Throughput Analysis in Asymmetric Two-way Relay Channel with Random Access

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Abstract—We consider the two-way relay channel with random access for the cases of symmetric and asymmetric channel statistics in the low SNR regime. We propose three different schemes implementing different physical layer techniques for collision recovery and channel adaptation and obtain analytical throughput expressions. We compare the proposed schemes with several benchmarks in order to study their bandwidth gains in practical scenarios.

I. INTRODUCTION

Cooperation in wireless networks can mitigate the disadvantages of communication over a shared medium with time-varying quality. In the two-way relay channel two-way data exchange between two terminals is enabled by a third one, the relay. Cooperation in this setup is achieved at the expense of time/frequency and energy resources of the relay. Resource optimization can lead to a greater global network efficiency with savings in terms of bandwidth and energy as well as a reduction in delay. Minimization of packet loss in both the multiple access (MA) and the broadcast (BC) phases is of primary importance in such a scheme as it saves retransmissions and improves communication rates.

In order to minimize packet loss in the MA phase, an ideal time division multiple access (TDMA) scheme would be needed. Although theoretically optimal, ideal TDMA is difficult to be implemented in real systems, as it requires network signalling and higher level coordination. For this reason in many practical systems such as ad-hoc networks and interactive mobile satellite systems a random access phase is often present [1].

In the two-way relay channel, four, three and two-phase schemes have been studied in the literature to reduce the number of retransmissions for data exchange (see Fig. 1). An information theoretical analysis for such schemes with amplify-and-forward and decode-and-forward (DF) relay as well as structured codes has been carried out in [2], [3], [4], [5] and [6]. In the four-phase scheme, TDMA is used in both the MA and BC phases. In the three-phase scheme, one transmission is saved in the BC phase by broadcasting a combination of the received signals, while in the two phase scheme terminals transmit simultaneously in the MA phase and the relay broadcasts a function of the received signal which is then used by terminals to try to decode their desired messages exploiting the knowledge of their own transmitted signal. In [7] the four, three and two-phase bidirectional relay schemes were considered. In particular several techniques have been studied for the two phase scheme, referred to as bidirectional amplification of throughput (BAT), and among these the denoise-and-forward (DNF) technique, in which the relay decodes the sum of the signals, was proposed. In [8] it was pointed out that a joint network layer and physical layer approach can significantly increase the throughput in two-way communications. Based on this idea [8] proposed the concept of physical layer network coding (NC) and a multiple access protocol leveraging on this concept was studied for different network topologies. Joint network and channel coding was also studied in [9], while in [10] and [11] the results were extended to the case of asymmetric channels between the terminals and the relay and the optimal time allocation in a three phase scheme was studied. In [12], the system outage behavior was studied in case of fading channels for various schemes.

In the present paper we consider DF relaying and propose a combination of physical layer NC techniques for the MA phase with asymmetric channel adaptation in the BC phase. In order to evaluate the performance in a realistic scenario, a random access phase is considered. This allows to measure the behavior of several protocols proposed in the literature for either MA or BC phases in a realistic scenario by removing the strong assumption of having two nodes always transmitting or perfect TDMA. We derive closed form analytical expressions for the average number of transmissions needed for delivering one packet to each of the terminals. We study symmetric as well as asymmetric channels, in which one of the terminals is shadowed.

![Fig. 1. Four, three and two-phase schemes for the two-way relay channel.](image-url)
A. System Model

Consider two terminal nodes $T_1$ and $T_2$ and a relay node $R$. The terminals can only communicate through the relay, i.e., there is no direct link between them. We assume that the channels between the terminals and the relay are affected by independent flat fading processes. Time is divided into slots each with a fixed number of symbols. We assume channel state information (CSI) at the relay when both transmitting and receiving, and CSI at the terminals when receiving only. We assume this is achieved through a brief signalling from the terminals after the MA phase, the influence of which in global throughput can be neglected if time slots are reasonably longer than the signalling periods. $T_1$ and $T_2$ have independent messages $u_1$ and $u_2$, respectively, to transmit to each other. Transmission is organized in two phases. In the MA phase, terminals transmit their messages to $R$, and in the BC phase $R$ relays the received messages to the terminals. We first consider the MA phase. As we assume no CSI at the transmitters during the MA phase, terminal $T_i$, $i = 1, 2$, encodes its message $u_i$ using a linear encoder with fixed rate $r$. The encoded message $x_i$ is then modulated and transmitted to the relay. We indicate with $s_i$, $i = 1, 2$, the modulated signals. Terminals randomly access the channel with probability $q$ independently from each other. Thus with probability $q^2$ a collision occurs at the relay, with probability $2q(1-q)$ only one node accesses the channel and with probability $(1-q)^2$ the channel remains idle.

When only one terminal accesses the channel $R$ receives the signal

$$y_R = h_i s_i + w,$$

where $h_i$ is a circularly symmetric zero mean unit variance complex Gaussian random variable, i.e., $h_i \sim CN(0, 1)$, modeling the fading coefficient between the transmitting terminal ($T_i$) and the relay, and $u_i \sim CN(0, \sigma^2)$. If the message is successfully decoded, $R$ broadcasts an acknowledgement (ACK) for message $u_i$. If the message is not correctly decoded a new random access phase takes place. $T_j$, after overhearing the ACK for message $u_i$, transmits its own packet $u_j$ to the relay. We assume that the duration of the ACK is negligible with respect to that of the time slot. Furthermore, we assume that ACK’s are always correctly received by both terminals.

If $T_1$ and $T_2$ access the channel simultaneously, a collision occurs. Assuming perfect symbol synchronization, the received signal at the relay is:

$$y_R = h_1 s_1 + h_2 s_2 + w.$$

In case of a collision we consider two different approaches. In the first one, $R$ tries to decode the XOR of the messages $u_1$ and $u_2$. By decoding the XOR we mean that $R$ tries to decode $x_{\oplus} = x_1 \oplus x_2$ starting from the received signal 2. Due to the linearity of the code, $x_{\oplus}$ is a codeword as well. Here, we consider LDPC codes at each of the terminals. In order to decode $x_{\oplus}$ the LDPC decoder at the relay is fed with the vector of log-likelihood ratios (LLR). In the case of BPSK signalling the LLR of the $t$-th symbol of $x_{\oplus}$ is

$$L[t] = \ln \left[ \frac{\cosh \left( \frac{y[t](h_1 + h_2)}{\sigma} \right)}{\cosh \left( \frac{y[t](h_1 - h_2)}{\sigma} \right)} \right],$$

where $y[t]$ is the $t$-th element of vector $y_R$. Extension of this expression to QPSK and 4-QAM modulations is straightforward, while for higher order modulations the calculation of the LLRs changes slightly [13], [14]. If decoding is not successful the relay sends out a NACK and the procedure starts over. If the relay can decode the XOR, one terminal is randomly chosen and acknowledged. Then the terminal that did not receive the ACK retransmits its message until the relay correctly decodes and acknowledges it. At this point $R$ can decode both messages $u_1$ and $u_2$. We will refer to this approach as de-noise (DN) [7].

In the second approach, the relay stores the received signal and acknowledges one randomly chosen terminal. As before, the terminal that does not receive the ACK retransmits until it receives an ACK. The relay then subtracts the signal corresponding to this message from the collided signal it received, and tries to decode the unknown message. If this is not possible, a retransmission is requested. We refer to this approach as interference cancellation (IC).

In both DN and IC approaches, at the end of the MA phase $R$ knows both $u_1$ and $u_2$. In the BC phase of both methods the relay makes an asymmetric channel adaptation by first channel-encoding and then network-encoding the two messages, as explained in the following. $u_1$ and $u_2$ are encoded by the relay according to $h_2$ and $h_1$, respectively, thus obtaining messages $x_{R1}^2$ and $x_{R2}^2$. Unlike in [10], we assume fixed slot duration as often occurs in practical situations. So channel encoding is done by keeping constant the length of the coded packets $x_R^2$ and varying the information rates based on the instantaneous channel states [15]. Finally, the relay calculates the XOR of the two encoded packets, [7][13]:

$$x_{R12}^2 = x_{R1}^2 \oplus x_{R2}^2.$$  

Packet $x_{R12}^2$ is finally modulated, producing signal $s_{R1}$, and broadcasted to the terminals. A block diagram describing the encoding process is depicted in Figure 2.

The signal received at terminal $i$ in the BC phase is

$$y_i = h_i^R s_{R} + w_i,$$

where $s_{R}$ is the signal transmitted by the relay and $w_i$ is the noise component at terminal $i$. Terminal $i$ wants to obtain message $x_{Rj}^i$, $i \neq j$. As stated above, the terminal has channel state information when receiving, which allows it to know the rate used by the relay to encode the message it is willing to receive. So it first calculates packet $x_{Rj}^i$ by using the same code rate of the relay, and then “strips” it directly from the received signal by inverting the sign of the log-likelihood-ratio (LLR) relative to the $t$-th symbol received if $x_{Rj}^i(t)$ is equal to 1, while taking the current value of the LLR if $x_{Rj}^i(t)$ is equal to 0 [10]. Note that the scheme we consider differs from the one
in [10] in that the terminals can not hear each other’s signals, channels are affected by fading (or fading and shadowing) and we consider random access in the first phase, which often occurs in real systems such as wireless LANs (access to the access point) and cellular networks (access to the base station).

The third scheme we propose is opportunistic DNF in which the channel adaptation in the BC phase is performed if no collision occurs in the MA phase. In case of a collision, DNF is used. We compare our proposed schemes with the following four benchmark schemes:

Two-phase scheme - Both terminals always transmit, \( R \) tries to decode the XOR of the messages, re-encodes at a fixed rate and broadcasts.

MA and NC after channel coding - No collision resolution technique is used in the MA phase, thus when a collision occurs packets are discarded, while in the BC phase there is asymmetric channel adaptation.

MA and NC before channel coding - No collision resolution technique is used in the MA phase, while channel encoding after network encoding is done in the BC phase.

DN and NC before channel coding - In this scheme collisions are addressed using DN while channel encoding after NC is used in the BC phase.

B. Throughput Analysis

We compare the various schemes based on their total throughputs defined as:

\[
\text{th} = \frac{1}{E\{N_s\}}
\]  

(6)

where \( N_s \) is the total number of time slots needed for both \( T_1 \) and \( T_2 \) to successfully decode their desired messages. In the rest of the section we derive analytical expressions for the average throughput of the various protocols, while in the next section we evaluate the total throughput by simulating a coded transmission scheme using multi-rate LDPC codes over different channel models.

De-noise + NC after channel coding - As described in the previous section this scheme deals with collisions in the MA phase using physical layer NC while asymmetric network-channel coding is performed in the BC phase. As there is channel adaptation in the BC phase, the rate of transmission to terminal \( i \) denoted by \( r_{iBC}, i = 1, 2 \), is a random variable depending on the state of the channel. BC rates are always less than or equal to the rate used in the MA phase (the relay can not transmit more data than it receives). The number of time slots \( N_s \) can be decomposed into \( N_{MA} \) and \( N_{BC} \), which are the number of slots needed to correctly decode both packets in the MA and BC phases, respectively. We denote by \( p_{M1} \) and \( p_{M2} \) the probabilities that the relay can not decode the message transmitted by a single terminal, or the XOR of the packets transmitted by both terminals, respectively. Also denote by \( p_B \) the probability that a terminal can not decode the message transmitted by the relay in the BC phase. Due to channel adaptation in the BC phase \( p_B \) can be made arbitrarily small assuming that the time slot is long enough; hence, we let \( p_B = 0 \) for our analysis.

We first calculate \( E\{N_{MA}\} \). Consider the Markov chain depicted in Fig. 3 in which the number of a state indicates the number of packets successfully decoded by the relay.

Fig. 3. Illustration of the Markov chain for the calculation of throughput. The number corresponding to a state indicates the number of packets successfully decoded by the relay.

\[
E\{N_{MA}\} = \frac{\beta + \delta}{1 - \gamma - \alpha} \left[ 1 - \gamma^2 + \frac{\gamma \alpha (\alpha - 2)}{(1 - \alpha)^2} \right].
\]  

(8)

If the SNR is high we can ignore the probability of decoding error. Letting \( p_{M1} = p_{M2} = 0 \) in (8) we find

\[
E\{N_{MA}\} = \frac{1 + 2q - q^2}{q(2 - q)}.
\]  

(9)
which goes to infinity as \( q \) goes to 0 and to 2 as \( q \) goes to 1, as expected. The minimum value of (9) is obtained for \( q = 1 \), which indicates that, if both \( p_{M1} \) and \( p_{M2} \) are negligible, the two nodes should always jointly transmit their packets to the relay. In this case the total number of transmissions needed is 3. If the two probabilities are not negligible, such as in the case of shadowing in one or both of the channels, (9) is no longer valid and the optimal transmit probability \( q \) could be different from one. As channel adaptation is performed in the BC phase, \( N_{BC} \) can be assumed to be 1.

Now, we consider the case in which one of the two nodes is shadowed, i.e., channel statistics are not the same. In this case we must consider the Markov chain in Fig. 4, where the states (0) \( (1, 0) \) \( (1 \oplus 1) \) and (2) indicate, respectively, the cases in which no message was decoded by the relay, only the message of \( T_1 \) has been decoded, the XOR of the messages has been decoded and both messages were decoded. In this case we have \( \alpha = (1-q)^2+q(1-q)(p_{Mtx1}+p_{Mtx2})+q^2p_{M2}, \beta_1 = q(1-q)(1-p_{Mtx1}), \beta_2 = q(1-q)(1-p_{Mtx2}), \gamma_1 = 1-p_{Mtx2}, \gamma_2 = 1-p_{Mtx1}, \delta = q^2(1-p_{M2})^2 \) and \( \epsilon = (\gamma_1+\gamma_2)/2 \), where \( p_{Mtx1} \) and \( p_{Mtx1} \) are the probabilities that the message from \( T_1 \) or \( T_2 \) is lost, respectively. Again \( p_{M2} \) is the probability that the XOR of the two messages is not correctly decoded. The probability that \( n \) transmissions are needed for both messages to be decoded is:

\[
P_r\{N_{MA} = n\} = \beta_1\gamma_1(1-\gamma_1)^{n-2} \sum_{i=0}^{n-2} \left( \frac{\alpha}{1-\gamma_1} \right)^i + \beta_2\gamma_2(1-\gamma_2)^{n-2} \sum_{i=0}^{n-2} \left( \frac{\alpha}{1-\gamma_2} \right)^i + \delta\epsilon(1-\epsilon)^{n-2} \sum_{i=0}^{n-2} \left( \frac{\alpha}{1-\epsilon} \right)^i,
\]

and \( P_r\{N_{MAC} = 1\} = 0 \). We then have

\[
E\{N_{MA}\} = \frac{\beta_1}{(1-\gamma_1-\alpha)} \frac{1-\gamma_1}{\gamma_1} \frac{1}{(1-\alpha)^2} + \frac{\beta_2}{(1-\gamma_2-\alpha)} \frac{1-\gamma_2}{\gamma_2} \frac{1}{(1-\alpha)^2} + \frac{\delta}{(1-\epsilon-\alpha)} \frac{1-\epsilon}{\epsilon} \frac{1}{(1-\alpha)^2}.
\]

**MA with IC and NC after channel coding** - The analysis for this scheme is the same as in the one with de-noise and NC after channel coding. The only difference is in the weights used in the Markov chain depicted in Fig. 3, in which we must put \( \alpha = (1-q)^2+2q(1-q)p_{M1} + q^2[p_{M1}^2 + p_{M1}(1-p_{M1})] \) and \( \delta = q^2(1-p_{M1})^2 + (1-p_{M1})p_{M1} \), for the symmetric case and \( \alpha = (1-q)^2+q(1-q)(p_{Mtx1}+p_{Mtx2})+q^2(p_{Mtx1}p_{Mtx2}+p_{Mtx1}(1-p_{Mtx1})/2 + p_{Mtx1}(1-p_{Mtx2})/2) \) and \( \delta = q^2(1-p_{Mtx1})(1-p_{Mtx2}) + (1-p_{Mtx1})(1-p_{Mtx2})/2 + (1-p_{Mtx2})(1-p_{Mtx1})/2 \) for the asymmetric case. All the other parameters remain the same in both the symmetric and the asymmetric cases. The analysis at high SNR is the same as in the previous scheme.

**Two-phase scheme** - The code rates in the MA and BC phases are assumed to be the same. In the MA phase of this scheme both nodes transmit with probability one, thus the only limiting factor is the probability of not decoding. So we have

\[
E\{N_{MA}\} = \frac{1}{1-p_{M2}}
\]

In the BC phase there can not be asymmetric channel adaptation because \( R \) only knows and broadcasts the XOR of packets received in the MA phase. In the symmetric channel case, the probability of packet loss in both relay to \( T_1 \) and relay to \( T_2 \) channels are the same, and we call it \( p_B \). Let us consider the Markov chain depicted in Fig. 5, where \( \alpha = p_B^2, \beta = 2p_B(1-p_B), \gamma = 1-p_B \) and \( \delta = (1-p_B)^2 \). From Fig. 5 and using (12) we obtain:

\[
E\{N_s\} = \frac{1}{1-p_{M2}} + \frac{1+2p_B}{1-p_B}
\]

In case one of the two nodes is shadowed we must consider the Markov chain in Fig. 4 for the BC phase, after substituting
the central two-hop branch with a one hop branch having weight \( \delta = (1-p_{BT1})(1-p_{BT2}) \), and using the corresponding expression (11) after substituting the last term in the sum with \( \delta/(1-\alpha)^2 \), putting \( \alpha = p_{BT1}p_{BT2}, \beta = p_{BT2}(1-p_{BT1}), \beta_2 = p_{BT1}(1-p_{BT2}), \gamma_1 = 1-p_{BT2} \) and \( \gamma_2 = 1-p_{BT1} \) where \( p_{BT1} \) is the probability that node \( T_i \) can not decode the XOR of the two messages. Finally for the two phase scheme we have \( E\{N_{s}\} = 1/(1-p_{M2}) + E\{N_{BC}\} \), where \( p_{M2} \) is the packet loss probability for XOR decoding in case one of the channels is affected by shadowing.

In the high SNR regime we can assume that the probability of packet loss is negligible. In this case the average number of transmissions needed can be calculated using (13) and (11), and letting \( \alpha = \beta = 0 \) and \( \gamma = \delta = 1 \), we get \( E\{N_{s}\} = 2 \):

**MA and NC after channel coding** - The analysis for this method is the same as in the case of denoise + NC after channel coding but with \( \delta = 0 \) and \( \alpha = (1-q)^2 + 2q(1-q)p_{M1} + q^2 \), which indicates that in case of a collision packets are lost with probability 1. In case of asymmetric channel statistics, (11) still holds for the MA phase with \( \delta = 0 \), while the BC phase remains unchanged. The rates used in the BC phase are always less than or equal to those in the MA phase.

**MA and NC before channel coding** - As in the previous scheme, no collision resolution is used in the MA phase while the BC phase is equivalent to the one in the two-phase system. Thus \( E\{N_{s}\} \) can be obtained from the previous scheme putting \( E\{N_{BC}\} = (1 + 2p_B)/(1 - p_B^2) \), and using \( \alpha = (1-q)^2 + 2q(1-q)p_{M1} + q^2, \beta = 2q(1-q)(1-p_{M1}) \) and \( \gamma = 1-p_{M1} \). In case of asymmetric channel statistics the number of transmissions in the MA phase is modified according to the scheme with MA and NC after channel coding, while the number of transmissions in the BC phase is modified according to the two phase scheme. The rate in the BC phase is the same as the rate in the MA phase.

**DN and NC before channel coding** - The MA phase of this protocol is the same as in the DNF with NC after channel coding, while the BC phase is the same as the MA and NC before channel coding protocol.

**Opportunistic DNF** - The expression for the average number of transmissions in the MA phase can be computed using the Markov chain in Fig. 5 letting \( \alpha = (1-q)^2 + 2q(1-q)p_{M1} + q^2(1-p_{M2}), \beta = 2q(1-q)(1-p_{M1}), \gamma = 1-p_{M2} \) and \( \delta = q^2(1-p_{M2}) \). Starting from state 0, the system goes in state 1 if the relay decodes a packet transmitted by either one or the other terminal or in state 2 if the relay decodes the sum of the received signals in case of a collision. In the BC phase, the number of transmissions is equal to the sum of the BC transmissions in the schemes with and without channel adaptation weighted by the factors \( \beta/(1-\alpha) \) and \( \delta/(1-\alpha) \), respectively. In case of asymmetric channels, the MA phase can be described by Fig. 4 after removing the state 1 \( \oplus 1 \) and directly connecting state 0 with state 2. In this case equation (11) must be modified substituting the third addend with \( \delta/(1-\alpha)^2 \). As in case of symmetric channels, the number of transmissions in the BC phase is the sum of those in the systems with and without channel adaptation each weighted for a factor that accounts for the probability that the first signal can be decoded by the relay was produced after a collision or not, respectively.

The analysis in the high SNR regime for the last four schemes can be obtained from those of the previous ones. The last one tends to behave as the two-phase scheme after optimizing the transmission probability. In the other three schemes the average number of transmissions needed after optimizing the transmission probability is 3. Considering the analysis at high SNR for the various schemes, we see how the two-phase scheme outperforms all the others, as expected, which confirms the correctness of our analysis. In the following section we show that the results are quite different in case of harsh channels conditions.

**C. Numerical Results**

We evaluate the performance of the various systems using BPSK modulation and variable rate LDPC codes. We use a fixed codeword length of \( N = 480 \) symbols, changing the message length \( K \in \{400, 360, 320, 240\} \) according to the desired rate \( r \in \{5/6, 3/4, 2/3, 1/2\} \) when doing channel adaptation. We compare the various schemes in two different scenarios by measuring the throughput defined in (6) normalized by the maximum throughput achievable in a two-phase scheme with frame error rate (FER) equal to zero (0.5 packets per time slot). The probability of transmission \( q \) is arbitrarily set to 0.5. As can be seen from the formulas in the previous section the throughput can be optimized by varying the transmission probability. This will be explored in a future work. Here, we are assuming that terminals do not know the packet error probabilities in the various links, which would be needed to maximize the throughput. By comparing schemes that differ only in one of the phases, we evaluate how the various techniques affect the total bandwidth.

We first consider a symmetric scenario. Fig. 6 depicts the normalized throughput. The two-phase scheme does not perform as well as it would be expected in a Gaussian channel. This is due to the higher FER of XOR decoding compared to decoding each message in the MA phase and to the lack of channel adaptation in the BC phase. In the second scenario, one of the channels is affected by lognormal shadowing in addition to fading. The shadowed channel has an average power which is 2 dB lower than that of the fading channel. In Fig. 7 the normalized throughput is shown.

The system with interference cancelation outperforms the others in the whole SNR range. This is because storing the received signal in case of a collision and stripping the decoded signal allows for separate decoding of the two messages and this leads to lower FER with respect to XOR decoding. Despite the better performance of the IC scheme, it must be pointed out that in a practical system storing the sampled message with sufficient accuracy requires memory resources which are much higher than those required by XOR decoding, which, moreover, can be implemented just modifying the L-values fed to the decoder without any further signal processing. From the results in both scenarios it can be seen that the use of
de-noising shows its benefits in the higher SNR range while adapting to each of the channels is particularly useful in case of strongly asymmetric channels in the lower SNR range of the region considered. Actually in Fig. 6 we observe that systems that use de-noising, and particularly opportunistic DNF, perform slightly better than the others as SNR grows; while in Fig. 7 we see how the systems performing asymmetric channel adaptation have better performances with respect to the others. This is due to the lower FER determined by the asymmetric channel adaptation. The analysis of the gain originated by asymmetric channel adaptation has been carried out in [15] in terms of ergodic capacity and capacity probability density function and a broadcast transmission for a generic number of nodes.

ACKNOWLEDGEMENT

This is the draft version of the following article: G. Cocco, D. Gunduz, C. Ibars, Throughput analysis in asymmetric two-way relay channel with random access, in Proceedings of the IEEE International Conference on Communications (ICC 2011), 5-9 June 2011, Kyoto (Japan).

G. Cocco is partially supported by the European Space Agency under the Networking/Partnering Initiative.

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