Weighted Sum Rate of Multi-Cell MIMO Downlink Channels in the Large System Limit

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Abstract—The optimization of the weighted ergodic sum rate is considered for the downlink of a cellular networks with multiple cells and multi-antenna base stations. We focus on the large system limit where the number of base station antennas and the number of users per cell go to infinity with a fixed ratio. We consider two extreme cases of full inter-cell cooperation (network MIMO) and no inter-cell cooperation (single-cell multi-user MIMO). Using the large random matrix theory and Lagrangian optimization, we obtain a numerical algorithm that exactly computes the maximum weighted sum rate in this asymptotic regime. Numerical results are presented for a simple case of two interfering cells in a linear arrangement.

I. INTRODUCTION

Multiuser multiple-input multiple-output (MU-MIMO) technology holds the potential to drastically improve the spectral efficiency of the next generation cellular systems by exploiting spatial multiplexing without requiring cumbersome multi-antenna user terminals. Many research efforts have been focused on the single cell setting with the full information theoretic understanding of the underlying MIMO Gaussian broadcast channel (see [1]–[3] and references therein).

In order to appreciate the full potential of such technology in a realistic wireless cellular setting, the following important aspects need to be considered: 1) a multi-cell coverage with realistic pathloss model and users’ spatial distribution; 2) the type of inter-cell cooperation (e.g., see [4]); 3) multiuser scheduling, taking into account fairness issues; 4) the type of downlink precoding and signal processing employed and the type of channel state information available at the transmitter (CSIT). While taking into account all the above aspects is very complicated and results in a model (MIMO Gaussian broadcast and interference channel with imperfect CSIT) that is not even fully understood from an information theoretic viewpoint, it makes sense to tackle a subset of these aspects at a time, seeking a clean closed-form or at least semi-analytic expressions towards a better understanding of the possible capacity gains and the tradeoffs involved to achieve them.

This work represents a step forward in the above direction, where we consider multiple cells, pathloss and users’ spatial distribution, and limit ourselves to two extreme cases of inter-cell cooperation: full multi-cell joint processing (akin to the so-called “Wyner model” [5]) and no inter-cell cooperation where inter-cell interference (ICI) is treated as noise. Fairness is a very important aspect in cellular networks, since users may be in very different conditions (near or far from the base station (BS), suffering from interference from adjacent BSs, etc.). The sum capacity is, in the sense of fairness, generally not a good measure for the system performance, since with a sufficiently large number of users it is typically obtained by scheduling only the users close to a BS (“center” users). While this maximizes the sum rate, it results in an unacceptably poor quality of service for the users in unfavorable locations (“edge” users). Since the ergodic achievable rate region \( \mathcal{R} \) is obtained as the convex hull of all achievable rates, it is easy to see that \( \mathcal{R} \) is a convex and bounded region. It follows that operating at any desirable point on the region boundary can be defined as a weighted sum rate maximization problem of

\[
\mathbf{R}^* = \arg \max_{\mathbf{R} \in \mathcal{R}} \sum_{k \in \text{users}} W_k \mathbf{R}_k \tag{1}
\]

where \( k \) runs over all the users in the system and \( \{W_k\} \) are suitable non-negative weights.

In this paper, we propose a method to compute (1) under the assumptions said above, in the large system limit where the number of antennas per BS and the number of users per cell become large with a fixed ratio. In order to achieve this goal, we combine the results of [6] on the asymptotic sum capacity for the MIMO multiple access channel and the well-known uplink-downlink duality [2], [7] with the input covariance matrix optimization technique of [8].

II. SYSTEM MODEL

We consider a multi-cell MIMO downlink system where \( M \) BSs with \( \gamma N \) antennas each communicate to \( KN \) single antenna users. \( KN \) users are divided into \( K \) co-located “user groups” of equal size \( N \) and \( \gamma \) indicates the ratio of the number of BS antennas to the number of users per group. Co-located users are characterized by (approximately) the same set of distances from the BSs. However, since (typically) a wavelength is much smaller than the distances between BSs and users \(^1\), we assume that users (also co-located users) are separated by a sufficiently large number of wavelength such that they undergo i.i.d. small-scale fading. Notice that this model accounts for an arbitrary placement of BSs’ and users’ locations. Specific examples will be given in Section IV. The received signal vector \( \mathbf{y}_k = [y_{k,1} \cdots y_{k,N}]^T \in \mathbb{C}^N \) for the \( k \)-th user group is given by

\[
y_k = \sum_{m=1}^{M} \alpha_{m,k} \mathbf{H}_{m,k}^H x_m + \mathbf{n}_k \tag{2}
\]

\(^1\)A wavelength here is referred to the carrier frequency \( f_0 \). For example, for \( f_0 = 2 \) GHz the wavelength is \( \lambda = 15 \) cm. On the other hands, distances between users and BS may range from 10 to \( 10^3 \) m.
where $\alpha_{m,k}$ and $H_{m,k}$ denote the (distance dependent) pathloss and the $N\times N$ channel matrix from the $m$-th BS to the $k$-th user group, respectively, $x_m = [x_m,1 \cdots x_m,N\gamma]^{T} \in \mathbb{C}^{N\gamma}$ is the transmitted symbol vector from the $m$-th BS and $n_k = [n_k,1 \cdots n_k,N]^{T} \in \mathbb{C}^{N}$ is additive white Gaussian noise (AWGN) at the $k$-th user group receivers. The elements of $n_k$ and $H_{m,k}$ are i.i.d. with $\mathcal{CN}(0,1)$. The input vectors $x_m$ satisfy the per-BS sum power constraint $\text{Tr}(E|x_m|x_m^{H}) \leq P_m$, where $P_m$ is the total power available at the $m$-th BS. We assume that both the BSs and the users know the CSI perfectly.

**A. No inter-cell cooperation**

We consider a fixed allocation of user groups to BSs, defined by the partition $\{\mathcal{K}_1, \cdots, \mathcal{K}_M\}$ of the set $\{1, \cdots, K\}$, such that if $k \in \mathcal{K}_m$, the $k$-th user group is served by the $m$-th BS. The users in the $k$-th user group for $k \in \mathcal{K}_m$ treat the ICI (signals received from the other BSs $n \neq m$) as additional Gaussian noise. We write:

$$y_k = \alpha_{m,k}H_{m,k}^{H}x_m + \sum_{n \neq m} \alpha_{n,k}H_{n,k}^{H}x_n + n_k. \quad (3)$$

We assume that the $m$-th BS is aware of the long-term average interference power at the user groups in $\mathcal{K}_m$, where “long-term average” indicates averaging with respect to the small-scale fading for fixed pathloss coefficients. It is immediate to show that this average interference plus noise power at any user terminal in group $k \in \mathcal{K}_m$ is given by

$$\sigma_k^2 = 1 + \sum_{n \neq m} \alpha_{n,k}^2 P_n. \quad (4)$$

Hence, from the $m$-th BS viewpoint, the system is akin to a single-cell MIMO downlink channel given by

$$y_m = H_{m}^{H}x_m + v_m \quad (5)$$

where $y_m \in \mathbb{C}^{N|\mathcal{K}_m|}$ is the stacking of $\{y_k\} \in \mathcal{K}_m$, $H_m \in \mathbb{C}^{N\gamma \times N|\mathcal{K}_m|}$ is the stacking of $\{\alpha_{m,k}H_{m,k}\} \in \mathcal{K}_m$, and $v_m \in \mathbb{C}^{N|\mathcal{K}_m|}$ is a Gaussian vector with independent components and block-diagonal covariance matrix $E[v_mv_m^{H}] = \text{diag}\{\sigma_k^2|\mathcal{K}_m|\}$. 

**B. Full inter-cell cooperation**

In this case, all users are simultaneously served by all BSs that form a distributed MIMO transmitter with $MN\gamma$ antennas. The relevant channel model is written as

$$y = H^{H}x + n \quad (6)$$

where the composite channel matrix $H \in \mathbb{C}^{MN\gamma \times KN}$ is

$$H = \begin{bmatrix} \alpha_{1,1}H_{1,1} & \cdots & \alpha_{1,K}H_{1,K} \\ \vdots & \ddots & \vdots \\ \alpha_{M,1}H_{M,1} & \cdots & \alpha_{M,K}H_{M,K} \end{bmatrix}, \quad (7)$$

$x \in \mathbb{C}^{MN\gamma}$ is the composite input signal vector and $n \in \mathbb{C}^{KN}$ is the composite noise vector. For simplicity, in this case we will consider the total sum power constraint of $\text{Tr}(E|xx^{H}|) \leq P$ where $P = \sum_{m=1}^{M} P_m$. It turns out that in a symmetric situation and in the large-system limit, the results for this relaxed constraint coincide with the per-BS power constraint with $P_m = P/M$.

**C. Dual uplink channel**

The downlink optimization is more easily computed via the dual uplink channel [2], [7], [9], although this duality was used mainly to solve for the instantaneous capacity, i.e., for fixed fading coefficients. Here, we shall use it to address the ergodic weighted sum capacity in the asymptotic regime. For both models (5) and (6), the corresponding dual uplink channel model is given in the form of

$$r = \tilde{H}s + w \quad (8)$$

where we define $s \in \mathbb{C}^{NA}$ with diagonal covariance matrix $Q$ subject to $\text{Tr}(Q) \leq Q$ and $w \sim \mathcal{CN}(0,\Sigma_{\text{N}})$, and $H \in \mathbb{C}^{NB \times NA}$ is given by the stacking of blocks $\beta_{m,k}H_{m,k}$. For the non-cooperative case (5), we have $A = |\mathcal{K}_m|$, $B = \gamma$, $Q = P_m$, $\beta_{m,k} = \alpha_{m,k}/\sigma_k$ and the stacking involves the blocks for $k \in \mathcal{K}_m$. Notice that in order to obtain the dual uplink channel, we have to incorporate the ICI plus noise powers (4) seen at each user terminal of the original downlink channel by scaling the channel coefficients. In the case of (6), we have $A = K$, $B = M\gamma$, $Q = P$, $\beta_{m,k} = \alpha_{m,k}$ and the stacking involves all blocks, as in (7). For the sake of space limitation and in order to avoid repetitions, we develop a single algorithm that will be applied to both cases, with the corresponding definitions.

**III. WEIGHTED ERGODIC SUM RATE MAXIMIZATION**

We assume that the distance-dependent pathloss coefficients $\alpha_{m,k}$ are fixed while the small-scale fading coefficients $H_{m,k}$ change in time according to some ergodic process with the assigned first-order Gaussian i.i.d. distribution (for simplicity, we may assume that they are i.i.d. also in time). This captures a typical situation where the distance between BSs and users undergoes significant variations over a time-scale of the order of the tens of seconds, while the small-scale fading decorrelates completely within a few milliseconds [10].

In this regime, the weighted sum rate maximization of (1), where $R = R(\alpha)$ denotes the region of achievable ergodic rates for fixed $\{\alpha_{m,k}\}$ and random $\{H_{m,k}\}$, captures in general any desirable “fairness” requirement in terms of ergodic rates (or long-term average “throughputs”). By symmetry, all users in the same user group are indeed indistinguishable (they can be interchanged without changing the achievable region). Therefore, it makes sense to assume that they are given the same priority and we shall assume that users in the same user group $k$ have the same weight equal to $W_k$. With an appropriate choice of the weights $\{W_k\}$, we can force the system to work at any fairness point, for example, the proportional fair point which maximizes the network utility function $\sum_k \log R_k \text{ over } R \in R$. The weights for which the solution of (1) corresponds to the optimization of some desired network utility function can be obtained through stochastic network optimization techniques [11] and in case of the large system limit, they can be found in a static way with a formulation of the Lagrange dual problem. Here we deal with the problem (1) for general arbitrary weights and the problem of how to determine the weights for a particular network utility function is treated in [12].
A. One user per location

We first look at the problem assuming $N = 1$, and then extend the approach to the case $N \to \infty$, for which an efficient algorithm is found. Setting $N = 1$, the channel matrix $\hat{\mathbf{H}}$ in (8) has a dimension of $B \times A$ and $\hat{h}_k$ denotes its $k$-th column. It is well-known that, for fixed (dual uplink) transmit powers $Q_1, \cdots, Q_A$, successive interference cancellation (SIC) achieves the weighted sum rate. In particular, let $\pi = (\pi_1, \cdots, \pi_A)$ denote the permutation that sorts the weights in non-decreasing order, $W_{\pi_1} \leq \cdots \leq W_{\pi_A}$. Then, the optimal SIC order decodes user $\pi_1$ first and user $\pi_A$ last.

We introduce the following notation: quantities with subscript $[k : A]$ define submatrices containing the columns from $\pi_k$ to $\pi_A$. Then, we define

$$
\mathbf{G}_{k:A} = \mathbf{I}_B + \hat{\mathbf{H}}_{k:A} \mathbf{Q}_{k:A} \hat{\mathbf{H}}_{k:A}^H,
$$

$$
\mathbf{C}_{k:A} = \mathbf{G}_{k:A} - Q_{\pi_i} \mathbf{h}_i \mathbf{h}_i^H,
$$

where $1 \leq i \leq A$. (9)

Next, we define the minimum mean square error (MMSE) and the signal-to-interference-plus-noise ratio (SINR) at the receiver for a given channel $\hat{\mathbf{H}}$, assuming that $\pi_1, \cdots, \pi_{k-1}$ have already been stripped off the received signal (i.e., at the $k$-th SIC decoding stage). For user $\pi_k$, we have

$$
\text{mmse}_{k:A}^{(i)} = 1 - Q_{\pi_k} \mathbf{h}_i^H \mathbf{G}_{k:A}^{-1} \mathbf{h}_i.
$$

Applying the matrix inversion lemma, the corresponding SINR is immediately obtained as

$$
\text{sinr}_{k:A}^{(i)} = \frac{1 - \text{mmse}_{k:A}^{(j)}}{\text{mmse}_{k:A}^{(i)}} = Q_{\pi_k} \mathbf{h}_i \mathbf{G}_{k:A}^{-1} \mathbf{h}_i.
$$

(10)

Now, the weighted sum rate objective function can be rewritten in the convenient form

$$
\sum_{k=1}^A W_{\pi_k} R_{\pi_k} = \sum_{k=1}^A \Delta_{\pi_k} \mathbb{E} \left[ \log \left( 1 + \text{sinr}_{k:A}^{(1)} \right) \right]
$$

$$
= \sum_{k=1}^A \Delta_{\pi_k} \mathbb{E} \left[ \log |\mathbf{G}_{k:A}| \right]
$$

(12)

where $\Delta_{\pi_k} \triangleq W_{\pi_k} - W_{\pi_{k-1}}$ with $W_{\pi_0} = 0$. Since the maximization of (12) with respect to $Q = \text{diag}(Q_1, \cdots, Q_A) \geq 0$ subject to $\mathbf{Tr}(Q) \leq Q$ is clearly a convex problem, the optimality condition is given in terms of the Karush-Kuhn-Tucker (KKT) conditions applied to the Lagrangian function

$$
\mathcal{L}(Q, \lambda) = \sum_{k=1}^A \Delta_{\pi_k} \mathbb{E} \left[ \log |\mathbf{G}_{k:A}| \right] - \lambda \left( \mathbf{Tr}(Q) - Q \right).
$$

(13)

Following the footsteps of [8, Appendix B], we assume that $Q$ is the matrix maximizing strictly concave objective function (12). Then, we look at the directional derivative of the Lagrangian function at $Q$. Letting $Q(\mu) = (1 - \mu)Q + \mu \mathbf{Q}$ ($0 \leq \mu \leq 1$), the one-side derivative of $\mathcal{L}' = \mathcal{L}(Q(\mu), \lambda)$ with respect to $\mu$ at $\mu = 0^+$ is given by

$$
\frac{d \mathcal{L}'}{d \mu} \bigg|_{\mu=0} = \sum_{k=1}^A \Delta_{\pi_k} \mathbb{E} \left[ \mathbf{Tr} \left( \mathbf{G}_{k:A}^{-1} \mathbf{G}_{k:A}^{-H} - \mathbf{I}_B \right) \right] - \lambda \mathbf{Tr}\left( -Q + \mathbf{Q} \right).
$$

(14)

where $\mathbf{G}_{k:A}^{-H} = \mathbf{I}_B + \hat{\mathbf{H}}_{k:A} \mathbf{Q}_{k:A} \hat{\mathbf{H}}_{k:A}$. For the $i$-th extreme point where $Q_{\pi_i} = Q$ and $Q_{\pi_j} = 0$ for $j \neq i$, after rearranging terms, some algebra and the application of the matrix inversion lemma, we arrive at

$$
\frac{d \mathcal{L}'}{d \mu} \bigg|_{\mu=0} = \sum_{k=1}^A \Delta_{\pi_k} \left( \mathbb{E}[\text{mmse}_{k:A}^{(j)}] - 1 \right)
$$

$$
+ \frac{Q}{Q_{\pi_i}} \sum_{k=1}^i \Delta_{\pi_k} \left( 1 - \mathbb{E}[\text{mmse}_{k:A}^{(j)}] \right) - \lambda \left( \mathbf{Tr}(\mathbf{Q}) + \mathbf{Q} \right).
$$

(15)

On the other hand, the KKT condition makes the equation (14) equal to zero. Then, we can solve the equation (15) in terms of $Q_{\pi_i}$. By imposing that the constraint holds with equality, the Lagrange multiplier is eliminated and we obtain

$$
Q_{\pi_i} = \frac{\sum_{k=1}^i \Delta_{\pi_k} \left( 1 - \mathbb{E}[\text{mmse}_{k:A}^{(j)}] \right)}{\sum_{j=1}^A \sum_{k=1}^j \Delta_{\pi_k} \left( 1 - \mathbb{E}[\text{mmse}_{k:A}^{(j)}] \right)}.
$$

(16)

B. Limit for $N \to \infty$

The amount of calculation in order to evaluate the solution of (16) via the iterative algorithm given in [8, Algorithm 1] is tremendous because the expectation in the terms $\mathbb{E}[\text{mmse}_{k:A}^{(j)}]$ must be computed by Monte Carlo simulation. In the following, using the asymptotic random matrix results of [13], we find a simple algorithm to compute (16) in the limit for $N \to \infty$. This, combined with the iterative algorithm of [8, Algorithm 1], yields the sought numerical method for the weighted sum rate maximization.

Because of symmetry and the fact that users in the same group have the same weights, we will consider a group-SIC decoder that successively decodes the groups, but treats jointly, in a single shot, all users in the same group. For simplicity, we arrange the user groups such that the order $1, 2, \cdots, A$ corresponds to non-decreasing weights (in other words, $\pi$ is the identity permutation). By symmetry we have that all users in group $k$ are allocated the same (uplink) power $Q_k$. After dividing the channel coefficients by $\sqrt{N}$, the power constraint for any $N$ remains as before: $\sum_{k=1}^A Q_k \leq Q$. Define the normalized user index $t \in [0, \nu]$ where $\nu = A/B$ is the ratio of the total user antennas to the total BS antennas, and the normalized BS antenna index $r \in [0, 1]$. Let $Q(t)$ denote the (dual uplink) transmit power profile, i.e., a piecewise constant function defined on $t \in [0, \nu]$ such that $Q(t) = Q_k$ for $\nu(k-1)/A \leq t < \nu k/A$. Also, let $G(r, t)$ denote the channel gain profile of the matrix $\hat{\mathbf{H}}$, that is, a piecewise constant function such that $G(r, t) = \beta_{m,k}^2$ for $\frac{\nu(k-1)}{A} \leq t < \frac{\nu k}{A}$ and $\frac{\nu(k-1)}{A} \leq t < \nu k/A$. Then, at the $k$-th group-SIC decoding stage, in analogy with the MMSE and the SINR of the $i$-th user at the $k$-th decoding stage defined in (10) and (11), we define the piecewise constant functions $\gamma_k(t)$ and $\Gamma_k(t)$ with domain $t \in [\nu(k-1)/A, \nu k/A]$, such that $\gamma_k(t)$ for $\frac{\nu(k-1)}{A} \leq t < \frac{\nu k}{A}$ is equal to the MMSE of any user in group $i \geq k$ after removing the users in all groups $1, \cdots, k - 1$, and $\Gamma_k(t) = 1/\gamma_k(t) - 1$ is the corresponding SINR. In the limit of large $N$, these functions satisfy a fixed-point equation given by the following lemma, which is a direct application of [13, Lemma 1].
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Lemma 1: As $N \to \infty$, for all $k = 1, \cdots, A$, the SINR functions $\Gamma_k(t)$ satisfy the equations

$$\Gamma_k(t) = \frac{AQ(t)}{\nu} \int_0^1 \frac{G(R, t)}{1 + (A - k + 1)E[\frac{G(R, t)Q(T)}{1 + \Gamma_k(t)}]} dR$$

(17)

where the expectations are taken over the two independent uniform random variables $R \sim \text{Uniform}[0, 1]$ and $T \sim \text{Uniform}[\frac{\nu(k-1)}{\nu}, \frac{\nu k}{\nu}]$. Also, the asymptotic $\Gamma_k(t)$ is given in terms of the asymptotic $\Gamma_k(t)$ as $\mathcal{Y}_k(t) = 1/(1 + \Gamma_k(t))$. □

Using the fact that $Q(t)$ and $G(r, t)$ are piecewise constant, we notice that for all $\frac{\nu(t-1)}{\nu} \leq t < \frac{\nu t}{\nu}$ with $i \geq k$, $\Gamma_k(t)$ must be piecewise constant, denoted by $\Gamma_k(i)$ and satisfying

$$\Gamma_k(i) = \frac{AQ_k}{\nu} \sum_{m=1}^{B/\nu} \int_{\frac{\gamma_m A}{\nu}}^{\frac{\gamma_m A}{\nu} + \frac{\gamma_m A}{\nu}} G(r, t) dr$$

$$= \gamma \sum_{m=1}^{B/\nu} \frac{\beta^2_{m,i}}{1 + \sum_{j=k}^{A} \beta^2_{m,j}}.$$

(18)

Combining (18) with the iterative algorithm [8, Algorithm 1] that converges to the KKT conditions in (16), we obtain the solution of Algorithm 1 reduces to the standard sum capacity.

Algorithm 1 Algorithm for weighted sum rate maximization

1) Initialize $Q(0) = \frac{Q}{A}I_A$.

2) For $\ell = 0, 1, 2, \cdots$, iterate until the following solution settles:

$$Q_i(\ell + 1) = Q_i - \sum_{j=1}^{A} \Delta_k(1 - \mathcal{Y}_k(i)),$$

(19)

for $1 \leq i \leq A$, where $\mathcal{Y}_k(i) = 1/(1 + \Gamma_k(i))$, and $\mathcal{Y}_k(i)$ is obtained as the solution (also obtained by the iterations) of (18) for powers $Q_k = Q_i(\ell)$.

3) Denote by $\mathcal{Y}_k(\infty)$, $\mathcal{Y}_k(\infty)$ and $Q_i(\infty)$ the fixed points reached by the iterations in step 2). If the condition

$$Q \sum_{k=1}^{A} \Delta_k \mathcal{Y}_k(\infty) \leq \sum_{k=1}^{A} \sum_{j=1}^{B} \Delta_k(1 - \mathcal{Y}_k(i)(\infty))$$

is satisfied for every $i$ such that $Q_i(\infty) = 0$, then stop. Otherwise, set $Q_i = 0$ for $i$ corresponding to the lowest value of $\sum_{k=1}^{A} \Delta_k \mathcal{Y}_k(i)(\infty)$ (and repeat steps 2) and 3).

Notice that the sum power constraint on $Q$ is always satisfied at every $\ell$-th iteration in step 2), when all the power levels are initialized to be strictly positive. If a particular power is initialized to zero, it remains at zero indefinitely [8]. Of course, if all the weights are the same (e.g., $W_k = 1, \forall k$), the solution of Algorithm 1 reduces to the standard sum capacity.

C. Calculation of Asymptotic Sum Rate

In this subsection, we compute the weighted sum rate from the solution $Q^*$ of Algorithm 1. We introduce a normalization by $1/N$. This normalization is rather arbitrary and any linear function of $N$ will produce similar results. Here we focus on the normalized weighted sum rate

$$R_{\text{warr}} = \lim_{N \to \infty} \frac{1}{N} \sum_{k=1}^{A} \Delta_k E[\log |Q_k^* A^*|]$$

(20)

where $|Q_k^* A^*|$ follows from (9) computed for the optimal $Q^*$. We will use the asymptotic analytical expression for the mutual information given in [6]. After some suitable modification of the results of [6, Result 1], we have the $k$-th term in the summation in (20) as

$$\sum_{j=k}^{A} \log \left(1 + Q_j^* \sum_{m=1}^{B} \beta_{m,j}^2 u_m \right)$$

$$+ \gamma \sum_{m=1}^{B} \sum_{j=k}^{A} Q_j^* \beta_{m,j}^2 v_j$$

(21)

where $\{u_m\}$ and $\{v_j\}$ are the unique solutions to the system of fixed point equations

$$u_m = \left(1 + \sum_{j=k}^{A} Q_j^* \beta_{m,j}^2 v_j \right)^{-1}, \quad m = 1, \cdots, B/\gamma$$

$$v_j = \left(1 + Q_j^* \sum_{m=1}^{A} \beta_{m,j}^2 u_m \right)^{-1}, \quad j = k, \cdots, A.$$  (22)

By applying (21) to all the terms in (20), the asymptotic weighted sum rate can be obtained for arbitrary user weights.

IV. SIMULATION RESULTS AND DISCUSSION

In this section, we provide some examples to show the validity of the proposed method in a simple one-dimensional 2-cell model ($M = 2$). We adopt the system parameters and pathloss model used in mobile WiMAX system evaluation [14] with the cell radius 1 km. Two BSs are located at position 1 km and $-1$ km, respectively and $K$ user groups are uniformly located between the BSs. The distances from the two BSs to the $k$-th user group are given by $d_{1,k} = \frac{2k-1}{K}$ km and $d_{2,k} = \frac{2(K-k)+1}{K}$ km, respectively. Based on the typical settings in [14], the distance-dependent pathloss coefficients are given by $\alpha_{m,k}^2 = 10^{-9.16428/(1 + (d_{m,k}/0.036)^{3.504})}$ and the BS transmit power normalized by the user terminal noise power is given as $P_1 = P_2 = 154$ dB.

In the followings, we assume $\gamma = 4$ aspect ratio, $K = 8$ user group locations, and, by symmetry, equal-sized user groups such that $K_1 = \{1, 2, 3, 4\}$ and $K_2 = \{5, 6, 7, 8\}$. Fig. 1 presents the individual group rate normalized by $|Q_k^* A^*|$ from (20) as

$$R_k = \lim_{N \to \infty} \frac{1}{N} \left(E[\log |Q_k^* A^*|] - E[\log |Q_k^{*+1} A^*|] \right).$$

It is shown that for given $W$, the fully cooperative system achieves higher rates in any location than non-cooperative system. In case of no cooperation with uniform weight $W = [1 1 1 1 1 1 1 1]$, the edge users (located around 0)
yield zero rates due to the strong interference from the other cell, while those rates are significantly higher in case of the full cooperation where two cells are coordinated and form a network MIMO. By granting arbitrary higher weights to the edge users, it is shown that their rates are improved at the sacrifice of the center user rates.

In Fig. 2, we compare the asymptotic rates in the large system limit with the achievable rates of dirty paper coding (DPC) in a finite dimension obtained by using Monte Carlo simulation. In the DPC simulation with finite \( N \), we assume that two BSs are equipped with \( \gamma N \) antennas and \( N \) users are located at each of \( K \) locations such that there are total \( KN/2 \) users in each cell. The channel vectors are randomly generated and the algorithm of [15] are applied to each realization of the channel vectors. In no cooperation case, each BS determines its DPC precoder based on the knowledge of the average interference power from the other cell, while in full cooperation case, the DPC is jointly precoded on the network MIMO with \( 2\gamma N \) antennas and \( KN \) users. Plot (a) and (b) illustrates the normalized individual rate for \( W = [1 2 4 8 8 4 2 1] \) in no cooperation and full cooperation case, respectively. Interestingly, the simulation results of DPC almost coincide with the proposed asymptotic analysis results even for very small \( N \). In the setting of \( \gamma = |K_m| \), the DPC precoder can serve most of users at each time and the weighted sum rate maximization problem reduces to the optimal power allocation problem which we have considered in the asymptotic analysis. Then the asymptotic analysis produce the results very close to the finite dimension DPC simulation which requires much higher complexity and longer runtime, especially in a large system dimension.

### References


