In this paper we propose a method, based on both physiologic and engineering considerations, for the motion planning of a prosthetic finger. In particular, we exploit a minimum jerk approach to define the trajectory in the Cartesian space. Then, cubic splines are adopted in the joint space. The redundancy problem arising from the presence of three links is solved by assuming that there is a constant ratio between the second and the third joint motion. The value of the proportional constant is determined by minimizing the maximum jerk in the joint space. It is found that this constant value can be suboptimally but effectively set to one for all the movements. This approach guarantees a natural movement of the finger as well as reduced vibrations in the mechanical structure and increased control performances.

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1. INTRODUCTION

In the last years, there has been a significant development in prosthetic hand devices and haptic perception. Many laboratories have produced multi-degrees-of-freedom artificial hands for robotic and/or prosthetic applications (see, e.g., refs. 1–3). In this context, it is of interest to reproduce the natural movement of fingers in order to perform different tasks, such as exploring an unknown object with artificial hands (for teleoperation use) or reaching and handling an object with prosthetic devices (for support of handicapped patients). However, this has to be done while taking into account also the design constraints of the mechanical structure and of the control architecture. From the physiological side, many studies have been published regarding the finger,5 arm,10,11 and human movements in general,12–14 from both a kinematic and a dynamic point of view. In general, it is concluded that natural movements are planned and executed by following optimization principles. In particular, we highlight the contribution of Flash and Hogan15 who show that the arm movement is executed in order to minimize the square of the magnitude of the jerk (i.e., the derivative of the acceleration function) of the hand in the Cartesian space over the entire movement. Following this result, Laczko et al.16 have investigated, in multi-joint kinematic chains, the relative contribution of the velocities, accelerations, and jerks in the individual joints to the total endpoint jerk; they conclude that the term related to the individual joint jerks dominates over the others.

From a robotic viewpoint, the problem of determining the motion of a multijoint finger has to face the redundancy of the system, i.e., there are an infinite number of joint configurations for a unique Cartesian fingertip position. In other words, the presence of the third joint/phalanx implies that we have three degrees-of-freedom, namely one more than that necessary to address a motion in a two-dimensional plane. In general, the additional degree-of-freedom is exploited in order to minimize some objective function.

Besides, it is recognized that the minimization of the joint jerk provides benefits to the mechanical structure of a robot manipulator, reducing the presence of vibrations and the joint wear and therefore increasing the robot life-span17 as well as permitting the increase of trajectory control performances.18,19 In this context, different approaches to minimize the joint jerk in the trajectory planning of robot manipulators have been proposed in the literature.20,21

However, when the coordinated movement of an artificial/natural arm consisting of a multi-link/multi-joint kinematic chain has to be programmed and controlled in order to reach a determined position in the proximal space with the end-effector (the fingertip) for grasping, touching, or exploring an object, it seems useful to maintain a smooth approach of fingertip along the straight line trajectory connecting its initial and final positions. This implies to assume a smooth control of the fingertip in the Cartesian space, with null acceleration at the beginning and at the end of the motion. This should prevent accidental mechanical shocks of the fingertip on the object, due to possible small errors in the joint actuators. Thus, in addition to physiological requirements (e.g., for aesthetic reasons which are of primary importance in the prosthetic devices field), this represents also an important mechanical feature, as a low impact is guaranteed in the fingertip approach to the object’s surface. Taking into account all these considerations, we propose a new approach for the motion planning of a prosthetic finger, based on the minimum-jerk principle. Basically, it consists of defining linear movements of the tip in the Cartesian space, defined in such a way to minimize the Cartesian jerk. Then, the inverse kinematic problem is solved by using cubic splines to interpolate between the knots and by fixing a constant ratio between the second and the third joint angle. The value of the constant ratio is found by minimizing the maximum jerk of the three joints. The obtained results are consistent with physiological evaluations.

The paper is organized as follows. In Section 2 the minimum jerk approach is explained, as well as the adopted optimization procedure. Results are presented in Section 3 and they are discussed in Section 4. Conclusions are drawn in Section 5.

2. THE MINIMUM JERK APPROACH

2.1. Motion Planning in the Cartesian Space

We consider a human middle finger, shown in Figure 1, where $l_1 = 54$ mm, $l_2 = 26$ mm, and $l_3 = 20$ mm are the lengths of the phalanges and $\theta_1$, $\theta_2$, and $\theta_3$ are the metacarpophalangeal (MP) joint, the proximal interphalangeal (PIP) joint, and the distal interphalangeal joint (DIP), respectively. From physiological considerations, it has to be taken into account that
-30° ≤ \theta_i ≤ 90°, \quad i = 1, 2, 3.

(1)

0° ≤ \theta_i ≤ 90°, \quad i = 2, 3.

Only straight movements in the X-Y plane are considered.

By applying the mathematical model developed by Flash and Hogan to the finger, it results that a natural movement from position \((x_0, y_0)\) to position \((x_f, y_f)\) starting at time \(t_0 = 0\) and ending at time \(t_f\) has to minimize the following objective function, i.e., the time integral of the square of the magnitude of jerk:

\[
C = \frac{1}{2} \int_{t_0}^{t_f} \left( \frac{d^3x}{dt^3} \right)^2 + \left( \frac{d^3y}{dt^3} \right)^2 dt,
\]

(2)

where \(x\) and \(y\) are the time-varying coordinates of the fingertip position. By solving the optimization problem, we have that the trajectory of the fingertip in the Cartesian space is uniquely determined as follows:

\[
x(t) = x_0 + (x_0 - x_f) \left( -6 \frac{t^5}{t_f^5} + 15 \frac{t^4}{t_f^4} - 10 \frac{t^3}{t_f^3} \right),
\]

(3)

\[
y(t) = y_0 + (y_0 - y_f) \left( -6 \frac{t^5}{t_f^5} + 15 \frac{t^4}{t_f^4} - 10 \frac{t^3}{t_f^3} \right).
\]

2.2. Motion Planning in the Joint Space

Once the movement of the fingertip in the Cartesian space has been determined, the inverse kinematics problem has to be solved in order to calculate the motion law in the joint space, to be directly applied, by using appropriate actuators, to the prosthetic finger. A practical solution, often adopted in industrial environments, is to select a sufficient number \(m\) of equally spaced knots along the Cartesian trajectory. For simplicity, we select the knots \(d_1, \ldots, d_m\), at time intervals equal to each other. Then, the corresponding joint configuration has to be determined by applying the inverse kinematics. In this context, the redundancy problem has to be faced. Namely, the presence of three joints causes that an infinite number of joint configurations solve the inverse kinematics problem.

To effectively tackle the redundancy problem, we propose to set the angle of the third phalanx proportional to that of the second phalanx, namely,

\[
\theta_3 = K \theta_2, \quad K > 0.
\]

(4)

Note that this assumption seems to be reasonable from a physiological point of view (see, e.g., refs. 4 and 8). Then, for each joint, cubic splines are adopted to interpolate the resulting displacements for each joint. The salient feature of the cubic splines is that they assure the continuity of the velocity and acceleration functions and, being of low order, they prevent large overshoots. Note that, in order to set a sensible number of initial and final conditions (i.e., to impose null velocity and acceleration at time \(t = 0\) and \(t = t_f\)), two additional “free” knots in the joint space have to be defined in second and penultimate positions. However, this does not influence significantly the results because of the relative high number of joint displacements to be interpolated. Formally, for each \(k\)th joint \((k = 1, 2, 3)\) we describe the displacement sequence of the knots in the joint space as follows: \(\theta_0, \ldots, \theta_n\), where \(\theta_k\) and \(\theta_{k,n-1}\) are free displacement parameters (i.e., \(n = m + 1\)). Joint velocity and acceleration at the \(i\)th knot are denoted by \(v_{ki}\) and \(a_{ki}\), respectively. Velocities \(v_{ko}, v_{kn}\) and accelerations \(a_{ko}, a_{kn}\) are set to zero. Denote by \(h = t_f / n\) the time interval necessary for the \(i\)th spline \(\Theta_{ki}(t)\) to connect knot \(i-1\) to knot \(i\) for \(t \in [0, h]\) (note that \(h\) is independent of the considered joint \(k\)). A convenient parametrization of spline \(\Theta_{ki}(t)\) that naturally incorporates the continuity of positions and velocities is the following:

\[
\Theta_{ki}(t) = \theta_{ki-1} + v_{ki-1} t + \frac{3}{h^2} (\theta_{ki} - \theta_{ki-1}) - \frac{1}{h} (v_{ki} + 2v_{ki-1}) t^2 + \frac{2}{h^3} (\theta_{ki} - \theta_{ki-1}) + \frac{1}{h^2} (v_{ki} + v_{ki-1}) t^3, \quad t \in [0, h].
\]

(5)

The unknown parameters in each spline can be...
determined by imposing the continuity of acceleration, i.e., by solving the following system of \( n + 1 \) linear equations \((k=1,2,3)\):

\[
\begin{align*}
\hat{\theta}_{k1}(0) &= a_{k0}, \\
\hat{\theta}_{k1}(h) &= \hat{\theta}_{k2}(0), \\
\vdots \\
\hat{\theta}_{k,n-1}(h) &= \hat{\theta}_{kn}(0), \\
\hat{\theta}_{kn}(h) &= a_{kn},
\end{align*}
\]

where \( a_{k0} = a_{kn} = 0 \). It can be easily seen that, once \( h \) has been fixed, the above system (6) admits a unique solution for any assigned data set.\(^2\) The jerk is evidently constant on each \( i \)th spline and its expression is given by

\[
j_{ki} := \ddot{\theta}_{ki}(t) = -\frac{12}{h^3} (\theta_{ki} - \theta_{k,i-1}) + \frac{6}{h^2} (v_{ki} + v_{k,i-1}).
\]

(7)

### 2.3. Optimization Problem

In the framework proposed in the previous subsections, the resulting trajectory in the joint space depends on the design parameters \( K \). An explicit solution of the inverse kinematic problem that depends on \( K \) cannot be computed because of its high complexity. Therefore, the value of \( K \) has to be fixed before the inverse kinematics and the cubic splines approach are applied. In this context, the inverse kinematic problem can be solved by applying a standard Newton–Raphson algorithm.\(^2\) Thus, once a value of \( K \) has been selected, taking into account the physical constraints on the joint angles (1) the inverse kinematic problem can be uniquely solved.

An appropriate method to find the value of \( K \) is to minimize the maximum jerk over the three joints. In other words, we have to solve the following constrained minimax optimization problem:

\[
\min_{K \geq 0} \max_{i=1\ldots3} \{ j_{ki} : k = 1,2,3 \}
\]

subject to

\[-30^\circ \leq \theta_i \leq 90^\circ, \]

\[0^\circ \leq \theta_i \leq 90^\circ, \quad i = 2,3.\]

From a practical point of view, an upper bound for the optimal value of \( K \) can be easily found by taking into account again physiological consideration.\(^2\) A conservative upper bound \( K^+=1.5 \) has been selected, while, as a lower bound, the value \( K^- = 0 \) has been retained. In order to find the optimal value \( K^* \) that minimizes (8), a tight gridding (with step equal to 0.01) over the interval \([K^-, K^+]\) has been performed, evaluating the maximum jerk for each value of \( K \) and then selecting the optimal one.

Therefore, the following algorithm has been applied for a single movement of the finger.

1. Define the endpoints and the time of the motion in the Cartesian space, i.e., assign a value to \( x_f, y_f, x_t, y_t, \) and \( t_f \).
2. Calculate the trajectory of the fingertip in the Cartesian space [see (3)].
3. Divide the movement segment in the Cartesian space into \( m - 1 \) equally spaced segments, determining \( m \) knots.
4. Divide the motion time \( t_f \) into \( n \) equal time intervals \( h = t_f/n (n = m + 1) \).
5. Set \( K^* = \infty \) and \( j^* = \infty \).
6. For \( K = 0:0.01:1.5 \)
   (a) For each knot \( d_i, i = 1\ldots m, \) determine \( \theta_{ki}, \) \( k = 1,2,3 \), by applying the Newton–Raphson algorithm. If constraints (1) are not satisfied or the Newton–Raphson algorithm does not converge, then set \( j = \infty \) and go to (e).
   (b) Determine \( \Theta_{ki}(t), i = 0\ldots n, k = 1,2,3 \).
   (c) Determine \( j_{ki}, i = 1\ldots n, k = 1,2,3 \).
   (d) Calculate \( j = \max \{|j_{ki}| : 1\ldots n, k = 1,2,3 \} \). If \( j < j^* \), then set \( j^* = j \) and \( K^* = K \).
   (e) End (for).
7. End.

Remark 1: The number of knots \( m \) is not a critical issue for the result of the algorithm (see Section 3). In particular, for all the movements we considered, we selected values of \( m \) spanning from 10 to 30 and we obtained practically the same values of \( K^* \).

Remark 2: The length of the phalanges is not a critical issue as well. Again, for each movement we considered, we applied the algorithm also with the average American female middle finger
($l_1=42.11\text{ mm}, l_2=20.27\text{ mm}, \text{ and } l_3=15.60\text{ mm})$ and with the average American male middle finger ($l_1=46.22\text{ mm}, l_2=22.26\text{ mm}, \text{ and } l_3=17.12\text{ mm}$), and we obtained the same results for all the three fingers.

3. RESULTS

The algorithm presented in Section 2.3 has been applied to a large number of movements, with different motion times. Here, for the sake of clarity, we focus on some significant results that illustrate the conclusions we draw. In particular, we consider straight movements performed in an interval time of $1\text{ s}$ (i.e., $t_f=1\text{ s}$) with $m=20$. As an example, in Figure 2 different configurations of the finger for a particular movement are reported.

First, we considered all the movements starting in $(100,0)$ and ending in the points of the workspace depicted in Figure 3. In all the cases we obtained that the optimal jerk is obtained by setting $K=1$. The resulting optimal jerk $j^*$ for each movement is shown in Figure 4. Slightly different results have been obtained for different starting points. Namely, solving the optimization problem does not always yield to the value of $K=1$. In any case, it is of interest to compare the values of the jerk $j^*$ achieved for the optimal values of $K$ and the values of the jerk achieved by fixing $K=1$. They have been reported in Figures 5 and 6, respectively, for movements starting in $(60,60)$ and in Figures 7 and 8, respectively, for movements starting in $(20,60)$.

It appears that the difference between the values of the maximum jerk achieved by selecting the optimal value of $K$ and $K=1$ is not very significant. This is confirmed by the fact that actually for many movements the resulting optimal value of $K$ is indeed $K=1$. For an illustrative example see Figure 9, where the final points of the movements (for the case $x_0=20$ and $y_0=60$) for which the optimal $K$ is equal to one are indicated. For a better evaluation of the results from an analytical point of view, the mean value of the maximum jerk and its standard deviation for the considered movements are reported in Table I.
4. DISCUSSION

The obtained results show that a suitable choice to solve the redundancy problem in the motion planning of the prosthetic finger consists of choosing a constant ratio equal to one between the PIP and the DIP joints. Although the proposed solution is theoretically suboptimal, from a practical point of view it is easy to implement and preserves the minimization of the maximum jerk in the joint space. In addition, the value $K=1$ is appropriate also from physiological considerations, as it reflects the behavior of a real human finger.

It also has to be noted that for $K=1$ the inverse kinematic problem can be analytically solved. In particular, we have

$$
\theta_1 = \arctan \left( \frac{y}{x} \right) - \arctan \left( \frac{\delta_2}{\delta_1} \right),
$$

$$
\theta_2 = \arctan \left( \frac{s_2}{c_2} \right),
$$

$$
\theta_3 = \theta_2.
$$
where

\[ c_2 = \pm \frac{\beta \pm \sqrt{\beta^2 - 4\alpha \gamma}}{2\alpha}, \]

\[ s_2 = \pm \sqrt{1 - c_2^2}, \]

\[ \delta_1 = l_1 + l_2 c_2 + l_3 \cos(2\theta_2), \]

\[ \delta_2 = l_2 s_2 + l_3 \sin(2\theta_2), \]

and

\[ \alpha = 4l_1 l_3, \]

\[ \beta = 2l_1 l_2 + 2l_2 l_3, \]

Hence, the overall proposed methodology satisfies both physiological and engineering requirements as the planned movement reflects that of a human finger, both in Cartesian and in the joint space, and it ensures the prevention of vibrations and the increasing of the control performances by keeping the maximum jerk of the joints at a low level.

5. CONCLUSIONS

In this paper we proposed a methodology for the motion planning of a prosthetic finger. It is based on the selection of the minimum jerk trajectory in the Cartesian space and the use of cubic splines in the joint space. This strategy ensures both a smooth approach to the object along a straight line trajectory (with a low shock of the fingertip when touching the object) and low jerk at the joint level, as required by physiological and mechanical considerations. The redundancy problem is solved by selecting a constant ratio (equal to one) between the PIP and the DIP joint, in order to minimize the maximum of the jerk functions of the three joints. This solution is suitable to be applied in a practical context since it satisfies engineering requirements (easiness of implementation, reduction of vibrations, increasing of the control performances, etc.) and at the same time it allows a physiological smooth natural movement.

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