Abstract—Fundamentally, benefits of Multiple Input Multiple Output (MIMO) in wireless systems rest on increasing the number of Degrees of Freedom (DoFs) that can be exploited for transmission. However, obtaining such DoFs in MIMO often requires knowledge of Channel State Information (CSI) either at the transmitter (CSIT) and/or at the receiver (CSIR). This itself has a cost since it requires use of wireless resources for CSI-estimation pilots and/or CSI feedback. Recently a new class of techniques known as “Blind Interference Alignment” (BIA) has demonstrated the possibility to grow DoFs without the CSIT costs of conventional systems. Despite this advantage, BIA does have challenges in cellular given that it requires a high Signal to Noise Ratio (SNR) to operate efficiently. By examining BIA, we present a method to reduce the SNR requirements and improve the application of BIA to cellular through power allocation and cluster-based transmission. We also examine the CSI overheads of BIA, and compare them to those of conventional CSIT-based MU-MIMO. We show that BIA can be applied successfully in cellular and can improve performance over conventional MU-MIMO, even for low SNR cell-edge users, in some scenarios.

I. INTRODUCTION

Many advances in wireless transmission have rested on the use of MIMO for transmission and reception. MIMO, fundamentally, provides an increase in the number of Degrees of Freedom (DoFs) that can be exploited by a system for transmission. Here, DoFs can be used for increased spectral efficiency and/or added diversity. In particular, a Single User MIMO (SU-MIMO) system with \( N_t \) transmit (TX) antennas serving a user with \( N_r \) receive (RX) antennas may be able to exploit up to \( \min(N_t, N_r) \) DoFs. These DoFs, for example under certain channel conditions, can be used to improve throughput by a factor that grows linearly with \( \min(N_t, N_r) \) [1]. Furthermore, to increase DoFs without requiring users (e.g., mobiles) to increase \( N_r \) or increase computational complexity in receivers, a Multi-User MIMO (MU-MIMO) system can be used. A MU-MIMO system with \( N_t \) transmission antennas at the base station (BS) and \( K \) single-antenna users (\( N_t = 1 \)) can provide up to \( \min(N_t, K) \) DoFs.

It is important to note that exploiting such DoFs often requires the knowledge of Channel State Information (CSI) between the transmit and receive antennas. Such CSI often has to be available to either the transmitter (such CSI is termed CSIT) and/or the receiver (such CSI is termed CSIR), each with implications to system design. For example, SU-MIMO systems such as single or multi coded-stream D-BLAST [1] require only CSIR. In contrast, conventional MU-MIMO requires additional knowledge of CSIT to define the required transmissions. Unfortunately, obtaining CSI creates overheads due to the required use of wireless resources for CSI-estimation pilots and/or CSI feedback [2]. This can create a “dimensionality bottleneck” that limits the growth in spectral efficiency, in particular for CSIT-based MU-MIMO [4].

Recently a new class of techniques, termed “Blind Interference Alignment” (BIA) techniques, has demonstrated the ability to grow DoFs without the CSIT overheads of conventional MU-MIMO systems. As demonstrated in [6], it is possible for a BIA MU-MIMO system with \( N_t \) TX antennas and \( K \) single-active-antenna users to achieve \( \frac{K \cdot N_t}{K + N_t - 1} \) DoFs without requiring CSIT. Thus, as \( K \) grows the system can approach the CSI-dependent upperbound of \( \min(N_t, K) \) DoFs, the same number as if there is perfect CSIT. This striking result goes against conventional intuition, and is of key interest as it may provide the potential to relieve the “dimensionality bottleneck” now faced by MU-MIMO.

However, BIA does have some inherent problems [9]. The first is that it often requires a high SNR or Signal to Interference plus Noise Ratio (SINR) to operate effectively, e.g., the original BIA scheme [6] may require SNRs on the order of 20 dB. This is due to the amplification of additive noise in the alignment process. As a consequence, the original BIA technique applied in a straight-forward fashion to cellular may not provide benefits to many cellular users.

In this paper we describe two novel advances and an extension of the analysis of BIA in cellular and cluster settings. The first advancement is a change in the power allocation to transmitted streams [9]. This can improve the operational SNR range of BIA significantly. The second advancement is the application of BIA transmissions to a scenario of overlapped-clusters of transmitting cells as a means to further boost users’ SINRs. While it has been shown that BIA is particularly attractive in such cluster-based systems [9], since BIA does not require CSIT and joint processing across cells, BIA does have additional CSIR requirements. We discuss CSIR related overheads of the BIA technique itself, and show how they may compare to the CSIT and CSIR overheads of conventional CSIT-based MU-MIMO. We show that, under certain assumptions of such overheads, BIA can be applied successfully in cellular and can improve over conventional MIMO for even cell-edge users in some scenarios. This provides perspective on the relationship between BIA, MU-MIMO and SU-MIMO, and what benefits BIA may ultimately have over conventional CSIT (and CSIR) based systems.
II. CSIT-BASED MU-MIMO AND BIA

A. Linear Zero-Forcing Beamforming

In the conventional downlink MU-MIMO, a multi-antenna BS uses CSIT to simultaneously transmit streams to multiple users. We consider the case \( N_r = 1 \) and assume \( K : K \leq N_t \) users are jointly served. We denote the channel between user \( k \) and the \( N_t \) TX antennas as \( h[k] \in \mathbb{C}^{1 \times N_t} \). Also, \( H \) stands for the \( K \times N_t \) matrix formed with \( k^{th} \) row being \( h[k] \).

In this paper we make comparisons to linear zero-forcing beamforming (LZFB) \cite{8} based on perfect CSIT\(^1\). LZFB is an attractive CSIT-based strategy that can achieve \( \min(N_t, K) \) DoFs. In LZFB the BS sends the codeword \( u[k] \) to user \( k \) in a direction orthogonal to the channel vectors \( h[j] \), \( j \neq k \). This can be done by applying a single information bearing stream for user \( k \) to a unit-norm beamforming vector \( w[k] \in \mathbb{C}^{N_t \times 1} \), where \( w[k] \) is a scaled version of \( k^{th} \) column of \( H^H \).

Higher DoFs do not necessarily translate to higher rates in the low SNR regime. For example, the case of \( K = N_t - 1 \) may yield higher rates than \( K = N_t \), although the latter case has more DoFs. Intuitively, this may happen since for \( K = N_t - 1 \) the choice of the beamforming vector \( w[k] \) carrying the message to user \( k \) is not unique, i.e., it lies in a two dimensional space orthogonal to the channels of other \( K - 1 \) users. Each user may therefore benefit in terms of useful received signal power from the flexibility in the beamforming. Benefits of creating less DoF will become more apparent when the performance BIA is examined.

B. Blind Interference Alignment

BIA \cite{6} enables the design of CSIT-free MU-MIMO systems using \( N_r = 1 \). For each user, BIA requires this “single” antenna to have multiple modes (up to \( N_t \)) that can change the CSI between the \( N_t \) TX antennas and the RX antenna in a linearly independent fashion. However, only one mode is active at any time, requiring only one Radio Frequency (RF) chain at the receiver. Thus for user \( k \) the system has multiple potential channels \( h[k](n) \), \( n = 1, \ldots, N_t \), whereby only one channel is available for use at each time slot. The BIA schemes of interest in this paper emanate from the original scheme in \cite{6}, and require that the values of \{\( h[k](1), \ldots, h[k](N_t) \)\} remain constant for the duration of a single instance of the BIA scheme. As shown in \cite{6}, the maximum number of the achievable DoF is described below:

**Theorem 1**: \cite{6} For the \( K \) user \( N_t \times 1 \) MISO BC, a total of \( \frac{N_r K}{N_t+K-1} \) DoF can be achievable, almost surely.

Next we present the original BIA scheme from \cite{6} in the case of \( K = 2 \) users and \( N_t = 2 \) antennas. The scheme achieves a total of \( 4/3 \) DoF (see \cite[Theorem 1]{6}) by means of sending two symbols to each user via a three-slot strategy that coordinates transmission with antenna-mode switching. In particular, letting \( x(t) \) denote the \( N_t \times 1 \) vector transmitted at time \( t \), and \( u_{k[i]} \) denotes the \( i^{th} \) data stream of user \( k \). Then the \( 6 \times 1 \) transmit signal vector is given by:

\[
\begin{bmatrix}
  x(1) \\
  x(2) \\
  x(3)
\end{bmatrix} =
\begin{bmatrix}
  I_2 & I_2 & O_2 \\
  I_2 & O_2 & I_2
\end{bmatrix}
\begin{bmatrix}
  u_{[1]} \ \\
  u_{[2]}
\end{bmatrix}
\]

(1)

and where \( I_M \) and \( O_M \) denote the \( M \times M \) identity and all-zero matrices, respectively. In the three time slots, user 1 cycles through modes 1, 2, 1, while user 2 through modes 1, 1, 2. Without loss of generality we consider the received signal vector at user 1, denoted as \( y[1] \). That is:

\[
\begin{bmatrix}
  y[1][1] \\
  y[1][2] \\
  y[1][3]
\end{bmatrix}
= \begin{bmatrix}
  h[1][1] & h[1][2] & 0 \\
  h[1][1] & 0 & h[1][2]
\end{bmatrix}
\begin{bmatrix}
  u_{[1]} \ \\
  u_{[2]}
\end{bmatrix}
+ z[1]
\]

(2)

Here \( y[1][t] \) denotes the received symbol of user 1 at the \( t^{th} \) time slot, \( t \in \{1,2,3\} \), and \( z[1] \) is the noise vector. From (2), it can be seen that the two interference beams are aligned in one dimension, leaving a two-dimensional interference-free space for the desired signal of user 1. Subtracting \( y[1][3] \) from \( y[1][1] \) yields an interference-free 2 × 2 MIMO channel:

\[
\begin{bmatrix}
  y[1][1] & y[1][3] \\
  y[1][2] & y[1][3]
\end{bmatrix} = \begin{bmatrix}
  h[1][1] & u_{[1]} \\
  h[1][2] & u_{[2]}
\end{bmatrix} + \begin{bmatrix}
  z[1][1] & z[1][3] \\
  z[1][2] & z[1][3]
\end{bmatrix}
\]

(3)

From (3), and even without CSIR, the interference is completely removed. However, the noise power in time slot 1 is doubled. For user 2, one can easily obtain a similar signal relationship as user 1. Thus, a total of \( 4/3 \) DOF is achieved. For the general \( K \) user \( N_t \times 1 \) MISO BC, extending the insight of this example as shown in \cite{6}, each user can on average achieve \( N_t \) DoF per \( N_t + K - 1 \) time slots. That is to say, within an \( N_t + K - 1 \) dimensional space at each receiver, \( N_t(K-1) \) interference beams align into \( K-1 \) dimensions, leaving an \( N_t \) dimensional space for the desired signal. Thus, a total of \( \frac{N_r N_t K}{N_t+K-1} \) DoF can be achieved.

We emphasize two properties of the above scheme: (1) There is no beamforming or joint-coding across the transmit antennas. (2) The \( K \) users are served simultaneously.

III. CSIT AND CSIR OVERHEADS

A key advantage of BIA is that it has no CSIT overhead. In contrast, in a Frequency Division Duplex (FDD) system using CSIT-based MU-MIMO such as LZFB each user has to first estimate CSI based on downlink estimation pilots (EPs) transmitted by the BS. The users then quantize and feed back descriptions of CSI estimates to the BSs via the uplink channel, providing CSIT. With this a BS can design a precoder (e.g. LZFB beams), and transmits both data and dedicated pilots using the precoder. The dedicated pilots provide CSIR to support coherent detection (CD) by users.

\(^1\)The case of imperfect CSIT is discussed in \cite{4}.
Under LZFB let \( \theta_{\text{csi}} \) and \( \theta_{\text{cd}} \) denote the fraction of the total downlink TX resource used for EPs per TX antenna, and used for CD pilots per user, respectively. Feedback (FB) overhead, while on a different frequency band in FDD, is a cost to the system (albeit in the uplink resource) that must be accounted for. We let \( \theta_{\text{fb}} \) be the fraction per user per TX-antenna of total downlink TX resources that has to be re-allocated to the uplink to support the uplink feedback\(^2\).

For example, if in an FDD-based LTE design each Resource Block of 12 × 14 OFDM symbols uses 8 symbols for EPs for each TX antenna, then \( \theta_{\text{csi}} = 8/168 = 4.76\% \). Thus if \( N_t = 4 \), and assuming the pilot structure scales, the total EP overhead is \( N_t \theta_{\text{csi}} = 19\% \), leaving 81\% of symbols for other signaling. FB overhead in conventional designs scales as the product of the number of transmit antennas and the number of users \( K \) from which feedback is collected. However if MIMO is exploited in the feedback link the FB overhead can be designed to scale linearly with the number of users. Finally, we assume the total CD pilot overhead grows as \( K \theta_{\text{cd}} \).

Since BIA does not require CSIT, \( \theta_{\text{csi}} = \theta_{\text{fb}} = 0 \). Let \( \theta_{\text{cd}} \), denote CD pilot overhead per TX antenna for BIA. Note that for CD each user has to estimate the channel between each transmit antenna and each of its receive antenna modes. Thus in the best case\(^3\) the CD pilot overhead grows as \( N_t \theta_{\text{cd}} \), and in the worst case it scales as \( N_t^2 \theta_{\text{cd}} \). Thus, the CSIR CD overhead of BIA may in fact be larger that of LZFB.

Table I summarizes the overheads of a cellular system with \( N_t \) antennas per BS serving \( K \) users, and a cluster system where clusters of \( C = 2 \) adjacent BSs, each with \( N_t \) antennas, serve \( K \) users using \( N_t^2 \) antennas. The exact values of \( \theta_{\text{csi}}, \theta_{\text{fb}}, \theta_{\text{cd}}, \) and \( \theta_{\text{cd}}' \), and the scaling of each, depend on the system design, SNRs and SINRs, coherence times and bandwidths of channels [4].

<table>
<thead>
<tr>
<th>Type</th>
<th>Cellular</th>
<th>Cluster ( C = 2 )</th>
<th>Cellular</th>
<th>Cluster ( C = 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>EPs</td>
<td>( N_t \theta_{\text{csi}} )</td>
<td>0</td>
<td>( 2N_t \theta_{\text{csi}} )</td>
<td>0</td>
</tr>
<tr>
<td>PB</td>
<td>( K \theta_{\text{fb}} ) to ( KN_t \theta_{\text{fb}} )</td>
<td>0</td>
<td>( 2KN_t \theta_{\text{fb}} )</td>
<td>0</td>
</tr>
<tr>
<td>CD</td>
<td>( K \theta_{\text{cd}} ) to ( N_t^2 \theta_{\text{cd}} )</td>
<td>( 2N_t \theta_{\text{cd}} ) to ( 4N_t^2 \theta_{\text{cd}} )</td>
<td>( 2N_t \theta_{\text{cd}} ) to ( 4N_t^2 \theta_{\text{cd}} )</td>
<td>( 2N_t \theta_{\text{cd}} ) to ( 4N_t^2 \theta_{\text{cd}} )</td>
</tr>
</tbody>
</table>

IV. REDUCING THE SNR REQUIRED BY BIA

The achievable rate with zero-forcing interference at each user is obtained in [6], and is restated next without proof.

**Theorem 2** [6] For the \( K \) user \( N_t \times 1 \) MISO BC, the achievable sum rate zero-forcing interference at each user is

\[
R = \sum_{k=1}^{K} \frac{1}{N_t+K-1} \mathbb{E} \left\{ \log \det \left( \mathbf{I} + \frac{(K+N_t-1)P}{N_t^2 K} \mathbf{H}^H[k] \mathbf{H}[k] \right) \right\}
\]

\[ (4) \]

\(^2\)In principle, FB overhead should be accounted for as an uplink resource constraint. In particular, it is more meaningful to look at the tradeoff in downlink achievable rate versus uplink FB resources [3]. This paper can crudely capture this uplink constraint by appropriate size of the factor \( \theta_{\text{fb}} \).

\(^3\)This may be achieved by the single receive RF chain, over-sampling pilots with factor \( N_t \) (with a loss in received pilot SNR). Alternatively, schemes such as e.g., [5] can be used to yield this linear in \( N_t \) dimensionality cost.

where \( \mathbf{H}[k] = \left[ \frac{1}{\sqrt{K}} \mathbf{h}[k]^H(1), \ldots, \frac{1}{\sqrt{K}} \mathbf{h}[k]^H(N_t - 1), \mathbf{h}[k]^H(N_t) \right]^T \)

As noted before, in BIA each user needs at least \( N_t \) modes. Without antenna-mode switching the channel to each user is statistically equivalent and the DoF collapses to one only. The maximum achievable rate in this case is equal to the capacity of a point-to-point SU-MISO channel without CSIT.

DoFs, however, do not fully describe the capacity in the low SNR regime. To illustrate this consider the first two examples in Fig. 1 which refer to the original BIA scheme in (3) and SU-MISO. It shows that if SNR is smaller than around 17 dB (for \( N_t = 2, K = 5 \)) or 16 dB (for \( N_t = K = 6 \)), then BIA would lose its superiority. Referring back to (3), this problem comes from noise enhancement due to the alignment frames.

Fig. 1. Rate Comparison in the Low SNR Regime

An “Improved BIA” scheme is shown in Fig. 1 that achieves reduced noise enhancement via a constant power transmission (other power allocations are given in [9]). To illustrate such a scheme take as an example the case \( N_t = 2, K = 2 \). In the original BIA scheme in (1), assuming independent equal-power data streams \( u_k^T \) the transmit power in time slot 1 is twice of that in time slot 2 and 3. In the equal-power Improved BIA scheme the transmission power made the same in all slots by scaling transmission blocks appropriately. Such a scheme is given by:

\[
\mathbf{x} = \begin{bmatrix}
\frac{1}{\sqrt{2}} \mathbf{I}_2 \\
\mathbf{I}_2 \\
\mathbf{O}_2 \\
\mathbf{I}_2
\end{bmatrix} u_1^T + \begin{bmatrix}
\frac{1}{\sqrt{2}} \mathbf{I}_2 \\
\mathbf{O}_2 \\
\mathbf{I}_2 \\
\mathbf{I}_2
\end{bmatrix} u_2^T.
\]

\[ (5) \]

The received signal at user 1 is:

\[
\mathbf{y}^{[1]} = \begin{bmatrix}
\frac{1}{\sqrt{2}} \mathbf{h}_1^{[1]}(1) \\
\mathbf{I}_2 \\
\mathbf{O}_2 \\
\frac{1}{\sqrt{2}} \mathbf{I}_2
\end{bmatrix} u_1^T + \begin{bmatrix}
\frac{1}{\sqrt{2}} \mathbf{h}_1^{[1]}(2) \\
\mathbf{I}_2 \\
\mathbf{O}_2 \\
\mathbf{I}_2
\end{bmatrix} u_2^T + \mathbf{z}^{[1]}
\]

and using a similar cancelation as for (1), user 1 obtains an interference-free \( 2 \times 2 \) MIMO channel, by zero-forcing the interference (now aligned along \( [1/\sqrt{2}, 0, 1]^T \)). Thus the DoF of such a system is still \( 4/3 \). The average transmit power can be made the same as in (1) by appropriate transmission power settings.
The ergodic rate for a scheme with general \( N_t, K \) is given by the following theorem (proof is omitted for brevity).

**Theorem 3** For the \( K \) user \( N_t \times 1 \) MISO BC, the achievable sum rate zero-forcing interference at each user is given by

\[
R = \sum_{k=1}^{K} \frac{1}{\text{max}(d_k - 1)} \mathbb{E} \left[ \log \det \left( I + \sum_{n=1}^{N_t} H[k] H[k]^\dagger \right) \right]
\]  

where \( H[k] = \left[ \frac{h[k]^\dagger (1)}{\sqrt{2K-1}}, \ldots, \frac{h[k]^\dagger (N_t - 1)}{\sqrt{2K-1}}, h[k]^\dagger (N_t) \right]^\dagger \).

As Fig. 1 reveals, such constant-power schemes show improved performance with respect to the original BIA scheme, with benefits increasing with larger \( N_t \) and \( K \). Specifically, the cross point moves from around 17 dB to 10 dB for \( N_t = 2, K = 5 \). For \( N_t = K = 6 \), the cross point moves from around 16 dB to 7 dB. Here, also, the rate loss is negligible even at low SNR with the improved BIA performing better than SU-MISO in nearly all SNR regimes. More details on the design and benefit of power allocations are given in [9].

V. CELLLULAR AND CLUSTER-BASED LZFB AND BIA

We consider an one-dimensional (1D) topology as shown in Fig. 2 on the interval \([0, B]\), with \( B = 8 \) cells and cell \( b \) spanning region \([b, b + 1]\) for \( b = 0, \ldots, 7 \). In cell \( b \), there is one \( N_t \)-antenna BS located at \( b + 0.5 \). In order to eliminate boundary effects, we assume a wrapped topology so that, e.g., the distance between points \( x_1, x_2 \in [0, B) \) is the wrapped topology distance, i.e., the minimum of \(|x_1 - x_2| \) and \( B - |x_1 - x_2| \). We assume a path loss model of the form [4], i.e.,

\[
g(d) = G_0 d^{-\kappa} / (\delta^\kappa + d^\kappa) \tag{8}
\]

where \( d > 0 \) is the distance between the transmitter and the receiver, \( \kappa \) is the propagation exponent, and \( \delta \) is the 3 dB breakpoint distance. The constant \( G_0 \) sets the transmit power at each BS and implicitly the received SNR.

In this paper we focus on architectures where groups of users in the same relative cell location are simultaneously and independently served by their BSs. Fig. 2 shows a cellular example, where BSs concurrently serve groups of users located at a distance 5 to their anchor BS.

Despite its improved (reduced) SNR requirement, the BIA scheme of Section IV still requires SNRs around 10 dB. One way to increase SNRs (SINRs) to cell-edge users is to use a Frequency Reuse (FR) 2 cellular system. Frequency reuse however reduces the number of transmission resources available to each station, and can hurt performance.

An attractive alternative is to use a cluster shifting approach [7]. Here all stations use all frequency resources. Fig. 3 illustrates such a system in 1D, where non-overlapping clusters of \( C = 2 \) adjacent stations are used to serve a group of users between the stations. Two overlapping patterns exist, and are applied either in different time or frequency resources. Consider the two groups of users \( G_1 \) and \( G_2 \). In Fig. 2 the two groups are independently served by their anchor and each served user experiences interference from the transmitting BSs in all the other cells. In contrast to FR-1 cellular, in both the FR-2 cellular and the cluster-based network in Fig. 3, \( G_1 \) and \( G_2 \) can be served on different bands (or slots). Unlike FR-2 cellular however, in the cluster-based network, on either band, each user is served by the two closest BSs. In this paper we will consider the performance of LZFB and BIA schemes in each of the following three architectural scenarios:

**Scenario 1:** (Cellular, FR-1) Cells transmit simultaneously with frequency reuse factor 1. All the cells work in the same manner, independently, serving on each slot a subset of users all residing at the same relative location within their cell.

**Scenario 2:** (Cellular, FR-2) A BS only transmits on half of the total bands, with doubled transmit power. Odd (even) indexed cells engage in simultaneous transmissions in odd (even) bands, and are silent in even (odd) bands. As in Cellular FR-1, cells transmit independently and serve on a given slot users residing at the same relative location within their cell.

**Scenario 3:** (Cluster-based, FR-1) On odd bands the \( B \) cells are split into \( B/2 \) clusters of the form \((2b, 2b + 1)\), while in the even bands, the cells are split into clusters of the form \((2b - 1, 2b)\), as in Fig. 3. Clusters work in a symmetric manner, independently. Each cluster on a slot transmits to a subset of users at a distance \( d \) from the left station (distance \( 1 - d \) from the right station).

For each scenario, we examine the achievable rates of the LZFB and BIA schemes, assuming equal power allocation to each data stream and equal transmission power in each time slot. Further performance improvements through power control on data streams and across BSs for both BIA and LZFB schemes are found in [7], [9].

Under Scenario 1 consider slots on which BSs serve users at location \( d \), with \( d \) uniform in \([-0.5, 0.5)\). For each \( b \), cell \( b \) serves simultaneously \( K = K(d) \) users located at \( b + 0.5 + d \), where \( K(d) \) is location specific.

For LZFB \( K(d) \leq \min(N_t, K_{\text{max}}) \) where \( K_{\text{max}} \) denotes the number of users at the same location. Due to symmetry and without loss of generality, we focus on cell 0 and consider serving a set of users at location \( d + 0.5 \). Then the received signal \( y_{[k]} \) at user \( k \) in cell 0 is given by:

\[
y_{[k]} = \sqrt{g[|d|]h_{[k]}^{[b]} x_{[b]}^{[b]} + \sum_{b \neq 0} \sqrt{g[|d - b|]h_{[k]}^{[b]} x_{[b]}^{[b]} + z_{[b]}}} \tag{9}
\]
where $x_i^{[k]} = w_i^{[k]}u_i^{[k]} \in \mathbb{C}^{N_t \times 1}$ is the transmit signal vector intended for user $i$ in cell $c_i$, and $w_i^{[k]}$ and $u_i^{[k]}$ are the unit-norm beamforming vector and the message-carrying stream of user $i$ in cell $c_i$ respectively, and due to LZFBI $h_i^{[k]}w_0^{[k]} = 0$ for $i \neq k$. In addition, $h_0^{[k]} \in C^{N_t \times 1}$ denotes the (power-normalized) $1 \times N_t$ channel vector between user $k$ in cell $0$ and the BS in cell $b$, and $|d-b|$ denotes the wrapped topology distance between BS in cell $b$ and users in cell $0$. The ergodic rate of users per cell per band by LZFBI is

$$R_{FBI-1}^Z(d) = \max_{1 \leq K \leq \min(N_t, K_{\text{max}})} R_{FBI}^Z(d; K) \tag{10}$$

whereby $R_{FBI}^Z(d; K)$ denotes the maximum achievable long-term rate provided at relative location $d$ when each BS serves $K$ users at relative location $d$ of its cell, and it is given by

$$R_{FBI}^Z(d; K) = \sum_{k=1}^{K} \log \left(1 + \frac{g(|d|)|h_i^{[k]}w_0^{[k]}|^2}{1 + \sum_{b \neq d} g(|d-b|)K |h_i^{[k]}w_b^{[k]}|^2} \right).$$

To illustrate the BIA operation in Scenario 1, first consider the case that two users are served simultaneously in each cell with $N_t = 2$ antennas. We use the same beamforming matrix structure in each cell as described in Section III, whereby the BS in each cell sends 2 symbols to each served user in its cell over three time slots. Denoting via $y^{[k]}$ the $3 \times 1$ received vector of user $k$ in cell $0$, we have

$$y^{[k]} = H_i^{[k]}x_0 + \sum_{b \neq 0} H_b^{[k]}x_b + z^{[k]} \tag{11}$$

where $H_i^{[k]} = \sqrt{g(|d-b|)} \left[ \text{diag}(h_i^{[k]}(1), h_i^{[k]}(2)) \right]$ and $x_0$ is given by $x_0 \in \mathbb{C}^{N_t \times 1}$ with the two data streams of user $k$ replaced by $u_i^{[k]} = [u_i^{[k]}(1), u_i^{[k]}(2)]^T$. $h_i^{[k]}(n)$ denotes the channel vector between the $n$th antenna mode of user $k$ in cell $0$ and the BS in cell $b$. User 1 can eliminate interference caused by user 2 by subtracting $R_{ZF}(t)w_0^{[k]}$ from $y^{[k]}$, whereby the BS in each cell sends 2 symbols to each served user and the BS in cell $b$ sends $2$ symbols to each served user in cell $0$. We use the same beamforming vector between the $k$th antenna of user $k$ in cell $0$ and the BS in cell $b$. User 1 can eliminate interference caused by user 2 by subtracting $\sqrt{2/3}$ times the third entry of $y^{[k]}$ from $\sqrt{2/3}$ times the first entry of $y^{[k]}$. In general at user $k$, we obtain a $2 \times 2$ effective MIMO channel of the form:

$$y^{[k]} = H_i^{[k]}u_i^{[k]} + \sum_{b \neq d} H_b^{[k]}u_b^{[k]} + z^{[k]} \tag{12}$$

where $H_i^{[k]} = \sqrt{g(|d-b|)} \left[ \begin{array}{c} h_i^{[k]}(1) \\ h_i^{[k]}(2) \end{array} \right]^T$ and the noise vector $z^{[k]} \sim \mathcal{CN}(0, I)$. Note that in addition to having interference eliminated from user 2 in cell 0, the BIA scheme at receiver 1 also has eliminated interference from user 2 transmissions from all other cells. In the general $N_t$, $K$, after user $k$ zero-forces interference, one can obtain an effective $N_t \times N_t$ measurement MIMO channel of the form (12), where $z^{[k]} \sim \mathcal{CN}(0, I)$, and

$$H_i^{[k]} = \sqrt{g(|d-b|)} \left[ \begin{array}{c} h_i^{[k]}(1) \\ \sqrt{2K-1} h_i^{[k]}(N_t-1) \\ \sqrt{2K-1} h_i^{[k]}(N_t) \end{array} \right]^T$$

Comparison of the BIA expression (12) against the associated LZFBI expression (9) reveals an important inherent advantage of BIA to eliminate both intra-cell interference and inter-cell interference from all but one stream from each of the operating cells [9]. Exploiting (12) in a manner similar to [6], by treating the remaining inter-cell interference as noise, the ergodic rate of the users per cell per band can be expressed as:

$$R_{FBI-1}^B(d; K) = \log \left| I + \frac{H_i^{[k]}H_i^{[k]}^H}{N_t} \right| \left( \text{E} \left[ 1 + \sum_{b \neq d} H_b^{[k]}H_b^{[k]}^H \right] \right) \right|.$$

Achievable rate expressions for LZFBI and BIA can be similarly derived for Scenarios 2 and 3. We omit the detailed analysis in this section due to the space limitation.

VI. SIMULATIONS AND RESULTS

In this section we compare the performance of LZFBI and BIA in cellular and cluster-based scenarios via Monte-Carlo simulations of the rate expressions previously defined. Ergodic rates are evaluated per location, assuming $N_t = 4$ and $K_{\text{max}} = 20$ users at each location. An eight cell 1D wrapped topology is used and the path loss parameters are $k = 3.8$, $\delta = 0.05$ and $G_0 = 50$ and $80$ dB. For LZFBI $K \leq \min(N_t, K_{\text{max}}) = 4$ are selected at random from the location being served for cellular transmission, and $K \leq \min(2N_t, K_{\text{max}}) = 8$ from each location being served for cluster-based transmission. For BIA we optimize $2 \leq K \leq K_{\text{max}}$ per location, selecting the $(M, K)$ BIA scheme, where $M = N_t$ for cellular and $M = 2N_t$ for clusters, which maximizes the total ergodic rate at that location.

Fig. 4 shows achievable ergodic rates at user locations in the interval $[0.5, 2.5]$ using LZFBI and BIA. Due to symmetry, the performance over any cell $b$, i.e., over $[b, b+1]$, is the same as cell 1 over $[1, 2]$. The rates shown in Fig. 4 ignore the inherent CSI overheads $\theta_{csi}, \theta_{fb}, \theta_{cd}$ and $\theta_{cd'}$ and are therefore upper bounds on net-rates for these scenarios.

For a cellular network with reuse factors 1 and 2, the (upper bound) LZFBI performance is uniformly better if $G_0 = 50$ dB. In contrast, if $G_0$ increases to 80 dB, then BIA performance is nearly uniformly better. The cluster-based scenario has some subtle differences. Here the LZFBI remains superior to BIA for cell center users. This is in fact due to the increased DoFs afforded by LZFBI coupled with the increased SINRs provided by cluster-based transmission. However, the benefits of BIA for, e.g., cell-edge users can be more pronounced than in cell-based transmission.

In practice, the benefit of BIA is seen when considering net-rates accounting for CSIT and CSIR overheads. For LZFBI the net overhead-cost ratio is between $\theta_{csi}, 2N_t + \theta_{fb}K + \theta_{cd}$ (referred to as “OH L”) and $\theta_{csi}, 2N_t + \theta_{fb}K + \theta_{cd}K$ (“OH U”). In contrast, for BIA the associated overhead cost ratio is between $\theta_{cd'}, 2N_t + \theta_{cd}K$ (“OH L”) and $\theta_{cd'}, (2N_t)^2$ (“OH U”). The net-rate of a given scheme is the rate ignoring CSI times one minus this net overhead-cost ratio. Since this ratio depends on

4Operationally, the rate expression (13) assumes that user $k$ in cell 0, possesses only statistical knowledge about the channels $H_b^{[k]}$, for $b \neq 0$. 

This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE ICC 2011 proceedings.
one can reconsider the optimization (e.g. reconsider $K$) of the scheme at each location accounting for the extra factor.

Fig. 5 depicts the net-rates considering such an optimization for the scenario Cluster FR-1 assuming $G_0 = 80$ dB and $\theta_{csi} = \theta_{cd'} = \theta_{cd''} = 1\%$. Overhead costs relative to “OH L” and “OH U” are considered for each case. In addition, two distinct $\theta_{fb}$ values are shown for LZFB: $\theta_{fb} = 1\%$ (“FB L”) and $\theta_{fb} = 3\%$ (“FB U”).

As the figure reveals, when overheads are taken into account, in the optimistic case (“OH L”) BIA is superior to LZFB at the cell edges (around integer locations). In the pessimistic case (“OH U”) two trends are evident. First, depending on the relative FB cost, LZFB can be uniformly inferior or superior to the BIA scheme. Second, in all cases the performance is substantially lower than that in the optimistic schemes, suggesting the importance of CSI-overhead efficient designs for both BIA and LZFB.

Finally, as Fig. 5 reveals, for cluster-based LZFB the worst-case rate may not be achieved at the cell edge. This is an artifact of location-based transmission schemes, whereby LZFB (and BIA) is constrained to serve users from the same location. For any location served, all users see the same nominal SINR from a given transmit BS, but the SINRs from the two transmitting BSs are different except at the cell edges. Although schemes jointly serving users from pairs of locations in cluster systems can be readily constructed, such studies are beyond the scope of this work.

VII. CONCLUSION

Blind interference alignment (BIA) schemes are of interest in MU-MIMO wireless systems, due to their ability to increase DoFs with reduced CSIT overheads and to simplify operation in cluster-based transmission while also aligning inter-cluster interference. In this paper we study and compare the performance of BIA and linear zero-forcing beamforming (LZFB) schemes in the downlink of FDD-based cellular and cluster-based architectures. The study focuses on location-based user scheduling with equal-power transmission to users and takes into account the overheads required to enable these transmission schemes. Simulations show that even under a small CSI overhead factor, BIA is able to outperform LZFB.

Several issues are worthy of further study. To begin, cellular BIA systems can be viewed more generally in terms of options of code-reuse and frequency-reuse [9], and this itself can be refined to consider unequal power allocation to codes and cells. In fact, for some users, code-reuse and cluster transmission lead to equivalent systems [9]. Furthermore, advances in BIA schemes, in particular for large $N_t$ and $K$ values, are required to reduce the joint-transmission block-lengths, which can be exceedingly large in existing systems [6].

REFERENCES