Vlasov simulations of plasma-wall interactions in a magnetized and weakly collisional plasma

S. Devaux
Laboratoire de Physique des Milieux Ionisés et Applications, CNRS and Université Henri Poincaré, F-54506 Vandœuvre-les-Nancy, France

G. Manfredi
Institut de Physique et Chimie des Matériaux de Strasbourg, UMR 7504 ULP-CNRS, 23 rue du Loess, BP 43, F-67034 Strasbourg Cedex 2, France

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A Vlasov code is used to model the transition region between an equilibrium plasma and an absorbing wall in the presence of a tilted magnetic field, for the case of a weakly collisional plasma ($\lambda_{\text{mfp}} \gg \rho_i$, where $\lambda_{\text{mfp}}$ is the ion-neutral mean-free path and $\rho_i$ is the ion Larmor radius). The phase space structure of the plasma-wall transition is analyzed in detail and theoretical estimates of the magnetic presheath width are tested numerically. It is shown that the distribution near the wall is far from Maxwellian, so that temperature measurements should be interpreted with care. Particular attention is devoted to the angular distribution of ions impinging on the wall, which is an important parameter to determine the level of wall erosion and sputtering. © 2006 American Institute of Physics. [DOI: 10.1063/1.2244533]

I. INTRODUCTION

The vast majority of plasmas produced in the laboratory are in contact with a material surface. Therefore, understanding the physical processes that are at play in plasma-wall interactions is a matter of paramount importance. In fusion devices, the surface can be either the material vessel that contains the plasma, or some $ad$ hoc device (limiter or divertor) specifically designed to optimize the interaction with the charged particles. The scrape-off layer, i.e., the region of plasma directly in contact with the wall, can erode the surface and release high-Z impurities, which migrate toward the bulk plasma and deteriorate its confinement.\(^1\) In low pressure plasmas, the understanding of plasma-wall interactions is even more fundamental, because treating a surface is often the very goal to achieve. Thus, the ion energy spectrum in front of the wall is a crucial factor for the quality of the surface treatment. Yet another research field where plasma-wall interactions cannot be ignored is probe measurements.\(^2\) Probes are small metallic objects introduced in the plasma in order to measure certain macroscopic properties (e.g., density, electric current). As the probe’s surface can disturb the plasma characteristics, its presence must be taken into account in order to interpret correctly the outcomes of a measurement.

It is well known that the transition region between an unmagnetized plasma at thermodynamic equilibrium and an unbiased and perfectly absorbing wall is composed of two different subregions: the Debye sheath (DS) and the collisional presheath (CP). The DS, located just in front of the wall, is positively charged and tends to shield the negative bias of the wall. Bohm showed in his work\(^3\) that the DS stability requires a supersonic ion flow at the DS entrance (a condition now known as the “Bohm criterion”). As the ion average velocity is generally smaller than the sound speed in the bulk plasma, an intermediate region is needed in order to increase the ion velocity up to the ion sound speed. This region, called the collisional presheath (CP), is quasineutral and dominated by ion-neutral collisions. The unmagnetized plasma-wall transition was investigated theoretically and numerically in several works—see, for instance, Refs. 4–7.

When an oblique magnetic field is applied,\(^8\) the plasma-wall transition is substantially modified. According to the theoretical study carried out by Ahedo,\(^9\) three fundamental length scales arise in the description of magnetized plasma-wall interactions. As in the unmagnetized case, the electron Debye length ($\lambda_{\text{De}}$) and the ion-neutral mean-free path ($\lambda_{\text{mfp}}$) determine the typical thickness of the DS and CP, respectively. The new important length scale is the ion Larmor radius ($\rho_i$), which governs physical phenomena due to the magnetic field. For the special case where $\lambda_{\text{De}} < \rho_i < \lambda_{\text{mfp}}$, Ahedo showed that the plasma-wall transition is composed of three separate regions. In the vicinity of the wall, the DS is dominated by space-charge effects and only mildly affected by the presence of the magnetic field. Farther from the wall, the magnetic field forces the charged particles to follow the field lines, so that ions are mainly accelerated in the direction parallel to $\mathbf{B}$. The crucial point is that Bohm’s criterion should still be satisfied at the Debye Sheath Edge (DSE), where the ions’ velocity normal to the wall should be at least equal to the acoustic velocity. Therefore, a new intermediate region (located between the DS and the CP) is needed in order to redirect the ionic flow toward the wall: this region is the magnetic presheath (MP), and its thickness is found to be of the order of the ion Larmor radius.

In the present paper, we investigate the magnetized plasma-wall interaction using a kinetic model that allows us to follow the ion dynamics along these three regions...
(CP, MP, and DS), from the bulk plasma to the wall. In the next section, we shall introduce the pertinent kinetic model and the related numerical techniques. Subsequently, several aspects of the plasma-wall transition will be illustrated through numerical simulations of the kinetic model.

II. THEORETICAL MODEL AND NUMERICAL TECHNIQUES

A. Kinetic model

Our aim in this work is to simulate the interactions between a plasma at thermodynamic equilibrium and a perfectly absorbing wall, when a tilted homogeneous magnetic field is applied. The geometry of the problem is given in Fig. 1. The unperturbed plasma is located at \( x > x_p \), and the wall at \( x = 0 \). The magnetic field lies in the xOy plane and makes an angle \( \alpha \) with the wall. Note that \( v_{ji} \) is defined as the component of the perpendicular velocity that lies in the xOy plane.

![FIG. 1. Geometry of the plasma-wall transition. The wall lies in the yOz plane, at \( x = 0 \), and the equilibrium plasma is located at \( x > x_p \). The magnetic field \( B \), in the xOy plane, makes an angle \( \alpha \) with the wall. Note that \( v_{ji} \) is defined as the component of the perpendicular velocity that lies in the xOy plane.
](image)

(\( \epsilon_p \)), from the bulk plasma to the wall. In the next section, we shall introduce the pertinent kinetic model and the related numerical techniques. Subsequently, several aspects of the plasma-wall transition will be illustrated through numerical simulations of the kinetic model.

\[
\frac{\partial f_i}{\partial t} + v_i \cdot \frac{\partial f_i}{\partial x} + \frac{e}{m_i} (E + v \times B) \cdot \frac{\partial f_i}{\partial v} = -v (f_i - f_0),
\]

where \( E \) and \( B \) are the electric and magnetic fields, \( x \) is the position coordinate normal to the wall, and \( v \) is the velocity vector. A generalized Bhatnagar-Gross-Krook (BGK) term has been added to the Vlasov equation in order to simulate the effects of ion-neutral collisions and ionization. The neutral distribution \( f_0(v) \) is Maxwellian with temperature \( T_{e0} \) and density \( n_{e0} \). The BGK term tends to rebuild the equilibrium distribution \( f_0(v) \) with a rate equal to \( v = v_{th} / \lambda_{mfp} \), where \( \lambda_{mfp} \) is the ion-neutral mean-free-path, and \( v_{th} \) is the ion thermal velocity.

Here \( f_0(v) \) also represents the ion distribution in the bulk plasma, where the ions are assumed to be at thermodynamic equilibrium with the neutrals, and therefore it is used as a boundary condition for \( f_i \) at \( x = x_p \). The wall, located at \( x = 0 \), is assumed to be perfectly absorbing, so that ions can leave the system, but not reenter (zero incoming flux).

The electrons are assumed to be at thermodynamic equilibrium with temperature \( T_e \). In order to obtain a self-consistent set of equations, Eq. (1) is coupled to Poisson’s equation:

\[
\frac{\partial^2 \phi}{\partial x^2} = -\frac{e}{\epsilon_0} [n_i - n_0 \exp(\epsilon \phi / k_B T_e)],
\]

where \( \int f_i dv \) is the ion density, \( \epsilon_0 \) is the vacuum dielectric constant, \( k_B \) is Boltzmann’s constant, \( n_0 \) is the equilibrium density in the unperturbed plasma, and \( \phi \) is the electric potential related to the electric field via the relation \( E_x = -\partial \phi / \partial x \). The following boundary conditions are adopted for the Poisson equation: (i) on the plasma side (\( x = x_p \)), the potential is set to zero; (ii) at the wall (\( x = 0 \)), a floating potential condition is assumed, given by the accumulation of electric charges on the wall. The floating potential is computed by integrating Ampère’s equation on the wall:

\[
\frac{\partial E_x}{\partial t} = -\frac{e}{\epsilon_0} (j_x - j_e).
\]

The ion flux toward the wall is given by \( j_x = \int v_i f_i dv \). The electron flux is estimated by assuming that the electron velocity distribution is half-Maxwellian on the wall, which yields

\[
j_e(0,t) = n_0 \left( \frac{k_B T_e}{2 \pi m_e} \right)^{1/2} \exp \left( \frac{\epsilon \phi(0)}{k_B T_e} \right).
\]

We also assumed that, under the physical conditions considered for our simulations, secondary electron emission at the wall is negligible. We are aware that one should be cautious with the latter hypothesis, as a small amount of secondary electrons can affect the structure of the plasma-wall transition.

B. Numerical methods—Nonuniform grid

The numerical method we use to solve the Vlasov-Poisson set of equations is based on a time-splitting technique developed by Cheng and Knorr.15 This method
amounts to splitting the Vlasov equation into a sequence of simpler equations, which are then solved one after the other (see Ref. 16 for further details):

$$\frac{\partial f_i}{\partial t} + v_i \frac{\partial f_i}{\partial x} = 0,$$

(5)

$$\frac{\partial f_i}{\partial t} + \frac{e}{m_i} (E + v \times B) \cdot \frac{\partial f_i}{\partial v} = 0,$$

(6)

$$\frac{\partial f_i}{\partial t} = -\nu(f_i - f_0).$$

(7)

Equations (5) and (6) possess exact solutions, which consist of a constant drift of the distribution function in the $x$ direction, or in each of the three velocity directions. Equation (7) corresponds to a “mixing” between $f_i$ and the equilibrium distribution $f_0$ and also has an obvious analytical solution. Usually, the drifts related to Eqs. (5) and (6) do not correspond to an entire number of grid steps, so that some kind of interpolation needs to be performed: in the present work; we make use of a finite-volume scheme.\(^\text{17}\) The resulting numerical scheme is only first order accurate in time, but this limitation is not too serious, as we are mainly interested in the steady state of the system.

Poisson’s equation must be solved immediately before Eq. (6) in order to obtain the electric potential and the electric field. The presence of the Boltzmann factor renders the corresponding equations and Poisson equations nonlinear, so that Poisson’s equation must be solved with an iterative method whose details can be found in Ref. 16.

Our aim in this work is to simulate the plasma-wall transition in the special case where

$$\lambda_{c,\psi} \ll \rho_i \ll \lambda_{\text{mfp}}.$$ 

(8)

As the various scale lengths differ considerably, an obvious numerical problem arises: if the grid step is small enough to correctly describe the DS, then too many points are wasted to mesh the entire CP. Typically, the grid step should be around $\lambda_{D_s}/T_s = 1/\lambda_{D_s}$ in the DS, with a simulation box approximately $15000\lambda_{D_s}$ long in order to contain the whole CP. For a temperature ratio $T_e/T_i = 10$, this yields roughly 5000 grid points in the $x$ direction. As we take 60 grid points for each velocity direction, a standard simulation with a uniform mesh would require more than a $10^9$ grid point.

To decrease the number of grid points, we resort to a nonuniform grid by transforming the “real” spatial coordinate $x$ into a new space coordinate $s$:

$$dx = g(s)ds,$$

(9)

$$g(s) = \Delta x_1 + \frac{\Delta x_2 - \Delta x_1}{2} \{1 + \tanh[c(s - s_0)]\},$$

(10)

where $\Delta x_1$ and $\Delta x_2$ are two constant grid steps, and $c$ and $s_0$ are two coefficients that control the shape of the grid function $g(s)$.

Using a constant step $\Delta s$ in the transformed coordinate, we get a nonuniform step in the original $x$ variable. By carefully choosing the grid function (see Fig. 2), we can obtain a small grid step $\Delta x_1$ in the DS near the wall and a larger one $\Delta x_2$ in the presheath. Typical values are $\Delta x_1 = 2\lambda_{D_s}$, $\Delta x_2 = 100\lambda_{D_s}$, $s_0 = 40\lambda_{D_s}$ and $c = 0.3\lambda_{D_s}$. Note that $s_0$ and $c$ represent, respectively, the position of the transition between the small and the large grid steps, and the steepness of the shape function $g(s)$.

The above change of variables is reflected in the Vlasov and Poisson equations (1) and (2), which become, respectively,

$$g \frac{\partial f_i}{\partial t} + v_i \frac{\partial f_i}{\partial s} + g \frac{e}{m_i} (E + v \times B) \cdot \frac{\partial f_i}{\partial v} = -g(\nu f_i - f_0),$$

(11)

$$\frac{\partial^2 \phi}{\partial s^2} - \frac{\partial \phi}{\partial s} = -\frac{e}{\epsilon_0} \left[ n_i - n_0 \exp(e\phi/k_BT_e) \right].$$

(12)

Because we are only interested in the steady state ($\partial f_i/\partial t = 0$), the factor $g(s)$ in front of the time derivative in Eq. (11) can be omitted: the resulting equation has the same stationary solutions and the same formal structure as Eq. (1), so that the same numerical method can be used to solve it. Of course, this modified Vlasov equation cannot be used to study time-dependent problems, as it is not equivalent to the correct Vlasov equation, except for the steady states.

The modified Poisson’s equation (12) is again solved using an iterative technique based on centered finite differences, which leads to a tridiagonal matrix to be inverted. The matrix inversion can yield a numerical instability if the following condition is not satisfied:

$$1 - \frac{g'(s)\Delta s}{g(s)} > 0,$$

(13)

where $\Delta s$ is the grid step for the $s$ coordinate and $g'(s) = dg/ds$. This condition is satisfied when the transition between the two grid steps $\Delta x_1$ and $\Delta x_2$ is sufficiently smooth. Indeed, Eq. (13) can be rewritten as $\Delta s < 2L_p$, where $L_p = g/g'$ is the typical scale length of variation of the grid.
shape function. This condition can be kept under control by carefully choosing the free parameters $c$ and $s_0$.

In summary, the use of a nonuniform grid allowed us to reduce the number of grid points in the $x$ direction to $N_x = 150$, which represents a gain of more than one order of magnitude compared to a regular mesh.

C. Physical conditions

In order to simulate realistic physical conditions, four dimensionless parameters can be adjusted. Two of them have already been mentioned: the ion-neutral collision frequency $\nu$ (normalized to the ion plasma frequency $\omega_{pi}$) and the angle $\alpha$ between the magnetic field and the wall. The two other parameters are $\tau = T_i/T_{0i}$, the ratio of the electron to the ion temperature in the bulk plasma, and $\omega = \omega_{ci}/\omega_{pi}$, the ratio of the ion cyclotron frequency to the ion plasma frequency. In the present work, our aim is to describe the plasma-wall transition in low-pressure plasma experiments, such as the MIRABELLE device based at the University of Nancy, France. Typical physical parameters in MIRABELLE (which works mainly with argon plasmas) are $n_0 = 2 \times 10^{15}$ m$^{-3}$, $T_e = 2$ eV, $T_i = 0.03$ eV, $\omega_{ci} = 10^4 - 10^5$ rad/s, $\omega_{pi} = 10^4$ rad/s, and $n_e = 10^3$ Hz. The neutrals are supposed to be at the same temperature as the ions, which is usual for low-pressure plasma devices. This yields the following dimensionless weights: $\nu = 10^{-3} - 10^{-2}$, $\tau = 60$, and $\nu = 10^{-5}$.

In our simulations, we used values in the following ranges: $\nu = 10^{-3} - 10^{-2}$, $\tau = 5 - 35$, $\omega = 5 \times 10^{-3} - 10^{-1}$, and $\alpha = 20^\circ$, $40^\circ$, and $60^\circ$. These values still allow us to respect the ordering of Eq. (8). For all cases, the electron-to-ion mass ratio is that of an argon plasma, $m_e/m_i = 1.351 \times 10^{-5}$. It must be noted that the above dimensionless parameters are relatively close to those found in the scrape-off layer (SOL) of tokamak devices, although the temperature ratio in a SOL plasma is generally of order unity.

III. NUMERICAL RESULTS

In this section, we present results from our numerical simulations. Each run begins with a spatially homogeneous plasma at thermodynamic equilibrium (ion Maxwellian distribution at temperature $T_{0i}$) in the whole transition region between $x = 0$ and $x = x_p$. Then, the plasma is allowed to evolve according to the Vlasov-Poisson system until it reaches a(n) (inhomogeneous) steady state. The run is stopped when the spatial profiles of physically relevant quantities (e.g., density, average velocity) do not evolve significantly anymore. As an example, the ion and electron densities for a typical run are plotted in Fig. 3. As expected, the plasma is quasineutral in the CP and MP, whereas a charge separation is observed in the DS near the wall.

A. Debye sheath

The Bohm criterion tells us that the ion velocity normal to the wall should be at least equal to the ion sound speed $c_s^i = \sqrt{k_B(T_e + T_i)/m_i}$ at the DS entrance. The original criterion was derived for a simple situation where the ions are cold and the plasma is collisionless and unmagnetized. A kinetic version of Bohm’s criterion was later derived, in which the average velocity $\langle v_i \rangle$ is replaced by $\langle v_i^2 \rangle^{1/2}$. However, the kinetic criterion does not work well for collisional plasmas, as the ion distribution at the DS entrance does not necessarily vanish at $v_i = 0$.

For a magnetized plasma, full kinetic simulations of the entire transition region showed that the Bohm criterion is not verified inside the DS. In order to check this point, we show the dependence of the Mach number $M_x = \langle v_i \rangle / c_s$ on the intensity and angle of incidence of the magnetic field (Fig. 4). The DS entrance is defined as the point where the charge separation $n_i - n_e$ is equal to 0.01$n_0$. The result is that Bohm’s criterion is indeed not satisfied when the magnetic field is large and its incidence grazing, in agreement with Ref. 19.

B. Magnetic presheath

The spatial extension of the MP is determined by a competition between the magnetic field, which tends to keep ions traveling along the field lines (as in the CP), and the electric field, which tends to reorient them along the direction normal to the wall (as in the DS). The MP is the intermediate region where these two effects are of the same order of magnitude.

Several theoretical and numerical studies have been carried out on the MP. In particular, Chodura provided the following expression for the spatial extension of the MP:

$$\lambda_{MP}^b \approx \sqrt{6} \cos \alpha \frac{\omega_{pi}^{3/2}}{\omega} \lambda_{DI}.$$  

This expression can be derived by projecting the Larmor radius on the direction normal to the wall. Here $\lambda_{MP}$ repre-

FIG. 3. Ion and electron density profiles along the plasma-wall transition. The zoom shows the positively charged layer in the DS.

FIG. 4. Mach number at the DS entrance, for various values of the magnetic field intensity $\omega$ and inclination $\alpha$. The collision rate is $\nu = 10^{-3}$ everywhere.
good test for the simulations: indeed, the behavior of Vlasov simulations of plasma-wall interactions appears to be consistent with our simulations as well. An experimental verification of Eq. (14) was performed in Ref. 20.

The above expression for the thickness of the MP is a good test for the simulations: indeed, the behavior of \( \lambda_{\text{MP}} \) with the various physical parameters (\( \tau, \alpha, \) and \( \omega \)) can be verified. In order to perform such tests, we need a reliable procedure to determine \( \lambda_{\text{MP}} \) from the simulation results. As we shall see later, the main influence of the magnetic field in the CP is to guide the ion acceleration along the field lines, without affecting the motion in the perpendicular direction. Therefore, the magnetic presheath edge (MPE) can be defined as the point where the point where the average perpendicular velocity equal to \( v_{\text{th}} \) intersect the wall and can thus be collected. The origin of the factor \( \sqrt{6} \) is less clear, but it was observed by Chodura in his numerical studies and appears to be consistent with our simulations as well. An experimental verification of Eq. (14) was performed in Ref. 20.

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temperature ratio—is given in Fig. 6(b), for \(\alpha=20^\circ\) and \(40^\circ\), and \(\nu=10^{-3}\). A good agreement with theory is also obtained in this case, except for the runs at very low ion temperature \((\tau=35)\), which lie slightly off the theoretical estimate. The general trend is, however, respected.

Chodura\(^8\) has suggested that the ion parallel velocity must become sonic at the entrance of the MP (a condition now known as the Bohm-Chodura criterion). In Fig. 7 (top), we plot the parallel Mach number as a function of the magnetic field intensity \(\omega\) and inclination \(\alpha\). Bottom: Parallel velocity distribution for three values of the collision rate, and \(\omega=0.01, \alpha=40^\circ\).

C. Phase-space distributions

The ion phase-space distribution function \(f_i\) contains all the information about the ion population and it is useful to inspect its modifications from the bulk plasma to the wall. Plots of the velocity distributions are rarely found in the existing literature, as most kinetic results were obtained by means of PIC simulations, which allow limited resolution in the phase space. A notable exception are the recent results by Sharma,\(^13\) which were obtained with a mesh-based code, although rather different from ours. The alternative method employed by Daube and Riemann\(^12\) samples the distribution function on a finite number of characteristics, yielding a highly singular distribution that is hardly comparable to our smooth results.

For ease of representation, we show two dimensional (2D) projections of the distribution function in velocity space, at different spatial locations within the plasma-wall transition. The projections are either in the xOy plane (where the magnetic field lies) or in the xOz plane (where the \(\mathbf{E}\times\mathbf{B}\) drift can be observed).

For comparison, we first display the results for an unmagnetized plasma (Fig. 8), which also apply to a magnetized case with normal incidence \((\alpha=90^\circ)\). We show contour plots of \(f_i\) in the \((v_x,v_y)\) plane along the plasma-wall transition, for \(\tau=10\) and \(\nu=10^{-3}\). In Fig. 8(a), i.e., in the bulk plasma, \(f_i\) is given by a Maxwellian distribution with zero average velocity. In the CP [Fig. 8(b)], the distribution core drifts slightly along the direction normal to the wall. As expected, the ion acceleration is much stronger in the DS [Fig. 8(c)].

Figure 9 shows the evolution of \(f_i\) in the \((v_x,v_y)\) plane, for a typical magnetized case with \(\alpha=40^\circ, \omega=0.010, \tau=10\), and \(\nu=10^{-3}\). The straight lines indicate the directions parallel
isodensity contours, the maximum value of \( f_i \) being given at the top left corner of each plot. (a) \( x=12000\lambda_{Di} \) (bulk plasma); (b) \( x=631\lambda_{Di} \) (CP); (c) \( x=196\lambda_{Di} \) (MPE); (d) \( x=40\lambda_{Di} \) (DSE); (e) \( x=10\lambda_{Di} \) (DS); and (f) \( x=0 \) (wall).

The above analysis of the phase space portraits clarifies the role of the magnetic field on the plasma-wall transition. The ions are first accelerated in the CP along the magnetic field lines; in the MP, their velocities are redirected toward the wall; and finally, in the DS, they are strongly accelerated in the direction normal to the wall. These results are in agreement with the 2D phase-space plots recently published by Sharma.\(^{13}\)

**D. Temperature and average velocity**

For laboratory plasmas in contact with a wall, the density and the temperature are crucial quantities to be measured. However, though measuring the plasma density is a relatively straightforward task, the interpretation of temperature diagnostics may present some ambiguity. Strictly speaking, the temperature is only defined for a population of particles with a Maxwellian distribution: the presence of walls distorts (often dramatically) the velocity distribution leading to non-Maxwellian profiles, so that the very concept of temperature may be meaningless. Of course, a temperature can always be defined from the velocity distribution in the usual way:

![Diagram](image-url)
\[ T_j = \int f_j(x,v)(v_j - \langle v_j \rangle)^2 \, dv, \]  

(15)

where the subscript \( j \) stands for the various directions (e.g., \( x, y, \) or parallel) along which we want to compute the temperature. Indeed, in a magnetized plasma, we do not necessarily expect perfect isotropy, so that the temperature can be different along different directions.

In order to interpret correctly the temperature profiles issued from the simulations, it is useful to plot 1D projections of the ion velocity distribution, particularly along the directions parallel to the magnetic field [Fig. 11(a)] and normal to the wall [Fig. 11(b)]. The distribution in the \( z \) direction is also plotted in Fig. 11(c).

The first observation is that these three directions do not present the same velocity profiles. The \( v_z \) distribution is more distorted and—between the MPE and the wall—develops a long tail extending for several ion thermal velocities. These profiles are similar to those obtained along the normal direction in the case of an unmagnetized transition (see Fig. 12). As a consequence of the distortion of the \( v_z \) distribution, the parallel temperature obtained from Eq. (15) may overestimate the “real” temperature. This is apparent in the temperature peak observed in Fig. 13, which corresponds to the region where the tail of the distribution is most prominent (i.e., the MP).

The physical origin of such a tail in the parallel distribution lies in the competition between collisions, which try to rebuild the equilibrium distribution, and the presence of the wall, which accelerates ions toward it. The ions are accelerated by the electric potential in the CP and the MP, so that their average velocity increases. Therefore, some ions will have a velocity larger (in absolute value) than what may be expected from the equilibrium Maxwellian \( f_0 \). During a collision, these ions will tend, on average, to slow down: the net outcome is to widen the ion distribution function, as shown in Fig. 11(a). Closer to the wall, the acceleration becomes more and more important compared to collisional effects. In the absence of collisions, a potential drop accelerates slower particles more efficiently than faster particles, so that the former will tend to catch up with the latter, leading to a narrowing of the velocity distribution: this is what happens in the DS, where the temperature decreases again. The temperature maximum corresponds to the location where the two competing effects (collisions and acceleration) are of the same order. A similar behavior was observed in previous numerical simulations,\(^{19}\) which showed a widening of the \( v_x \) distribution at the MPE and subsequent narrowing within the DS. Recent experimental studies on unmagnetized plasmas have also confirmed this pattern.\(^{22}\)

In order to confirm this picture, we performed several runs in the unmagnetized regime (see Fig. 14). In these simulations, the wall is no more at the floating potential, but instead it is polarized to a certain negative potential. By increasing the polarization, we increase the electric field near
the wall and thus the extension of the DS. The simulations show that the temperature peak is shifted to the right (i.e., toward the CP) as the polarization increases. This result supports our conjecture: for a larger polarization (in absolute value), the effect of the ion acceleration and the corresponding narrowing of the distribution occur at a larger distance from the wall.

Temperature measurements are more meaningful in the direction normal to the wall: as it can be seen in Fig. 11(b), the distribution does not develop a tail along this axis. In the z direction (normal to the magnetic field), the distribution stays even closer to a Maxwellian. Therefore, it appears that the distortion of the velocity distribution is maximum in the direction where the electric field dominates (the parallel direction), whereas it is negligible when the magnetic field dominates (i.e., in the directions normal to B). The overall variations of the different temperatures (Fig. 13) reflect this behavior: the peak is much less pronounced in the $T_e$ temperature and has almost disappeared for the $T_i$ temperature.

The wall has also an important effect on the $E \times B$ drift. In our situation, for which the magnetic field is uniform, this drift is simply proportional to the electric field and directed along the z axis:

$$v_E = \frac{E \times B}{B^2} = \frac{E_y(x) \cos \alpha_y}{B} z.$$  \hspace{1cm} (16)

We expect the $E \times B$ drift to play an important part in the ion dynamics in the z direction. In order to verify this, we plot in Fig. 15 both the theoretical expression (16) for the $E \times B$ drift and the average velocity along the z axis as measured from the simulations.

The two curves are identical in the CP and start to differ around $650 \lambda_D/e$, which corresponds to the MPE. This discrepancy results from the collection of high-speed ions by the wall. Indeed, the MPE corresponds to the location where the orbits of ions with perpendicular velocity equal to $v_{thi}$ intersect the wall. But wall collection is more effective for faster ions (with larger Larmor radii), which are then removed from the distribution, leading to a decrease in the absolute average velocity of the population. Further, in the DS the electric field varies so rapidly that the ion dynamics can no longer be separated into a gyromotion and a drift. A corollary of this result is that approximations based on the guiding center motion (drift-kinetic, gyrokinetic) can only be applied in the CP.

**E. Ion distribution at the wall**

An accurate knowledge of the ion distribution at the wall is crucial to determine the effect of the ions on the material surface.\cite{3} Indeed, sputtering and physical adsorption by the surface strongly depend on the energy and angle of incidence of the ions impinging on it.

Here, we focus on the angular distribution $f(\theta)$, where $\theta$ is the angle of incidence, defined as the angle between the velocity vector $v$ and its projection on the $(v_x, v_y)$ plane, as can be seen in Fig. 16. This definition of $\theta$ is consistent with that of $\alpha$, the angle of incidence of the magnetic field: $\theta=90^\circ$ corresponds to ions impinging on the wall along the normal direction whereas $\theta=0^\circ$ describes ions with a grazing incidence. From Fig. 16, it easily comes that

$$\theta = \arctan\left(\frac{|v_x|}{\sqrt{v_{y}^2 + v_z^2}}\right),$$  \hspace{1cm} (17)

where $v_x, v_y,$ and $v_z$ are the components of the velocity vector of an ion striking the wall.

In Figs. 17 and 18, we show the ion distribution functions at the wall for several sets of physical parameters. In each figure, we plot the 2D distribution function in the plane

$$v_x$$

$$v_y$$

$$v_z$$

$$v$$

where $v_x, v_y,$ and $v_z$ are the components of the velocity vector of an ion striking the wall.
of the magnetic field $f(v_x, v_y)$, and the corresponding angular distribution $f(\theta)$. All simulations of Fig. 17 were performed with a temperature ratio and a collision frequency kept constant at $\tau = 10$ and $\nu = 10^{-3}$.

We notice that the distribution is sometimes composed of two peaks—see, for instance, Fig. 17(e): the lower peak is centered at a relatively large $v_y$ velocity, whereas the upper peak is centered around $v_y = 0$. The existence of the two peaks can be explained in terms of the competition between the magnetic field and the effects due to collisions.

When the magnetic field is strong compared to the collision rate ($\omega \gg \nu$), the ions are mainly accelerated along the parallel direction, which entails acceleration toward negative $v_y$; this explains the presence of the lower peak in Fig. 17. In the opposite case ($\omega \ll \nu$), the ions are still accelerated along the field lines, but the isotropizing collisions tend to redirect the ion flow in the direction normal to the wall, so that only one peak around $v_y = 0$ is visible [see Fig. 8(c) for the case of an unmagnetized transition, $\omega = 0$]. For comparable values of $\omega$ and $\nu$, some ions will be accelerated almost without collisions and form the lower peak; other ions will undergo several isotropizing collisions and contribute to the upper peak, thus leading to a distribution of the type observed in Fig. 17(e) (the DS only shifts the distribution in the $v_x$ direction, which does not affect the previous conclusions). This effect is clearly visible for $\alpha = 20^\circ$, but less so for $\alpha = 40^\circ$, because the parallel direction is closer to the $x$ direction. These two ion populations, because of their different velocities and angles of incidence, may display different behaviors concerning physical adsorption and sputtering at the wall.

In order to validate the above analysis, we performed two more simulations: the first with a large collision rate, $\nu = 10^{-2}$, shown in Fig. 18(a); and the second with no collisions [$\nu = 0$, 18(b)]. In both cases, we took $\alpha = 20^\circ$ and $\omega = 0.100$. As expected, the peak at $v_y = 0$ disappears when the collision rate is zero, whereas it becomes dominant when collisions are important. It is interesting to note that the distributions of Figs. 18(a) and 17(e) are very similar to each other, and indeed the ratio $\omega / \nu$ is the same in both cases.

The above phenomenon is reflected in the angular distribution, which also displays two peaks. The peak close to $\theta = 90^\circ$ corresponds to ions having experienced many collisions, whereas the peak at lower $\theta$ corresponds to ions having been accelerated without collisions along $\mathbf{B}$. In Table I, we give the value of $\theta_{\text{max}}$ for several sets of parameters, $\theta_{\text{max}}$ being the angle corresponding to the maximum of the angular distribution $f(\theta)$. As expected, $\theta_{\text{max}}$ is very sensitive to the value of $\omega$ for $\alpha = 20^\circ$. It can also be noted that, even with a strong magnetic field ($\omega = 0.1$), $\theta_{\text{max}}$ is still significantly larger than $\alpha$, the angle of incidence of the magnetic field. This difference between $\theta_{\text{max}}$ and $\alpha$ shows that the electric field is always dominant in DS, and manages to redirect the
ions to an angle of incidence closer to the normal to the wall. A similar dependence of $\theta_{\text{max}}$ on $\alpha$ was already suggested by Chodura.$^{24}$

IV. CONCLUSION

In this paper, we studied plasma-wall interactions in the case of a magnetized and weakly collisional plasma ($\lambda_{De} \ll \rho_i \ll \lambda_{\text{mfp}}$). The physical regimes chosen for the simulations are relevant to low-pressure laboratory plasmas and tokamak edge plasmas. Kinetic Vlasov simulations were performed using an accurate Eulerian code. The use of a non-uniform grid allowed us to simulate the entire transition—from the equilibrium plasma to the wall—with a moderate number of grid points. The resulting code enabled us to obtain smooth phase-space distributions along the entire plasma-wall transition region.

The results provided us with an increased understanding of the phase space dynamics of the plasma in the transition region. It is now clear that the plasma is first accelerated along the magnetic field lines in the CP, then is redirected along the axis normal to the wall in the MP, and finally is strongly accelerated in the DS. The thickness of the MP was measured from the simulation results, and was found to be in good agreement with a theoretical estimate.

The ion velocity distribution is considerably deformed in the various sheaths, due to the combined action of the electric and magnetic fields. Therefore, any measure of the ion temperature obtained as the width of the velocity distribution must be taken with care. This deformation is particularly significant in the MP, and corresponds to a spurious peak in the ion temperature. We also showed that the velocity in the direction normal to both the electric and magnetic field is well described by the $E \times B$ velocity in the CP, but departs from it in the MP and even more in the DS.

Finally, we studied the angular distribution of the ions striking the wall. This is an important quantity, as the angle of incidence determines the level of sputtering and erosion of the surface. The results showed that, even for relatively strong magnetic fields, the angle of incidence of the impinging ions is never as grazing as the angle between the magnetic field and the wall. In the regimes considered here, the electric field always manages to partially redirect the ions normally to the surface. Interestingly, we observed that two different ion populations can be present at the wall, giving rise to a two-peak velocity distribution. These two populations correspond to ions that have been drifting without collisions along the magnetic field lines and to ions that have experienced several collisions before hitting the wall. Further work will be necessary to relate more precisely the angular ion distribution to the erosion of the material surface.

### Table I. Values of $\theta_{\text{max}}$, defined as the angle under which most ions strike the wall, for $\tau=10$ and $\nu=10^{-3}$.

<table>
<thead>
<tr>
<th>$\omega$</th>
<th>$\alpha=20^\circ$</th>
<th>$\alpha=40^\circ$</th>
<th>$\alpha=60^\circ$</th>
<th>$\Delta \theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.100$</td>
<td>$52.8^\circ$</td>
<td>$70.8^\circ$</td>
<td>$82.1^\circ$</td>
<td>$1.25^\circ$</td>
</tr>
<tr>
<td>$0.050$</td>
<td>$59.6^\circ$</td>
<td>$73.1^\circ$</td>
<td>$86.6^\circ$</td>
<td>$1.25^\circ$</td>
</tr>
<tr>
<td>$0.010$</td>
<td>$70.8^\circ$</td>
<td>$79.8^\circ$</td>
<td>$86.6^\circ$</td>
<td>$1.25^\circ$</td>
</tr>
<tr>
<td>$0.005$</td>
<td>$78.8^\circ$</td>
<td>$86.6^\circ$</td>
<td>$86.6^\circ$</td>
<td>$1.25^\circ$</td>
</tr>
</tbody>
</table>

FIG. 18. Ion distribution function in the $(v_x,v_y)$ plane and corresponding angular distribution for (a) a large collision rate ($\nu=10^{-2}$) and (b) the vanishing collision rate ($\nu=0$). Other parameters of the simulation are $\alpha=20^\circ$, $\omega=0.100$, and $\tau=10$. 

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