can join and leave at any time causing service interruption for their descendants; 2) in real-time multimedia communications an additional problem is the stringent quality of service (QoS) requirements, for example in terms of delay and jitter; 3) for many applications like video-conferencing and tele-teaching, there is a need for centralized control of the P2P network.

The instability problem affecting peers participating in a network has been addressed by several researchers, for example by using multiple distribution-tree structures to distribute multimedia traffic among multiple paths. Two protocols of this kind are CoopNet [1] and Splitstream [4], which use multiple multicast trees and multiple description coding (MDC) [5]. However, this kind of coding requires a large amount of computation overhead, and so these sources are rarely implemented.

All these constraints and challenges, in combination with the delay- and loss-sensitive nature of interactive multimedia applications, make video and audio communication over P2P networks a challenging proposition.

With all this in mind, the target of this paper is twofold: on the one hand, a tree construction algorithm to support real-time video communications over a P2P network is proposed; on the other hand, an analytical framework is defined to evaluate performance and provide the designer with a tool to choose some project parameters, for example the bandwidth required by each peer, the maximum number of children in the tree network, and the maximum number of tree levels, determining the best trade-off between the number of users and performance provided to them.

The proposed tree-construction algorithm takes into account the presence of two kinds of peers: fertile peers and sterile peers. Sterile peers are peers that do not share any incentives, allow data forwarding to other peers, which become their children in the tree.

The paper is structured as follows. Section 2 describes the proposed tree-construction algorithm and peer admission strategy. Moreover, the same section defines a Service Provider revenue parameter which will be used as the target for the network parameter optimization process. Section 3 proposes the analytical Markov model. Section 4 analytically derives some important performance evaluation parameters. Section 5 applies the model to achieve a numerical analysis of the behavior of the system, discussing the impact of the peer admission parameters on the revenue parameter and performance guaranteed to platform users. Finally, Section 6 concludes the paper.

This work was partially supported by the Italian MIUR project PRIN 2006 “Profiles”.

1-4244-0981-0/07/$25.00 ©2007 IEEE.
2. System Description

The system we consider in this paper is a multipoint video communication platform allowing video sources to send a video stream to a community of peers simultaneously. The application we will consider here is any real-time video application, for example video-conference, video-chat or tele-teaching. The platform is managed by a Service Provider which tries to find the best trade-off between its revenue and performance provided to them.

In the following, we will focus our attention on one video session, constituted by one source and many receivers. As mentioned in the previous section, the solution most widely accepted today is the construction of a video distribution tree, where each client constitutes a node forwarding received streams to its children.

In the scenarios considered, as known, the main requirements are related to delay and loss performance, rather than scalability for huge numbers of users. In fact, if video streaming is directed towards large communities, interaction is very difficult. Interactive real-time video applications are therefore usually reserved for small or medium size groups. In addition, the reduced bandwidth of participating clients, who for example can access the platform at home from a consumer level ADSL link, limits the number of children of each peer; only by using P2P approach can the number of users reach acceptable levels. However, this number is limited again due to delay constraints, which limit the maximum tree depth. In other words, for this kind of applications the number of users is limited by both technical problems and scenario requirements. We will define the tree building protocol with this in mind.

We will consider two classes of users. Users belonging to the first class, in exchange for priority over the other users, and a reduced service cost, provide the network with bandwidth resources, allowing forwarding to other peers. These users can be located as intermediate nodes in the distribution tree, and will have children in the tree structure. For this reason, in the following we will refer to them as fertile nodes or fertile peers. Users belonging to the second class, on the other hand, do not provide other users with resources, and therefore can only be leaves in the distribution tree structure. For this reason they will be referred to as sterile nodes or sterile peers.

As said so far we will differentiate the cost per unit of time to access the multipoint content distribution platform. Fertile peers will be charged with a lower cost because they pay in bandwidth, providing the network with their bandwidth resources. On the contrary, sterile peers only use network resources, and therefore pay a greater amount of money. Let us indicate the cost per unit for the two classes of peers as $c_r$ and $c_s$, respectively, with $c_r < c_s$.

The number of children that a fertile peer can support depends on the bandwidth available to each peer, and is a topology parameter. For the sake of simplicity, we will assume that the maximum number of children that can be supported by each fertile peer is the same for all peers, and will be indicated as $F$.

To match the delay requirements, another important topology parameter is the maximum distance between a peer and the video source, which coincides with the maximum depth of the distribution tree. We will indicate it as $L$.

Given that the considered system is not best-effort, but aims at guaranteeing quality, the proposed tree management protocol is based on peer admission control (PAC). When a new peer wants to enter the network, irrespective of the class it belongs to, it has to issue an admission request to the admission controller. The admission controller may be the source node, but not necessarily.

When a sterile peer requests admission, it is inserted in the level with the closest available room to the source. On the other hand, when a fertile peer requests admission, the admission controller looks for the level $\hat{l}$ which is the nearest to the source in the tree, where there is some room not occupied by fertile peers. If in this level there is a free room unoccupied by any peers, then the new peer will be inserted there. Otherwise the admission controller behaves differently according as the level $\hat{l}$ is equal or not to $L$:

- if $\hat{l}$ is different from $L$, the entering fertile peer will either preempt a sterile peer in this level with probability $p_1$, or be treated as a sterile peer during the tree insertion process with probability $1 - p_1$ (i.e., one of the sterile peers in the $\hat{l}$ level is randomly chosen, extracted by the tree and, after the insertion of the entering peer in the $\hat{l}$ level, reinserted in the first available position for sterile nodes);

- if $\hat{l}$ is equal to $L$, the entering fertile peer will either preempt a sterile one in this level (causing the rejection of this sterile peer) with probability $p_s$, or rejected by the system with probability $1 - p_s$.

Therefore, $p_1$ is the preemption probability of a sterile peer by an arriving fertile peer when the highest level of the tree not fully occupied by fertile peers have no free rooms because of the presence of some sterile peer, and this level is not the $L$-th level; the probability $p_s$ is the rejecting probability of a sterile peer when a fertile peer can be inserted only in the last level and this level is full.

Of course, this approach can cause the rejection of admission requests made by both fertile and sterile peers, and service interruption for sterile peers due to preemption by fertile peers.

With all this in mind, we define the Service Provider revenue as follows:

$$ G = \bar{N}_f \cdot (1 - p_{s}^{\text{insert}}) \cdot c_r + \bar{N}_f \cdot c_r,$$  \hspace{1cm} (1)

where:

- $\bar{N}_f$ and $\bar{N}_s$ are the mean number of sterile and fertile peers present in the system, respectively;
\( (\bar{\lambda}_{\text{sterile}})^{-1} \) and \((\bar{\lambda}_{\text{fertile}})^{-1} \) are the mean sterile and fertile peer lifetimes, respectively;

- \( P_{\text{sterile \ interchange}}^{(\text{sterile} \rightarrow \text{fertile})} \) is the sterile peer interruption probability, due to preemption by a fertile peer.

The target of this paper is to define an analytical tool to optimize the revenue defined in \( \text{(1)} \) while guaranteeing a given level of quality to the platform users.

### 3. System Model

The target of this section is to define the model of a P2P video distribution session according to what has been described so far. The proposed model is a continuous-time Markov chain which will be described by its infinitesimal generator matrix. Section 3.1 will introduce some notation and assumptions which are needed to derive the model. Then, in the same section, the state of the Markov chain will be defined, and some observations will be made with the aim of reducing its state space. Sections 3.2 and 3.3 will model the two possible events determining a state change: peer arrival and peer departure, respectively. Finally, Section 3.4 will present the definition of the infinitesimal generator matrix.

#### 3.1 Notation and State Definition

Before defining the analytical model, we will introduce some notation. Let \( L \) be the maximum number of levels in the P2P distribution tree. As said in the previous section, it is a project parameter, and has to be carefully chosen given that it has an important impact on system performance. Let us assume that all peers accommodate at most \( F \) children, that is, \( F \) is the maximum number of branches for each node in the tree structure.

The Markov chain state at the generic instant \( t \) can be defined as follows:

\[
S(t) = (N_1(t), \Phi_1(t), N_2(t), \Phi_2(t), \ldots, N_L(t), \Phi_L(t))
\]

where:

- \( N_i(t), \) for each \( i \in [1, L] \), is the total number of peers at the generic level \( i \);
- \( \Phi_i(t), \) for each \( i \in [1, L] \), is the number of fertile peers at the generic level \( i \);

Moreover, let us indicate the generic value of the Markov chain state variable as \( \mathbf{s}(t) = (n_1(t), f_1(t), n_2(t), f_2(t), \ldots, n_L(t), f_L(t)) \). The state space of the Markov chain as defined in \( \text{(2)} \) may appear to be enormous. However, component variables are linked to each other, and they cannot take any value. Below we will present some rules defining the interdependence among the above component variables, which determine a notable state space reduction.

1. The number of fertile peers in each level cannot be greater than the total number of peers in the same level, that is:

\[
\Phi_i(t) \leq N_i(t) \quad \forall i \in [1, L]
\]

2. The number of fertile peers at the first level in the tree is equal to 1, that is:

\[
N_1(t) = 1 \quad \text{and} \quad \Phi_1(t) = 1
\]

3. Given that only fertile peers can have children, the total number of peers at the generic level \( i \) cannot be greater than the number of fertile peers at the upper level, multiplied by the maximum number of children, \( F \), that is:

\[
N_i(t) \leq F \cdot \Phi_{i+1}(t) \quad \forall i \in [2, L]
\]

#### 3.2 Peer Arrival Modeling

In this Section we model the peer arrival event. Let us note that two possible arrivals can occur: sterile peer arrival and fertile peer arrival. They can only be accepted if the conditions described in Section 2 are verified.

More specifically, a new sterile peer can only be accepted in the network if room is available in the tree. Let \( \tilde{l} \) be the level where it is accommodated, which is the highest level where room is available. The sterile peer acceptance condition is therefore:

\[
\text{a sterile peer is accepted if: } \exists \tilde{l} = \min_i \{ \Phi_i(t) < F^{\tilde{l}+1} \}
\]

In this case, the total number of peers at level \( \tilde{l} \) increases by 1, that is:

\[
n_{\tilde{l}}^{(0)} = n_{\tilde{l}}^{(0)} + 1
\]

As for the insertion of a fertile peer, let \( \tilde{l} \) be the first level where there is room unoccupied by fertile peers, that is:

\[
\tilde{l} = \min \{ \Phi_i(t) < F^l \}
\]

If \( N_{\tilde{l}}(t) < F^{\tilde{l}+1} \), i.e. some peers in the level \( \tilde{l} - 1 \) are able to accommodate children, the entering peer is inserted in the level \( \tilde{l} \) and the state variable evolves as follows:

\[
n_{\tilde{l}}^{(0)} = n_{\tilde{l}}^{(0)} + 1 \quad \text{and} \quad f_{\tilde{l}}^{(0)} = f_{\tilde{l}}^{(0)} + 1
\]

If \( N_{\tilde{l}}(t) = F^{\tilde{l}+1} \) (i.e. the level \( \tilde{l} \) is full), and \( \tilde{l} \neq L \), the entering peer will either preempt a sterile peer with probability \( p_{\tilde{l}} \), or its admission is managed as for a sterile node. In the first case the state variable evolves as follows:

\[
n_{\tilde{l}}^{(0)} = n_{\tilde{l}}^{(0)} \quad \text{and} \quad f_{\tilde{l}}^{(0)} = f_{\tilde{l}}^{(0)} + 1 \quad \text{and} \quad n_{\tilde{l}+1}^{(0)} = n_{\tilde{l}+1}^{(0)} + 1
\]

In the second case the admission condition is:

\[
\text{a fertile peer is accepted if: } \exists \tilde{l} = \min_i \{ \Phi_i(t) \}
\]

such that

\[
N_{\tilde{l}}(t) < F \cdot \Phi_{\tilde{l}+1}(t),
\]

and the state variable evolves as:

\[
n_{\tilde{l}}^{(0)} = n_{\tilde{l}}^{(0)} + 1 \quad \text{and} \quad f_{\tilde{l}}^{(0)} = f_{\tilde{l}}^{(0)} + 1
\]
Finally if \( N_f(t) = F^{(i)} \) and \( \bar{l} = L \), the entering peer will either be rejected leaving the state unchanged, or preempt a sterile peer with probability \( p_s \). In this second case the state variable evolves:
\[
n_i^{(l)} = n_i^{(l)} \quad \text{and} \quad f_i^{(l)} = f_i^{(l)} + 1
\]  
(13)

### 3.3 Peer Departure Modeling

Let us now model the departure of a peer. In order to describe this event more clearly, we will introduce further notation to explicitly represent the tree structure. Let us note that the generic level \( l \) can accommodate \( F^{(i)} \) nodes. The state of the node in the generic position \( z \) at level \( l \), with \( l \in [1, L] \) and \( z \in [1, F^{(i)}] \), can be represented by a ternary variable defined as follows:
\[
\beta_{z,l} = \begin{cases} 
0 & \text{if the position } (l,z) \text{ is empty} \\
1 & \text{if a sterile node in } (l,z) \\
2 & \text{if a fertile node in } (l,z)
\end{cases}
\]  
(14)

Therefore a tree \( A \) with a maximum length \( L \) and a maximum number of children per node \( F \) can be exhaustively represented by the set \( A = \{\beta_{z,l}\} \), for each \( l \in [1, L] \) and \( z \in [1, F^{(i)}] \).

Now let us define the state evolution. Let us indicate the generic tree structure at the transition start instant as \( B' = \{\beta'_{z,l}\} \), with \( l \in [1, L] \) and \( z \in [1, F^{(i)}] \).

To this end we have to distinguish between the following two occurrences:

1. **sterile** peer departure. This can happen in the levels where \( N_s(t) > \Phi_s(t) \). So, if the sterile peer leaving the network was at the generic level \( \bar{l} \), with \( n_i^{(l)} > f_i^{(l)} \), the state variable evolves as follows:
\[
n_i^{(l)} = n_i^{(l)} - 1
\]  
(15)

2. **fertile** peer departure. Given a tree structure \( B' = \{\beta'_{z,l}\} \) consistent with the \( \bar{s}^{(l)} \) state let us consider the sets of all nodes and fertile nodes, respectively, which are present at the generic level \( l \) of \( B' \):
\[
\Omega_i^{(l)} = \{\beta'_{z,l} : \beta'_{z,l} \neq 0, \forall z \in [1, F^{(i)}]\}
\]  
(16)
\[
\Omega_i^{(l,f)} = \{\beta'_{z,l} : \beta'_{z,l} = 2, \forall z \in [1, F^{(i)}]\}
\]  
(17)

If we indicate the number of elements of these sets as \( n_i' \) and \( f_i' \) respectively, it follows that:
\[
n_i' = n_i^{(l)} \quad f_i' = f_i^{(l)} \quad \forall l \in [1, L]
\]  
(18)

Let \( \bar{l} \) be the level and \( \bar{z} \) the position within the level where the leaving node was.

The depth of the sub-tree having the leaving peer as its root, henceforward referred to as the leaving subtree, \( \forall z \in [\bar{z} - 1, F^{(i)}], \forall F^{(i)} \), is:
\[
\bar{z} = \min \{ \beta'_{z,l} : 0 \}
\]  
(19)

Let us indicate the leaving sub-tree as \( \bar{B} = \{\beta_{z,l}\} \), with \( l \in [1, \bar{L}] \) and \( z \in [1, F^{(i)}] \). Let \( \Omega_{\bar{B}}^{(l)} \) and \( \Omega_{\bar{B}}^{(l,f)} \) be the sets of all nodes and fertile nodes, respectively, which are present at the generic level \( l \) of \( \bar{B} \), and \( \bar{p}_i \) and \( \bar{f}_i \) the number of elements in these sets.

After the departure of the fertile node, we will re-insert all its direct children, which are \( \alpha^f = \bar{f}_i \) fertile peers, and \( \alpha^s = \bar{p}_i - \bar{f}_i \) sterile peers, and the relative sub-trees of the \( \alpha_j \) fertile peers.

Let us indicate the intermediate state of the system immediately after the departure of the leaving sub-tree, but before the re-insertion of its elements, as \( \dot{s}^{(l)} \). The state variable evolves as follows:
\[
n_i^{(l)} = n_i^{(l)} - \bar{p}_i, \quad \forall m \in [0, \bar{L} - 1]
\]
\[
f_i^{(l)} = f_i^{(l)} - \bar{f}_i, \quad \forall m \in [0, \bar{L} - 1]
\]  
(20)

Now we have to insert the \( \alpha_j \) fertile peers with their sub-trees, starting from the system state \( s^{(l)} \). To this end, let us indicate the sub-tree with its root being the generic \( k \)-th fertile child of the \( \alpha_j \) children of the leaving peer, as \( A_k = \{\beta_{z,l}\} \), with \( k \in [1, \alpha_j] \), and \( l \in [1, L_k] \), where \( L_k \) is the depth of \( A_k \). Moreover, let \( z_k \) be the position of this generic \( k \)-th fertile child with respect to the original leaving sub-tree \( B \) (in particular, it is at the second level of \( B \), since it is a direct child of the leaving peer). We can link the structure of the sub-tree \( A_k = \{\beta_{z,l}\} \) to the structure of \( B \) as follows:
\[
\beta_{z,l}^{(l)} = \bar{B}_{(l + 1)\{z - 1, F^{(i)}, z_k\}}
\]

Now, starting from the deepest sub-tree, we insert all the \( \alpha_j \) sub-trees \( A_k = \{\beta_{z,l}\} \) one by one. At the first step of this insertion, we use the system state \( s^{(l)} \) as the start state, and calculate the new intermediate state \( \hat{s}^{(l)} \) by using the function \( \hat{s}^{(l)}(\alpha_j, A_k, A_{\alpha_j}) \). It calculates the new state of the tree, \( s^{(l)} \), after insertion of the sub-tree \( A_k = \{\beta_{z,l}\} \), when the tree starts from the state \( s^{(l)} \). The term \( \alpha_j \) represents the number of unaccommodated sterile nodes after the \( A_k \) tree insertion. In the same way, after \( \alpha_j \) consecutive steps we will calculate the final state \( s^{(l)} \). Finally, we have to insert the \( \alpha_j \) unaccommodated sterile peers after the insertion of the \( \alpha_j \) sub-trees. The final state, \( s^{(l)} \), is then calculated as follows:
\[ s(l) = \Gamma^{\text{uste}, \text{ins}}(\xi(\omega), \alpha_{\omega, \delta}) \]. It calculates the new state of the tree, \( s(l) \), after insertion of \( \alpha_{\omega, \delta} \) sterile peers, when the tree starts from the state \( \xi(\omega) \). The functions \( \Gamma^{\text{ins}}(\xi(\omega), \alpha_{\omega, \delta}) \) and \( \Gamma^{\text{uste}, \text{ins}}(\xi(\omega), \alpha) \) are derived in [6].

### 3.4 Markov Chain Definition

In order to define the Markov chain, let us calculate its infinitesimal generator, \( Q \). To this end, let us consider two generic states \( s_0 = (n_0^f, f_0^f, n_0^i, f_0^i, \ldots, n_0^l, f_0^l) \) and \( s_1 = (n_1^f, f_1^f, n_1^i, f_1^i, \ldots, n_1^l, f_1^l) \) belonging to the state space \( \mathbb{Y} \), representing the start and arrival states, respectively. The infinitesimal generator can be calculated taking into account that four possible events can modify its state: sterile and fertile peer arrivals, occurring with frequencies of \( \lambda_0^s \) and \( \lambda_0^p \), respectively, and sterile and fertile peer departures, occurring with frequencies of \( \lambda_0^d \) and \( \lambda_0^m \), respectively. So the generic element of the infinitesimal generator is:

\[
Q_{s_0, s_1} = \begin{cases} 
\gamma(s_0, s_1) & \text{if } s_0 \neq s_1 \\
- \sum_{s_0 \sim s_1} Q_{s_0, s_1} & \text{if } s_0 = s_1
\end{cases}
\]

\( \gamma(s_0, s_1) = \lambda_0^s \delta_0^s (s_0, s_1) + \lambda_0^p \delta_0^p (s_0, s_1) + \lambda_0^d \delta_0^d (s_0, s_1) + \lambda_0^m \delta_0^m (s_0, s_1) \)

where:

- \( \delta_0^s (s_0, s_1) \) is the indicator function of the feasibility of a transition from the state \( s_0 \) to the state \( s_1 \) for a sterile peer arrival. It can be defined by taking into account that an arriving sterile peer is accepted at level \( \bar{l} \) if the level \( \bar{l} \) is the first level where some free room is available. In this case the number of nodes at level \( \bar{l} \) is increased by one, while the number of fertile nodes remains the same. Therefore we have:

\[
\delta_0^s (s_0, s_1) = \begin{cases} 
1 & \text{if } \exists \bar{l}: n^s_{\bar{l}} = F \cdot f^s_{\bar{l}} \forall l < \bar{l} \\
n^s_l < F \cdot f^s_{l} & \text{if } \exists \bar{l} \text{ and } n^s_{\bar{l}} = F \cdot f^s_{\bar{l}} \forall l < \bar{l} \\
(n^s_l, f^s_{l}) = (n^s_l + 1, f^s_{l}) & \text{otherwise}
\end{cases}
\]

In the above definition, for the sake of conciseness, we have emphasized only the state variables subject to changes. The same will be done hereafter.

- \( \delta_0^p (s_0, s_1) \) is the indicator function of the feasibility of a transition from the state \( s_0 \) to the state \( s_1 \) for a fertile peer arrival. It can be defined by taking into account that an arriving fertile peer is accepted at level \( l \) in three different cases:

1. the fertile peer enters a level with free space; this happens if the level \( l \) is the first level where some room is available;
2. the fertile peer enters the system throwing out a sterile peer. This happens with probability \( p_2 \), if the first full level \( \bar{l} \) with a sterile peer is not the \( L_s \) level; in this case the fertile peer is accommodated in the \( l+1 \) level
3. the fertile peer acts as a sterile peer during the admission process; this happens with probability \( 1-p_1 \), if the first full level \( \bar{l} \) with sterile peers is not the \( L_s \) level; in this case the entering fertile peer is accommodated in the first level with free room.

4. the fertile peer enters the system in the \( L_s \) level throwing out a sterile peer. This happens with probability \( p_2 \), if all the levels of the tree are full and only the \( L_s \) level has sterile peers.

Therefore we have:

\[
\eta_0^s (s_0, s_1) = \begin{cases} 
1 & \text{if } \exists \bar{l} \in [1, L] : F_{\bar{l}} = F_{\bar{l}+1} \\
n^s_l < F^s_{l} & \text{if } \exists \bar{l} \in [1, L-1] : F_{\bar{l}} = F_{\bar{l}+1} \\
(n^s_l, F^s_{l}) = (n^s_l + 1, F^s_{l}) & \text{if } \exists \bar{l} \in L \text{ and } \exists \bar{l} \in [1, L] : F_{\bar{l}} = F_{\bar{l}+1}
\end{cases}
\]

\[
\delta_0^f (s_0, s_1) = \begin{cases} 
1 & \text{if } \exists \bar{l}: n^f_{\bar{l}} = F \cdot f^f_{\bar{l}} \forall l < \bar{l} \\
n^f_l < F \cdot f^f_{l} & \text{if } \exists \bar{l} \text{ and } n^f_{\bar{l}} = F \cdot f^f_{\bar{l}} \forall l < \bar{l} \\
(n^f_l, f^f_{l}) = (n^f_l + 1, f^f_{l}) & \text{otherwise}
\end{cases}
\]
departure can happen at level \( \bar{I} \) if the level \( \bar{I} \) contains a sterile peer. In this case the number of peers at level \( \bar{I} \) is decreased by one, while the number of fertile nodes remains the same. Therefore we have:

\[
\delta_{\text{oct}}^{(i)}(\underline{s}^{(i)}, \underline{s}^{(i)}) = \begin{cases} 
1 & \text{if } \exists l; n_{l}^{(i)} > f_{l}^{(i)} \text{ and } \sum_{n \in \mathcal{L}(\underline{s}^{(i)})} n_{l}^{(i)} f_{l}^{(i)} = (n_{l}^{(i)} - 1, f_{l}^{(i)}) \\
0 & \text{otherwise}
\end{cases} 
\]

(24)

- \( \eta_{\text{oct}}^{(i)}(\underline{s}^{(i)}, \underline{s}^{(i)}) \) is the transition probability from \( \underline{s}^{(i)} \) to \( \underline{s}^{(i)} \) after a fertile peer departure.

To calculate \( \eta_{\text{oct}}^{(i)}(\underline{s}^{(i)}, \underline{s}^{(i)}) \) we have to take into account that the state \( \underline{s}^{(i)} \) represents the set of all the possible trees with a number of nodes and fertile nodes matching the variables of \( \underline{s}^{(i)} \), that is, with \( n_{l}^{(i)} \) nodes and \( f_{l}^{(i)} \) at the generic level \( l \). Therefore \( \underline{s}^{(i)} \) can be considered as a macrostate made up of many microstates, each representing a particular tree structure. Let us indicate the set of all the tree structures contained in the macrostate \( \underline{s}^{(i)} \) as \( \Lambda(\underline{s}^{(i)}) \), and the number of elements in this set as \( N^{\text{ALL}}(\underline{s}^{(i)}) \). According to the notation introduced so far to represent each specific tree, \( \Lambda(\underline{s}^{(i)}) \) can be expressed as the set of all the possible tree structures \( B^{(m)}(\underline{s}^{(i)}) \) consistent with the \( \underline{s}^{(i)} \) state, with \( m \in [1, N^{\text{ALL}}(\underline{s}^{(i)})] \). If we indicate the number of all nodes and fertile nodes in the generic level \( l \) of the specific tree \( B^{(m)}(\underline{s}^{(i)}) \) as \( n_{l}^{(m)} \) and \( f_{l}^{(m)} \), they are linked to the respective variables \( n_{l}^{(i)} \) and \( f_{l}^{(i)} \) of the state \( \underline{s}^{(i)} \) as follows:

\[
n_{l}^{(m)} n_{l}^{(i)} = n_{l}^{(i)} \quad \text{and} \quad f_{l}^{(m)} = f_{l}^{(i)}
\]

(25)

Let \( B^{(m)} \) and \( B^{(i)} \) be two microstates belonging to the sets \( \Lambda(\underline{s}^{(i)}) \) and \( \Lambda(\underline{s}^{(i)}) \), respectively, that is, two tree structures compatible with the states \( \underline{s}^{(i)} \) and \( \underline{s}^{(i)} \). According to the Markov chain state aggregation theory, the probability of transition from the macrostate \( \underline{s}^{(i)} \) to the macrostate \( \underline{s}^{(i)} \) (with \( \underline{s}^{(i)} \) and \( \underline{s}^{(i)} \) belonging to the state space \( \mathcal{Y} \)) can be calculated as a function of the probabilities of transition between all the microstates belonging to \( \underline{s}^{(i)} \) and \( \underline{s}^{(i)} \) as follows:

\[
\eta_{\text{oct}}^{(i)}(\underline{s}^{(i)}, \underline{s}^{(i)}) = \sum_{B^{(m)} \in \Lambda(\underline{s}^{(i)})} \sum_{B^{(n)} \in \Lambda(\underline{s}^{(i)})} P_{B^{(m)}, B^{(n)}}^{(i)} \nu_{B^{(m)}, B^{(n)}}^{i} \nu_{B^{(m)}, B^{(n)}}^{i}
\]

(26)

where:

- \( P_{B^{(m)}, B^{(n)}}^{(i)} \) is the probability of transition from the microstate \( B^{(m)} \) to the microstate \( B^{(n)} \);
- \( \nu_{B^{(m)}, B^{(n)}}^{i} = \frac{1}{\pi_{B^{(m)}}^{i}} \sum_{B^{(n)} \in \Lambda(\underline{s}^{(i)})} \pi_{B^{(n)}}^{i} \) is the conditional probability that the Markov chain is in the microstate \( B^{(m)} \) provided that it is in the macrostate \( \underline{s}^{(i)} \), \( \nu_{B^{(m)}, B^{(n)}}^{i} \) being the steady-state probability of the microstate \( B^{(m)} \). Given that all the specific tree structures in the same state \( \underline{s}^{(i)} \) have the same probability, we have:

\[
\nu_{B^{(m)}, B^{(n)}}^{i} = \frac{1}{N^{\text{ALL}}(\underline{s}^{(i)})} \quad \forall \underline{s}^{(i)} \in \mathcal{Y}
\]

(27)

Now, noting that \( \sum_{s^{(i)} \in \Lambda(\underline{s}^{(i)})} P_{s^{(i)}, \underline{s}^{(i)}}^{i} \) is the probability of a transition from the microstate \( B^{(m)} \) to any microstate belonging to \( \underline{s}^{(i)} \), and indicating this probability as \( \operatorname{Prob}[B^{(m)} \rightarrow \underline{s}^{(i)}] \), (26) becomes:

\[
\eta_{\text{oct}}^{(i)}(\underline{s}^{(i)}, \underline{s}^{(i)}) = \sum_{B^{(m)} \in \Lambda(\underline{s}^{(i)})} \operatorname{Prob}[B^{(m)} \rightarrow \underline{s}^{(i)}] \nu_{B^{(m)}, B^{(n)}}^{i}
\]

(28)

Finally, indicating the set of fertile nodes in the state \( \underline{s}^{(i)} \) as \( \mathcal{Z}_{\underline{s}^{(i)}} \), and applying the theorem of total probability, we have:

\[
\eta_{\text{oct}}^{(i)}(\underline{s}^{(i)}, \underline{s}^{(i)}) = \sum_{B^{(m)} \in \Lambda(\underline{s}^{(i)})} \sum_{p \in \mathcal{Z}_{\underline{s}^{(i)}}} \operatorname{Prob}[B^{(m)} \rightarrow \underline{s}^{(i)}|p \text{ leaves the tree}] \cdot \operatorname{Prob}[p \text{ leaves the tree}] B^{(m)} \nu_{B^{(m)}, B^{(n)}}^{i}
\]

(29)

Let us note that the probability that \( p \) leaves the tree only depends on the number of fertile peers in the tree. In fact, we have:

\[
\operatorname{Prob}[p \text{ leaves the tree}] B^{(m)} = \frac{1}{\sum_{l} f_{l}^{(m)}}
\]

(30)

Moreover, let us note that the term \( \operatorname{Prob}[B^{(m)} \rightarrow \underline{s}^{(i)}|p \text{ leaves the tree}] \) is a Boolean function which is true if, after the departure of the peer \( p \) when the tree structure is \( B^{(m)} \), the new tree structure belongs to the macrostate \( \underline{s}^{(i)} \). It is defined as follows:

\[
\operatorname{Prob}[B^{(m)} \rightarrow \underline{s}^{(i)}|p \text{ leaves the tree}] = \begin{cases} 
1 & \text{if } \Gamma_{\text{ARR}}(p, B^{(m)}) = \underline{s}^{(i)} \\
0 & \text{if } \Gamma_{\text{ARR}}(p, B^{(m)}) \neq \underline{s}^{(i)}
\end{cases}
\]

(31)

where \( \Gamma_{\text{ARR}}(p, B^{(m)}) \) is the deterministic function calculating the arrival macrostate reached starting from the tree structure \( B^{(m)} \) when the peer \( p \) leaves the tree. It
can be calculated according to the procedure described in section 3.3.

4. Performance Analysis

In this section we use the steady-state probability array derived in the previous section to evaluate the main performance indices of the system being considered. According to the previous notation, let us indicate the generic state of the system as $s^{(i)} = (n_i^{(0)}, f_i^{(0)}, \ldots, n_i^{(0)}, f_i^{(0)})$. In the following section we will evaluate the main QoS parameters regarding the number of peers in the network (Section 4.1), the peer rejection/interruption probability (Section 4.2), and the peer position in the tree (Section 4.3).

4.1 Number of peers in the network

The first parameters we evaluate regard the number of peers present in the network at the generic instant, considering all of them, fertile and sterile, separately. Specifically, the probability density function (pdf) of the number of fertile and sterile peers, respectively, can be calculated as follows:

\[
\varphi_s(p) = \sum_{s^{(i)}} \pi(s^{(i)}) \cdot \delta_p(s^{(i)})
\]

\[
\varphi_s(p) = \sum_{s^{(i)}} \pi(s^{(i)}) \cdot \delta_p(s^{(i)})
\]

where $\delta_p(s^{(i)})$ and $\delta_p(s^{(i)})$ are defined as follows:

\[
\delta_p(s^{(i)}) = \begin{cases} 1 & \text{if } \sum f_i^{(0)} = p \\ 0 & \text{otherwise} \end{cases}
\]

\[
\delta_p(s^{(i)}) = \begin{cases} 1 & \text{if } \sum (n_i^{(0)} - f_i^{(0)}) = p \\ 0 & \text{otherwise} \end{cases}
\]

From the above pdf’s we can easily derive the mean number of all peers, fertile and sterile, in the network:

\[
\bar{N}_s = \sum_{s^{(i)}} p \cdot \varphi_s(p) \quad \bar{N}_s = \sum_{s^{(i)}} p \cdot \varphi_s(p)
\]

4.2 Peer rejection/interruption probability

Another important set of performance parameters regards peer admission control. In fact, as described in Section 2, the tree management mechanism has been defined so as to guarantee the maximum distance from the source. However, for this reason, loss of peers is possible. Therefore, let us calculate the following probabilities:

- Fertile peer admission rejection probability, $P_{f^{(j)}}$
- Sterile peer admission rejection probability, $P_{s^{(j)}}$
- Sterile peer interruption probability, due to preemption by a fertile peer, $P_{s^{(j),(f \rightarrow s)}}$

Specifically, they are defined as follows:

\[
P_{f^{(j)}} = \sum_{s^{(j)}} \pi(s^{(j)}) + \sum_{s^{(j)}} \pi(s^{(j)}) (1 - p_i)
\]

\[
P_{s^{(j)}} = \sum_{s^{(j)}} \pi(s^{(j)})
\]

\[
P_{s^{(j),(f \rightarrow s)}} = \sum_{s^{(j)}} \pi(s^{(j)}) (1 - p_i)
\]

where:

- $s^{(j)}$ is the system state at which all the tree rooms are occupied by fertile peers, that is, $n_i^{(f \rightarrow s)} = f_i^{(f \rightarrow s)} = F^{(i)}$, for each $i \in [1, L]$;
- $S^{(REJ)}$ is the set of states where we have to preempt a sterile node in the $l$ level for the insertion of a fertile peer and, when the system denies (with probability $(1 - p_l)$) this preemption, there is no free space in the lower levels for the entering peer. Therefore we have:

\[
S^{(REJ)} = \left\{ s^{(i)} \text{ such that } \exists l < L : n_i^{(l)} = F^{(l)}, f_i^{(l)} < N^{(l)} \text{ and } n_i^{(l)} = F^{(l)} \cdot f_i^{(l)} \forall l \in [1, L] \right\}
\]

- $S^{(REJ)}$ is the set of states where no rooms are available for sterile peers, that is, the number of nodes (both sterile and fertile) at any level is equal to the maximum number of nodes that can be accommodated. Therefore, we have:

\[
S^{(REJ)} = \left\{ s^{(i)} : n_i^{(l)} = F^{(l)} \forall l \in [1, L] \right\}
\]

- $S^{(Lost, f \rightarrow s)}$ is the set of states where a sterile peer is expelled by an arriving fertile peer. As discussed in Section 2, it comprises all the states where the tree is completely full, and the last level contains some sterile peers (let us note that the fact that the tree is completely full means that there can be no sterile peers at levels above the last one).

4.3 Position in the tree and delay from the source

Finally, another important set of parameters regards the delay between the source and each peer. To this end first we will derive the probability distribution of the position occupied by the generic fertile peer, hereafter indicated as the tagged fertile peer (TFP).

More specifically, the pdf of the position occupied by a fertile peer is:

\[
\varphi^{(f \rightarrow s)}(l') = \text{Prob}[\text{TFP is at level } l' \mid N_f \neq 0] = \sum_{l'c} \text{Prob}[\text{TFP is at level } l' \mid S(l') = s^{(o)}] \cdot \pi[l']
\]

where $l'$ represents the level occupied by the generic fertile peer and $N_f$ represents the number of fertile peers. We are considering the case $l' \in [2, L]$ because we are focusing our analysis on receiver peers only. The term $\sum_{s^{(o)}} \text{Prob}[\text{TFP is at level } l' \mid S(l') = s^{(o)}]$ can be calculated as the number of fertile peers in the level $l'$ over the total number of fertile peers, that is:
\[
\text{Prob}\{\text{TFP is at level } l' \mid S(t) = \xi_n^{(\rho)}\} = \frac{\int_{l_n^{(\rho)}}^{\int_{l_n^{(\rho)}}} \sum_{l_n^{(\rho)}} f_{l_n^{(\rho)}} \, dl_n^{(\rho)}}{\sum_{l_n^{(\rho)}} f_{l_n^{(\rho)}}} \quad \text{if } \sum_{l_n^{(\rho)}} f_{l_n^{(\rho)}} \neq 0
\]
\[
0 \quad \text{otherwise}
\]

Likewise, the probability density function of the position occupied by the generic sterile peer, hereafter indicated as the tagged sterile peer (TSP) is:

\[
\phi^{(\rho)}(l') = \text{Prob}\{\text{TSP is at level } l' \mid N_r \neq 0\} = \sum_{l_n^{(\rho)}} \text{Prob}\{\text{TSP is at level } l' \mid S(t) = \xi_n^{(\rho)}\} \cdot \pi_{l_n^{(\rho)}}
\]

\[
1 - \text{Prob}\{N_r = 0\}
\]

where the term \(\text{Prob}\{\text{TSP is at level } l' \mid S(t) = \xi_n^{(\rho)}\}\) can be calculated as follows:

\[
\text{Prob}\{\text{TSP is at level } l' \mid S(t) = \xi_n^{(\rho)}\} = \frac{n_{l_n^{(\rho)}} - f_{l_n^{(\rho)}}}{\sum_{l_n^{(\rho)}} (n_{l_n^{(\rho)}} - f_{l_n^{(\rho)}})}
\]

The terms \(\text{Prob}\{N_r = 0\}\) in (39) and \(\text{Prob}\{N_r = 0\}\) in (41) are calculated as (32) and (33), respectively.

Finally, we can easily derive the mean delay from a fertile and a sterile node to the source as follows:

\[
\Delta_s = \Theta \cdot \sum_{l_n^{(\rho)}} l'. \phi^{(\rho)}(l')
\]

\[
\Delta_s = \Theta \cdot \sum_{l_n^{(\rho)}} l'. \phi^{(\rho)}(l')
\]

where \(\Theta\) is the mean delay of a single-hop in the P2P overlay network, that is, the mean delay between two generic nodes in the Internet.

5. Numerical Results

In this section we will apply the analytical model presented so far to show how it can be used to perform a numerical analysis of the Quality of Service (QoS) provided by a real-time multimedia transmission P2P-based network. To this aim we will analyze the impact of the two preemption probabilities \(p_1\) and \(p_2\).

The case study we are addressing is a very common scenario, characterized by a real-time communication network where peers participate by using domestic ADSL access with an upload bandwidth of 256 kbit/s and a download bandwidth of 2 Mbit/s. Let us note that domestic access, which is unfortunately asymmetric, is not the best way to participate in a peer-to-peer communication network, because the number of children in the communication tree is mainly limited by the upload bandwidth, while the download bandwidth is only used for one download connection. However, this is the most widely used kind of access and is therefore a very important case to consider.

Let us assume that the source transmits a multimedia stream comprising the superposition of an MPEG rate-controlled video flow encoded with a QCIF format and a frame rate of 10 frame/s. The output rate of the MPEG encoder is set to 56 kbit/s. Therefore each peer can support at most \(F = 4\) upload connections, leaving a portion of bandwidth available for signaling and protocol messages. The maximum tree depth is set to \(L = 3\) to limit the end-to-end delay on the overlay network. The arrival and departure rates of fertile peers are \(\lambda_{\text{OUT}} = 0.02\) departures/min while the arrival and departures rate of fertile peers are \(\lambda_{\text{IN}} = 0.08\) departures/min, respectively. We are considering \(\lambda_{\text{IN}} < \lambda_{\text{OUT}}\) and \(\lambda_{\text{IN}} < \lambda_{\text{OUT}}\) because we suppose the the system supplies a certain number of reliable fertile peers to improve its capacity.

The numerical analysis is carried out versus \(p_1\) and \(p_2\). Figs. 1 and 2 represent the mean number of fertile and sterile peers in the network, respectively, calculated as in (35). In Fig. 1 we observe that, as expected, the mean number of fertile peers increases when both the fertile node preemption probabilities \(p_1\) and \(p_2\) increase. In Fig. 2 we can see that the mean number of sterile peers increases when \(p_1\) increases, since the number of free rooms for sterile nodes increases. Given that more fertile peers are accepted at the highest levels in the tree. On the contrary, it decreases when \(p_2\) increases, because fertile peer preemption of a sterile node produces a sterile peer rejection. Therefore, in a system where sterile peers pay
Figure 3: Mean delay of a fertile peer from the source for $p_{1} = 0.0\div1.0$ (step: 0.1)

Figure 4: Mean delay of a sterile peer from the source for $p_{1} = 0.0\div1.0$ (step: 0.1)

Figure 5: Fertile peers admission rejection probability for $p_{2} = 0.0\div1.0$ (step: 0.1)

Figure 6: Sterile peer interruption probability due to preemption of an arriving fertile peer for $p_{2} = 0.1\div1.0$ (step: 0.1)

...to participate in the network only if they are able to complete the received service, low values of $p_{2}$ improve performance in terms of achieved revenue.

Figs. 3 and 4 show the mean delay for fertile and sterile peers. This delay has been calculated considering a mean delay of 170 ms between two peers; this value has been taken from statistical delay measures available in [7]. Fig. 3 points out that the mean delay grows with $p_{1}$. In fact, when $p_{1}$ is high, the arriving fertile nodes will pre-empt sterile nodes in the level $L$ with a high probability, causing an increase of the mean delay. Moreover, the fertile node delay trend versus $p_{1}$ changes for different values of $p_{2}$:

- for low values of $p_{2}$ and high values of $p_{1}$, it decreases when $p_{1}$ increases because more entering fertile peers will be able to preempt sterile nodes in the upper levels
- for high values of $p_{2}$ and low values of $p_{1}$, the mean delay increases with the increase of $p_{1}$ because the probability of preemption of a sterile node by a fertile peer in the levels $l < L$ grows, causing an increase in the number of available rooms for fertile nodes in all the levels of the tree; in particular, this also occurs in the last level and, as $p_{1}$ is high, in this level many fertile peers will be accommodated.

From Fig. 4 we can also see that the mean delay of sterile peers from the source can be reduced by using low values of both $p_{1}$ and $p_{2}$.

Fig 5 presents the admission rejection probability for fertile nodes. Obviously this rejection probability decreases when $p_{1}$ and $p_{2}$ increase. In fact, when a fertile peer cannot preempt a sterile peer in a level $l < L$, it is treated as a sterile peer during the insertion process. So, it could not find any free room in the lower levels. Similarly, a fertile peer is rejected when it cannot preempt a sterile peer in the level $L$.

Another important performance parameter is the sterile peer interruption probability due to preemption by fertile peers. Fig. 6 shows this parameter, whose importance is due to the fact that in our proposed strategy only sterile peers who are not interrupted pay for the received service. Of course this probability is zero when $p_{1} = 0$, and therefore is not shown in the figure. This is because a fertile node arrival causes the discarding of a sterile node when it preempts a sterile node at the lowest level of the tree; if $p_{1}$ is zero, this event will never happen.

Therefore, to reduce the sterile peer interruption...
probability due to preemption by fertile peers, we have to choose small values for both $p_1$ and $p_2$. In fact, a preemption of a sterile peer happens only when the first $L-1$ levels are full of fertile nodes; if $p_1$ is low many fertile peers will be inserted in the lower levels of the tree, leaving free rooms in the higher levels. Similarly, low values of $p_2$ lowers the probability of a sterile peer service interruption in the $L$-th level when a fertile node arrives and tries to preempt it. Finally, Fig. 7 presents the normalized revenue for the Service Provider as defined in Section 2, in a scenario with fertile peers receiving the stream for free (taking into account that they pay in bandwidth resources, contributing to the life of the network) and sterile peers paying some money to receive the stream. Therefore, according to the defined notation, we have $c_s = 0$.

Sterile peers, on the contrary, pay $c_s = C$ units per time unit, but only if its service is not interrupted by a fertile peer. In order to carry out an analysis independently of the particular value of $C$, we show the normalized revenue, defined as follows:

$$\hat{G} = \frac{G}{C \cdot \lambda_0} \cdot (1 - P_s(L, f, \lambda_0))$$  \hspace{1cm} (44)

where:

- $\overline{N}_s$ is the mean number of sterile peers in the system, calculated as in (35), and shown in Fig. 2 in this case study;

- $P_s(L, f, \lambda_0)$ is the sterile peer interruption probability, due to preemption by a fertile peer, calculated as in (36), and shown in Fig. 6 in this case study.

The revenue is also normalized by $\lambda_0$ to make it independent of the departure rate of sterile peers.

The curve of the normalized revenue is presented in Fig. 7. As we can see, the revenue increases when $p_1$ increases and $p_2$ decreases. This curve, together with the curves presented in the previous figures, constitutes a very powerful tool, because provides the Service Provider the possibility of choosing the best values for $p_1$ and $p_2$, while taking into account the quality assured to the platform users in terms of mean delay from the source, admission request rejection probability for fertile peers, and service interruption probability for sterile peers.

6. Conclusions and future work

P2P systems are very suitable for multipoint video content distribution over IP networks. According to the P2P approach, peers that participate in the same group organize themselves into an application layer multicast tree. This paper, on the one hand, has proposed a tree construction algorithm and a peer admission strategy to support real-time video communications over a P2P network; on the other hand, an analytical framework is defined to evaluate performance and provide the designer with a tool to choose some project parameters, determining the best trade-off between the Service Provider revenue and performance provided to users in a video communication session. Two classes of users have been considered: fertile peers and sterile peers.

The model has been used to analyze the behavior of the proposed algorithm, and applied in a case study. As future work, we intend to apply the model to a multiple-tree structure like that used by some MDC mechanisms as, for example, Coopnet and Splitstream.

REFERENCES


