Mapping Synergies from Human to Robotic Hands with Dissimilar Kinematics: an Approach in the Object Domain

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Abstract—In the last two decades, studies in neuroscience concerning the sensorimotor organization of the human hand demonstrated that, notwithstanding the complexity of the hand, a few variables are able to account for most of the variance in the patterns of configurations and movements. The reduced set of parameters that humans effectively use to control their hands is known in the literature as set of synergies. The synergistic organization of the human hand is the theoretical foundation of this work which focuses on the problem of mapping human hand synergies on robotic hands with dissimilar kinematic structures. The mapping from human to robotic hands is designed in the manipulated object domain where the role of the object is played by a virtual sphere, whose radius and center position change dynamically, and the role of the human hand is played by a hand model referred to as “paradigmatic hand,” able to capture the idea of synergies in human hands. Mapping sensorimotor synergies onto robotic hands independently from their kinematic structures allows to simplify the control of robotic hands which as the human hands can be controlled using a few parameters. Borrowing the terminology from software engineering, the proposed mapping represents a middleware solution to control the robotic hands independently from their specific structure, and focusing only on the manipulation tasks.

I. Introduction

A. Motivation and objective of the work

Robotic hands have many degrees of freedom (DoFs) distributed among several kinematic chains: the fingers. The complexity of the mechanical design is needed to adapt hands to the many kinds of tasks required in unstructured environments, such as surgical rooms, industry, house, space and other domains, where robotic grasping and manipulation have become crucial. Some remarkable example of robotic hand design are the UTAH/MIT hand [1] with 16 actuated joints, 4 per each finger, the Shadow hand, with 20 actuated joints [2], and the DLR hand II with 12 actuated joints [3]. One of the main issues in designing and controlling robotic hands is that a large number of motors is needed to fully actuate the DoFs. This makes both the mechanical and the control system design of robotic hands dramatically more complex when compared to simple grippers often used in industrial applications [4], as pictorially represented in Fig. 1: the larger is the number of DoFs, the lower is the number of possible applications of the robotic hands in industries. This is one of the major limitations to the use of advanced robotic hands in flexible automation together with the raise of costs and the decrease of the robustness of such devices.

As far as control is concerned, it is our belief that the development of a unified framework for programming and controlling of robotic hands will allow to extend the use of these devices in many areas. Borrowing the terminology of software engineering, we believe that there is a need for middleware solutions for manipulation and grasping tasks to seamlessly integrate robotic hands in flexible cells and in service robot applications.

This paper is a first contribution in the direction of developing a unified framework for programming and controlling robotic hands based on a number of fundamental primitives, and abstracting, to the extent possible, from the specifics of their kinematics and mechanical construction. Finally, the ultimate goal of our approach is to define a device independent control framework for robotic hands, based on synergies of human hands, and this work presents a possible solution to project the synergy based control of human hands to robotic hands with dissimilar kinematics.

B. Synergies

This work is inspired and supported by neuroscience studies which have shown that the description of how the human hand moves during grasping is dominated by trajectories in a configuration space of much smaller dimension than the kinematic structure would suggest. Such configuration space is referred to as the space of postural synergies. Santello et al. [5] investigated this hypothesis by collecting a large set of data containing grasping poses from subjects that were asked to shape their hands in order to mime grasps for a large set \( N = 57 \) of familiar objects. Principal Components Analysis (PCA) of this data revealed that the first two principal components account for more than 80% of the variance, suggesting that a satisfying characterization of the recorded data can be obtained using a much lower-dimensional subspace.
of the hand DoF space. These and similar results seem to suggest that, out of the more than 20 DoFs of a human hand, only two or three combinations can be used to shape the hand for basic grasps used in everyday life.

These ideas can be brought to use in robotics, since they suggest a new and principled way of simplifying the design and analysis of hands different from other more empirical, sometimes arbitrary design attempts, which has been the main roadblock for research in artificial hands in the past [4]. The application of synergy concepts has been pioneered in robotics by [6], [7]. In [6], and later on in [8], the idea has been exploited in the dimensionality reduction of the search space in problems of automated grasp synthesis, and has been applied effectively to derive pre-grasp shapes for a number of complex robotic hands. In [7], authors designed a robotic hand consisting of groups of mechanically interconnected joints, with a priority inspired by resemblance to postural synergies observed in human hands. More recently, in [9] a synergy impedance controller was derived and implemented on the DLR Hand II.

The synergy approach to analysis and design of artificial hands is the focus of some recent works on grasp theory. In [10] the authors investigated to what extent a hand with many DoFs can exploit postural synergies to control force and motion of the grasped object. In [11] numerical results were presented showing quantitatively the role played by different synergies (from the most fundamental to those of higher-order) in making possible a number of different grasps. In [12] the analysis of underactuated hands was integrated by extending existing manipulability definition [13]–[15]. The main aspect considered there is that in underactuated hands often the force problem cannot be univocally solved within a rigid-body framework, because of static indeterminacy [10], [11]. This problem can be solved considering the hand and the contact compliance, as discussed in [16], [17].

C. Mapping

The main contribution of this work is that of studying a mapping function between the postural synergies of the human hand and synergistic control action in robotic devices. This mapping leads to an interesting scenario, where control algorithms are designed considering a paradigmatic hand model, and without referring to the kinematic of the specific robotic hand. The proposed approach could also represent a possible control paradigms to be used to robustly solve manipulation tasks for parallel jaw grippers that are simple to program but also limited in function. The paradigmatic hand [11], [18] is a model inspired by the human hand that does not closely copy the kinematical and dynamical properties of the human hand, but rather represents a trade-off between the complexity of the human hand model, accounting for the synergistic organization of the sensorimotor system, and the simplicity and accessibility of the models of the robotic available hands.

This paper focuses on the mapping of human synergies onto robotic hands by using a virtual object method, as shown in Fig. 2. Different mapping procedures have been developed, mainly for tele-manipulation and learning by demonstration tasks. They are summarized in the following section, and can be substantially divided in two approaches: Joint space and Cartesian space. Both of them are focused on the hand and not on the task. They impose constraints on the choice of the reference joints or points, thus their applicability to hands with very different kinematics is not simple.

The mapping method proposed in this paper is focused on the task and in particular on the reproduction, on the robotic hand, of the movement and deformation that the human reference hand would perform on a virtual object whose geometry is defined by the hand posture itself. This allows to work directly on the task space avoiding a specific projection between different kinematics.

The paper is organized as it follows. In Section II a review of the literature related to the mapping between human and robotic hand is presented. Section III summarizes the main results concerning grasp properties in synergy actuated hands. Section IV describes the mapping method while in Section V some numerical simulations are shown, to confirm the proposed approach effectiveness and the applicability of the method is tested in a manipulation task. Section VI discusses how the proposed mapping procedure could be used to define a two–layer control strategy, in which the high level controller is substantially independent from the robotic hand and depends only on the specific operation to be performed. At the end, in Section VII, the advantages and drawbacks of the proposed method are summarized, conclusion and future work are outlined.

II. RELATED WORK

In the literature there are several examples where a mapping between a human hand and a robotic hand is required and these examples belong usually to two different categories: tele-manipulation and learning by demonstration. In the former case, data gloves are typically used to capture human hand motion to move robotic hands. In [19], a DLR Hand is controlled using a CyberGlove. In the learning by demonstration case, human data are used to improve the grasping performances by teaching to the robot the correct posture necessary to obtain stable grasps. In [20], authors evaluated and modelled human grasps during the arm transportation sequence in order to learn and represent grasp strategies for different robotic hands, while in [21] Do et al. proposed a system for vision-based grasp recognition, mapping and execution on a humanoid robot to provide an intuitive and natural communication channel between humans and humanoids.
Since the kinematics and configuration spaces of a human hand and an artificial robotic hand are typically different, especially when the robotic hand is not anthropomorphic, a mapping function is needed. The main approaches that have been used in the past to deal with this problem are: joint to joint mapping, fingertip mapping and pose mapping. The first method consists on a direct association between joints on the human hand and joints on the robotic hand. Intuitively, this solution is quite efficient for anthropomorphic hands, while some empirical solutions have to be adopted for non-anthropomorphic device. An example were joint values of a human hand model are directly used to move joints of a robotic hand can be found in [22]. Although it represents the simplest way to map movements between hands, this method presents some drawbacks: since the joint correspondence is set by empirical and heuristic considerations, in each specific case it has to be defined according to the kinematic characteristics of the hands, in other terms it is not generalizable, and its performance notably decreases with non-anthropomorphic structures.

Cartesian space mappings focus on the relation between the two different workspaces. This solution is more suitable for representing the fingertip positions and it is a natural approach when, for example, precision grasps are considered. In [23] a point-to-point mapping algorithm is presented for a multi-fingered telemanipulation system where fingertip motion of the human hand is reproduced with a three-finger robotic gripper. In [24] authors use a virtual finger solution to map movements of the human hand onto a four-fingered robotic hand. In [25] authors propose a mapping approach divided in three different steps. In the first step, a virtual finger approach is used to reduce the number of fingers. Then, an adjustable or gross physical mapping is carried out to get an approximate grasp of the object which is geometrically feasible. Finally, a fine-tuning or local adjustment of grasp is performed. Even if this method presents some advantages with respect to the joint to joint mapping, it is still not enough general to guarantee a correct mapping in terms of forces and movements exerted by the robotic hand on a grasped object.

The pose mapping can be considered as a particular way of indirect joint angle mapping. The basic idea of pose mapping is to try to establish a correlation between human hand poses and robot hand poses. For example, Pao and Speeter [26] developed an algorithm that tries to translate human hand poses to corresponding robotic hand positions, without loss of functional information and without the overhead of kinematic calculations, while in [27] neural network are used to learn the hand grasping posture. Anyway, the proposed solutions can produce unpredictable motions of the robot hand, and thus in our opinion are only exploitable in cases where basic grasp postures are required.

Besides the above mentioned methods, Griffin et al. proposed in [28] a 2D virtual object based mapping for telemanipulation operations. The object based scheme assumes that a virtual sphere is held between the thumb of the user and the index finger. Important parameters of the virtual object (the size, position and orientation) are scaled independently and non-linearly to create a transformed virtual object in the robotic hand workspace. This modified virtual object is then used to compute the fingertip locations of the robotic hand, which in this case is a two-finger, four DoFs gripper.

A 3D extension of the last method is presented in [29]. Even if this extension allows to analyse more cases, this method is still not enough general for our purposes. In particular, it is constrained by the kinematics of the master and slave hand, the number of contact points (three) and their locations (the fingertips) which have to be the same for both the hands. Then it can be used only for a given pair of human and robotic hands and for precision grasp operations.

The approach proposed in this paper is inspired by the last two mentioned methods. The main contributions of our work with respect to the methods proposed in [28], [29] is a generalization to a generic number of contact points that can be different in the human and robotic hands and there are no constraints on positions of contact points on the master and on the slave hand. We will describe in detail this aspect in the following.

### III. BACKGROUND ON GRASP MODELLING AND SYNERGY ACTUATED HANDS

This section summarizes the main equations necessary to study hands controlled by synergies. A more detailed presentation of the problem is described in [10], further details on grasp theory can be found in [30], [31].

Consider a generic hand grasping an object as sketched in Fig. 3. The hand and the object have \( n_l \) contact points.

Let \( \{N\} \) indicate a reference frame on the hand palm, and \( \{B\} \) a reference frame on the object. Let \( o \in \mathbb{R}^3 \) denote the position of \( \{B\} \) origin with respect to \( \{N\} \), and let \( \phi \in \mathbb{R}^3 \) a vector describing the relative orientation between the frames.
(e.g. Euler angles). Let furthermore $u = [\omega^T \theta^T]^T \in \mathbb{R}^6$ collect information on position and orientation between the above mentioned frames.

Compliance can be considered at different levels on the system: the contact between the object and the hand is generally not stiff and the contact induces a local deformation of the surfaces. Furthermore, often the hand links and joint actuators have compliance that have to be considered [32]. In [11], the compliance was introduced also at the synergy level, however, we did not consider it in this study.

In a quasi-static condition, the force and moment balance for the object can be described by the equation:

$$w = -G\lambda$$

(1)

where $w \in \mathbb{R}^6$ is the external load wrench applied to the object, $\lambda \in \mathbb{R}^{n_c}$ is the contact force vector, $G \in \mathbb{R}^{6 \times n_c}$ is the grasp matrix. In this paper, for the sake of simplicity, we consider a single point with friction contact model, often referred to as hard finger contact model [31]. With this type of contact model, at each contact point the force $\lambda$ has three components and no moments are transmitted. Then the dimension of the contact force vector $\lambda$ is $n_c = 3n_j$.

By applying the static-kinematic duality relationship, we can express the velocities $\dot{\nu}^o \in \mathbb{R}^{6 \times 1}$ of the contact points on the object as a function of the object twist $\nu^o \in \mathbb{R}^6$:

$$\dot{p}^b = G^T \nu^o$$

(2)

If a hard finger contact model is assumed, the transpose of the grasp matrix can be expressed as [31]

$$G^T = \begin{bmatrix} I & -[p_1 - o]_x \\ \vdots & \vdots \\ I & -[p_n - o]_x \end{bmatrix}$$

(3)

Solving eq. (1) for the contact forces introduces the definition of internal forces, i.e. the contact forces included in the nullspace of matrix $G$. In particular

$$\lambda = -G^\# w + A\xi$$

(4)

where $G^\#$ is the pseudoinverse of grasp matrix, $A \in \mathbb{R}^{n_c \times h}$ is a matrix whose columns form a basis for the nullspace of $G$ ($\mathcal{N}(G)$) and $\xi \in \mathbb{R}^h$ is a vector parametrizing the homogeneous part of the solution to eq. (1). The generic solution of the homogeneous part $\lambda_o = A\xi_o$ represents a set of contact forces whose resultant force and moment are zero, referred as internal forces.

The relationship between hand joint torques $\tau \in \mathbb{R}^{n_q}$, where $n_q$ is the number of actuated joints, and contact forces is:

$$\tau = J^T \lambda$$

(5)

1. $\nu^o = [\omega^T \theta^T]^T$, where $\omega$ is the object centre linear velocity, while $\theta$ is the object angular velocity.

2. In the following, we will indicate with the lower index $o$ the skew matrix that can be used to evaluate the cross product, i.e. for an arbitrary vector $v = [v_x \ v_y \ v_z]^T$, $v_o = \begin{bmatrix} 0 & -v_z & v_y \\ v_z & 0 & -v_x \\ -v_y & v_x & 0 \end{bmatrix}$.

where $J \in \mathbb{R}^{n_c \times n_q}$ is the hand Jacobian matrix [31].

The hand Jacobian relates the contact point velocities on the hand $\dot{p}^b$ to the joint velocities $\dot{\theta}$:

$$\dot{p}^b = J\dot{\theta}$$

(6)

We suppose that the hand is actuated using a number of inputs whose dimension is lower than the number of hand joints. These inputs are then collected in a vector $z \in \mathbb{R}^{n_z}$ that parametrizes the hand motions along the synergies.

In this paper soft synergies are defined as a joint displacement aggregation corresponding to a reduced dimension representation of hand movements according to a compliant model of joint torques which can be for instance obtained with tendons [33]. In other terms, the reference values of joint angular velocities $\dot{\theta}_{ref} \in \mathbb{R}^{n_q}$ is a linear combination of synergy velocities $\dot{z} \in \mathbb{R}^{n_z}$ with $n_z \leq n_q$

$$\dot{\theta}_{ref} = S\dot{z}$$

(7)

through the synergy matrix $S \in \mathbb{R}^{n_q \times n_z}$, whose columns describes the shapes, or directions, of each synergy in the joint space.

As detailed in [10], [16], to solve the problem of force distribution we need to introduce other equations, usually referred to as constitutive equations, that model the system compliance. Starting from an initial equilibrium condition, the contact force variation is expressed as

$$\delta \lambda = K_s (J\delta \theta - G^T \delta u)$$

(8)

where $\delta \theta$ and $\delta u$ are the variations of joint angles and object motion respectively, and $K_s \in \mathbb{R}^{n_c \times n_c}$ represents the contact stiffness matrix. If we consider the joint compliance, as discussed for instance in [32], also the joint torques can be expressed as

$$\delta \tau = K_q (\delta \theta_{ref} - \delta \lambda)$$

(9)

where $K_q \in \mathbb{R}^{n_q \times n_q}$ is the joint stiffness matrix and $\delta \lambda$ is the joint reference value variation.

Starting from an equilibrium configuration and applying a small change of the synergy reference value $\delta z$, according to a procedure similar to those described in [16] and detailed in [10], it is possible to evaluate the corresponding configuration variation. In particular, object displacement $\delta u$ is given by

$$\delta u = V \delta z$$

(10)

where $V = (GKG^T)^{-1}GKS$, in which the equivalent stiffness $K$ is evaluated as $K = (K_s^{-1} + JK_qG^T)^{-1}$ [32]. From eq. (8) one gets the contact force changes $\delta \lambda$ as

$$\delta \lambda = P \delta \xi$$

(11)

where $P = (I - G^h_k G) KJS$, with $G^h_k$ pseudoinverse of grasp matrix $G$ weighted with the stiffness matrix $K$ [16].

Among all the possible motions of the grasped objects, rigid-body motions are relevant since they do not involve visco-elastic deformations in the contact points. Considering eq. (8) and imposing $\delta \lambda = 0$, the rigid body motion can be obtained computing $\mathcal{N} [JS - G^T]$. Let us then define a matrix $\Gamma$, whose columns form a basis of such subspace. Under the
hypothesis that the object motion is not indeterminate neither redundant [31] matrix $\Gamma$ can be expressed as

$$\Gamma = N \begin{bmatrix} JS & -GT \end{bmatrix} = \begin{bmatrix} \Gamma_{\text{CSS}} & \Gamma_{\text{ACS}} \end{bmatrix}$$

(12)

where the image spaces of $\Gamma_{\text{CSS}}$ and $\Gamma_{\text{ACS}}$ consist of coordinated rigid–body motions of the mechanism, for the soft synergy references and the object position and orientation, respectively. It is possible to show that

$$\mathcal{R}(\Gamma_{\text{ACS}}) \subseteq \mathcal{R}(V).$$

(13)

i.e. rigid–body motions of the object are not all the possible motions of the object controlled by synergies as in (10). The subspace of all synergy controlled object motions $\mathcal{R}(V)$ also contains motions due to deformations of elastic elements in the model. For a complete discussion on rigid body motions, including kinematic indeterminacy and redundancy, refer to [10].

In Section V we will analyse the performance of the proposed mapping procedure both in free-hand motions and during grasping operations. In the latter case we will use the mapping procedure to command the reference values of the joint variables and there we will have $q_{\text{ref}} \neq q$, because of system compliance.

IV. DESCRIPTION OF THE MAPPING METHOD

The target of this study is to define a way to map a set of synergies defined on a reference human hand, henceforth the paradigmatic hand, onto a generic robotic hand (Fig. 4). However, the method can be used to map also arbitrary motions of the human hand, including the case where all the joints are independently controlled.

The kinematic analysis of the paradigmatic hand with postural synergies is reported in [11] and is briefly summarized here. Similar analysis can be found in [34], [35]. With respect to the model presented in [11], here the model has been enriched adding the distal interphalangeal joints for the index, middle, ring and pinkie fingers, and the abduction/adduction DoF of the middle finger metacarpal joint. The resulting kinematic model has 20 DoFs.

The hand’s fingers are modelled as kinematic chains sharing their origin in the hand wrist. Fig. 5 shows a scheme of bone linkage. The bone lengths have been chosen according to the anatomy of the real hand skeleton [36], [37]. The metacarpophalangeal (MCP) joint of the index, middle, ring and pinky fingers have two DoFs each (one for adduction/abduction and another flexion/extension). The proximal interphalangeal (PIP) and distal interphalangeal (DIP) joints of the other fingers have one DoF each. The thumb has four DoFs: two DoFs in trapeziometacarpal (TM) joint, one DoF in metacarpophalangeal (MP) joint, and one DoF in interphalangeal (IP) joint.

Let the paradigmatic hand be described by the joint variable vector $q_h \in \mathbb{R}^{q_h}$ and assume that the subspace of all configurations can be represented by a lower dimensional input vector $z \in \mathbb{R}^z$ (with $n_z \leq n_{qh}$) which parametrizes the motion of the joint variables along the synergies, i.e. $q_h = S_h z$, being $S_h \in \mathbb{R}^{q_h \times n_z}$ the synergy matrix. In terms of velocities one gets

$$\dot{q}_h = S_h \dot{z}.$$  

(14)

Let furthermore $q_r \in \mathbb{R}^{q_r}$ represent the joint variable vector of the robotic hand. In general we have $n_{qr} \neq n_{qh}$. The ultimate goal of this work is to control the reference joint velocities $q_r$ of the robotic hand in a synergistic way using the same vector of synergies $z$ of the paradigmatic hand.

We developed a mapping framework in which the kinematic structures of the hands, the hand configurations, and the number of reference points and their relative positions are arbitrary decided. Working in task space is a possible way to get such flexibility. To the best of our knowledge all previous synergy mapping strategies [6], [38] do not explicitly take into account the task to be performed by the robotic hand. In this work, we propose a method of projecting synergies...
from paradigmatic to robotic hands which explicitly takes into account the task space. One of the main advantages of designing a mapping strategy in the task space is that the results can be used for robotic hands with very dissimilar kinematics. To define the mapping we assume that both the paradigmatic and the robotic hands are in given configurations $q_{0h}$ and $q_{0r}$.

**Remark 1:** Note that there is not a fixed a-priori relationship between initial reference configurations, that can be independently chosen. This feature allows to obtain a more general and adaptable mapping procedure, with respect to the methods defined in the configuration space. However, the choice of very dissimilar initial configuration may lead to hand trajectory that appears very different in the configuration space, although they produce, on the virtual object, the same deformation. Very dissimilar initial space, although they produce, on the virtual object, the same deformation as a function of the synergy vector velocity $\dot{z}$ of the paradigmatic hand

\[
\begin{bmatrix}
\dot{p}_h \\
\dot{o}_h \\
\dot{r}_h
\end{bmatrix} = A_h^{-1} p_h = A_h^{-1} J_h S_h \dot{z}
\]

where $A_h^{-1}$ denotes the pseudo-inverse of matrix $A_h$. In this work, we suppose that $N(A_h) = \emptyset$. If this hypotheses is not verified, possible virtual sphere motions and/or deformations do not involve reference point displacements. In this case, the problem is indeterminate and a unique solution for the mapping problem could not be identified. This situation is similar to indeterminate grasps, in which the transposes of grasp matrix $G$ defined in eq. (2) and evaluated considering a hard finger contact model, as shown in eq. (3). Matrix $A_h$ depends on the type of motion that we decide to reproduce on the robotic hand and then it depends on the task. From (6), (14) and (16) we can evaluate the virtual sphere motion and deformation as a function of the synergy vector velocity $\dot{z}$ of the paradigmatic hand

\[
\tilde{p}_{ih} = \dot{p}_h + \omega_h \times (p_{ih} - o_h) + \dot{r}_h (p_{ih} - o_h).
\]

Grouping all the reference point motions one gets

\[
\tilde{p}_h = A_h \begin{bmatrix}
\dot{p}_h \\
o_h \\
\dot{r}_h
\end{bmatrix},
\]

where matrix $A_h \in \mathbb{R}^{n_h \times 7}$ is defined as follows

\[
A_h = \begin{bmatrix}
I & -[p_{1h} - o_h] \times (p_{1h} - o_h) \\
\vdots & \vdots & \ddots & \vdots \\
I & -[p_{n_h} - o_h] \times (p_{n_h} - o_h)
\end{bmatrix}
\]

It is worth to note that, the first six columns of $A_h$, i.e. its first two column blocks in eq. (17), correspond to the transpose of grasp matrix $G$ defined in eq. (2) and evaluated considering a hard finger contact model, as shown in eq. (3). Matrix $A_h$ depends on the type of motion that we decide to reproduce on the robotic hand and then it depends on the task. From (6), (14) and (16) we can evaluate the virtual sphere motion and deformation as a function of the synergy vector velocity $\dot{z}$ of the paradigmatic hand

\[
\begin{bmatrix}
\dot{p}_h \\
o_h \\
\dot{r}_h
\end{bmatrix} = A_h^{-1} p_h = A_h^{-1} J_h S_h \dot{z}
\]

where $A_h$ denotes the pseudo-inverse of matrix $A_h$. In this work, we suppose that $N(A_h) = \emptyset$. If this hypotheses is not verified, possible virtual sphere motions and/or deformations do not involve reference point displacements. In this case, the problem is indeterminate and a unique solution for the mapping problem could not be identified. This situation is similar to indeterminate grasps, in which the transposes of grasp matrix has a non-trivial nullspace, as discussed in [31].

Virtual sphere motions and deformations have to be mapped on the robotic hand. The robotic hand is in a given configuration $q_{0r} \in \mathbb{R}^{n_r}$ with resulting reference point location vector $p_r \in \mathbb{R}^{n_r}$. Recall that no hypothesis were imposed on the number of reference points on the paradigmatic human and robotic hands, in general we can consider $n_{ch} \neq n_{cr}$, neither on their locations, and neither on the initial configuration of the two hands. A virtual sphere is computed also in the robotic hand on their configuration.

Although the virtual sphere does not represent an object grasped by the paradigmatic hand, it can be easily shown that with a suitable model of joint and contact compliance, the rigid-body motion of the virtual sphere corresponds to the motion of a grasped spherical object and that the non-rigid motion approximately accounts for the normal components of the contact forces for a spherical object grasp.

Representing the motion of the hand through the virtual object, the motion of the generic reference point $p_{ih}$ can be expressed as

\[
\tilde{p}_{ih} = \dot{p}_h + \omega_h \times (p_{ih} - o_h) + \dot{r}_h (p_{ih} - o_h).
\]

This factor is necessary to scale the velocities from the paradigmatic to the robotic hand workspace. Note that the scaling factor depends on the dimension of the hands, but also on their configuration.

Then, the motion and deformation of the virtual sphere generated by the paradigmatic hand are scaled and tracked by the virtual sphere referred to the robotic hand.
\[
\begin{bmatrix}
\dot{r}_r \\
\dot{\omega}_r \\
\ddot{r}_r
\end{bmatrix} = K_c
\begin{bmatrix}
\dot{h}_r \\
\dot{\omega}_h \\
\ddot{h}_r
\end{bmatrix}
\]  
(20)

where the scale matrix \( K_c \in \mathbb{R}^{7 \times 7} \) is defined as
\[
K_c = \begin{bmatrix}
k_{x1}I_{3,3} & 0_{3,3} & 0_{3,1} \\
0_{3,3} & I_{3,3} & 0_{3,1} \\
0_{1,3} & 0_{1,3} & 1
\end{bmatrix}
\]  
(21)

According to eq. (16) and (17), the corresponding robot reference point velocity is given by
\[
p_r = A_r
\begin{bmatrix}
\dot{r}_r \\
\dot{\omega}_r
\end{bmatrix},
\]  
(22)

where matrix \( A_r \in \mathbb{R}^{q_r \times 7} \) is defined as follows
\[
A_r = \begin{bmatrix}
I & -[p_{1r} - o_r]_x & (p_{1r} - o_r) \\
\vdots & \vdots & \vdots \\
I & -[p_{nr} - o_r]_x & (p_{nr} - o_r)
\end{bmatrix}
\]  
(23)

Recalling eq. (18) and (20) we can express the robotic hand reference point velocities \( p_r \) as a function of the synergy velocities \( \dot{z} \) as
\[
p_r = A_r K_c A_p^T J_p S_h \dot{z}
\]  
(24)

and, considering the robot hand differential kinematics \( \dot{p}_r = J_r \dot{q}_r \), where \( J_r \in \mathbb{R}^{q_r \times 7} \) is its Jacobian matrix, the following relationship between robot hand joint velocities and synergy velocities is defined
\[
\dot{q}_r = J_r^T A_r K_c A_p^T J_p S_h \dot{z},
\]  
(25)

where \( J_r^T \) indicates the robot hand Jacobian pseudoinverse. If the robotic hand in the selected grasp configuration is redundant, i.e. if \( N(J_r) \neq \emptyset \) [31], eq. (25) can be furthermore generalized [39] as
\[
\dot{q}_r = J_r^T A_r K_c A_p^T J_p S_h \dot{z} + (I - J_r^T J_r) \dot{q}_0
\]  
(26)

with \( \dot{q}_0 \) arbitrary. The vector \( \dot{q}_0 \) can be chosen in order to conveniently use the redundant DoFs. The discussion on how to manage the robotic hand redundancy is not deepened in this paper.

Remark 3: The vector of synergies actuation \( \dot{z} \) is mapped onto the robotic hand joint variable \( q_r \) through matrix \( J_r^T A_r S_c A_p^T J_p S_h \) which is function of

- paradigmatic and robotic hand configurations \( q_{0h} \) and \( q_{0h} \)
- location of the reference points for the paradigmatic and robotic hands, \( p_h \) and \( p_r \).

The proposed method allows to define a non-linear mapping between the paradigmatic human-like hand and the robotic hand.

V. SIMULATIONS AND PERFORMANCE ANALYSIS

The proposed mapping algorithm was validated through numerical simulations considering two different robotic hand models. The first model was a three-fingered fully-actuated robotic hand with the kinematic structure that resemble the Barrett Hand [40], with two joints in the thumb (that is fixed with respect to the palm, no abduction/adduction joint) and three joints in the other two fingers (two flexion/extension, one abduction/adduction). The second model was the DLR/HIT II Hand [41]. It has an anthropomorphic structure with five fingers and 15 DoFs. The simulations were realized using Matlab R2009a over a 2.4 GHz Intel Core i5, 4 GB RAM.

The joint to joint mapping and the fingertip mapping methods were compared with the proposed virtual sphere algorithm. Other mapping methods [28], [29] were not taken into account since they can not be easily extended to kinematic structures that differ from those proposed in the relative papers.

It is worth noting that differently from other approaches where the joint to joint mapping only concerns pre–grasps [22], we extended its use to grasp analysis including in the hand model both contact and joint compliance. This allows to easily extend the pure kinematic model to the quasi–static analysis of grasp.

The actual grasp of different objects was considered. In this analysis we evaluated what happens if we use the described mapping strategy to control the grasp of real objects. We compared the mapping algorithms in terms of internal force variations and object motions. Both these aspects are fundamental to evaluate the performance of a grasping and manipulation task [42]. In the end, a simulation of a possible application of the method to a manipulation task is reported.

A. Internal force evaluation

The proposed mapping method is independent from the number of contact points considered in the paradigmatic and in the robotic hands. A direct comparison of the force exerted on the object is, thus, not possible when different number of contact points are selected. We adopted a measure of the whole object deformation produced by the activation of synergies, in a compliant context as described in Section III, to evaluate and compare the performance of the mapping procedure. We considered the energy variation of the hand-object system due to contact forces.

For each mapping method and for each considered robotic hand, a synergy variation \( \delta \lambda \) was imposed both to the paradigmatic and to the robotic hand. Note that the fingers not involved in the grasp were moved according to the synergies’ activation on the paradigmatic hand, while they were left at their respective initial positions on the robotic hands, for all the simulations. According to eq. (11) the corresponding internal force variation \( \delta \lambda \) was evaluated. Assuming a linear compliant model for the contacts as those described in eq. (8), the contact force variation corresponds to a deformation of the contact springs. By indicating with \( \delta x \) the vector containing the deformation components of each contact point evaluated as \( \delta x = K^{-1} \delta \lambda \), the elastic energy variation produced by the
activation of synergies can be computed as
\[ \delta E_{el} = \frac{1}{2} K_s \| \delta x \|^2 = \frac{1}{2} K_s^{-1} \| \delta \lambda \|^2, \]  
(27)

where \( K_s \) is the contact stiffness matrix defined in eq. (8). The sign of the energy variations has been taken into account during the simulations.

Tables I and II show how far is the Barrett Hand from replicating the energy obtained with the paradigmatic hand moving the hand along the first, second and third synergy respectively, when a precision/fingertip grasp is considered. The reported values indicate, in percentage, the energy variation difference. Two cases were considered, with three and four reference points respectively, on the paradigmatic hand. In both cases three reference points were assumed on the robotic hand. The size of the grasped object (a sphere in this case) was selected as to rather fit the average position of the fingers of the paradigmatic hand. The sphere grasped by the Barrett Hand was scaled using the scaling factor defined in eq. (20). As it can be seen from the tables, energy values obtained in the robotic hands using the proposed virtual sphere method are closer to that obtained for the paradigmatic hand, for all the evaluated cases. It is worth to note that when the number of contact points is different, a correction factor \( k_{cp} = \frac{s_{ch}}{s_{cr}} \) has to be considered to compare the different values. Then deformation energy on the robotic hand was computed as
\[ \delta E_{el,r} = \frac{1}{2} k_{cp} K_s \| \delta x \|^2. \]  
(28)

The result trends obtained with three and four reference points on the paradigmatic hand are quite similar, and show the robustness of the proposed mapping procedure with respect to the number of reference points chosen on the paradigmatic hand.

Tables III and IV present the results of similar simulations performed with the DLR-HIT II Hand model. We considered the same number of reference points (four) in the robotic hand, while two cases, four and three reference points respectively, were considered for the paradigmatic hand. Also in this case, the performances of the proposed mapping method are better with respect to other methods, since the energy stored in the contact springs of the robotic hand is closer to the value obtained with the paradigmatic one. This means that the deformation imposed to the grasped object by the robotic hand is similar to the deformation that would be impressed by the paradigmatic hand when the same synergy input is applied. Note that, in Table IV the fingertip method is not considered since, in that example we considered more fingers in the robotic hand (four) than those considered in the paradigmatic hand (three) and the fingertip method is not directly applicable.

Additional tests were performed to analyse the sensitivity of the obtained results, in terms of elastic energy variation, with respect to variation of the object dimension. We considered spheres of different sizes, manipulated by the Barrett Hand, driven by the paradigmatic hand grasping always the same sphere. Results obtained by activating only the first synergy are shown in Fig. 6. On the x-axis, the scaling factors computed as the ratio between the two sphere radii are reported. On the y-axis the elastic energy variation percentage difference between the paradigmatic and the robotic hand is shown. As it is clear from the diagram, the virtual sphere algorithm has always a lower difference value, and, moreover, its performance is substantially independent from the the scaling factor, and thus from the object size. Similar results are obtained when other synergies are activated.

In Table V the results of a power grasp are presented. Six contact points were considered on the paradigmatic hand while four and six points were considered on the Barrett and DLR/HIT II Hand, respectively. Fig. 7 shows the considered grasps. The better results obtained with the DLR/HIT II Hand are due to the highest dexterity of the device that present 15
DoFs instead of the 8 DoFs considered for the Barrett Hand.

B. Object motion during manipulation

In order to investigate how the mapping procedure influences the grasped object trajectory, we simulated the motion $\delta u$ of an object grasped by the robotic hands, while the first synergy is activated on the paradigmatic hand. As summarized above, $\delta u$ is due both to the displacement of contact point due to the and configuration variation (rigid body motion), and to the different deformation of the contact equivalent springs that can be defined to take into account compliance [12]. The vector $\delta u$ is composed of the object centroid displacement and object rotation, and was evaluated according to eq. (10). For the sake of simplicity, in the presented results only the translational component of the object motion was considered. We evaluated the difference between the centroid motion directions of three different objects grasped by the paradigmatic and robotic hands: a sphere, a cube and a cylinder.

The results for a cube object motion are reported in Table VI. In this case we considered the motion due to the activation of the first three synergies separately using the virtual sphere mapping onto the Barrett Hand.

Concerning the cylinder, in Table VII are reported the angular differences in motion direction obtained mapping onto the DLR-HIT II Hand the first three synergies separately. In Fig. 10 the consider grasps are shown.

For the spherical object, we evaluated also the sensitivity of the mapping procedure with respect to hand initial configuration changing the starting position of the robotic hands by varying its palm orientation with respect to the palm of the paradigmatic hand. The obtained results show that the Fingertip method and the virtual sphere mapping are substantially independent from the different orientations of the hands. The virtual sphere mapping behaves better in terms of difference between object directions. The joint to joint mapping method, instead, obtains sensibly worse results, and its sensitivity with respect to hand orientation is evident. In particular, in the DLR-HIT II Hand, we observe that the joint to joint angular error is approximately linearly dependent on the orientation difference between the hands.

C. Manipulation task

The virtual sphere mapping algorithm previously described was also tested in a manipulation task, as an example of its applicability. In this example, we considered the grasp of a cubic object. In the initial configuration the paradigmatic hand grasped the cube with three contact points placed at the fingertips of the thumb, the index and the middle finger, as sketched in Fig. 11(a). We suppose that the internal forces,
in the reference configuration, are sufficient to guarantee a stable grasp, then they satisfies force closure criteria. [31]. The same cube was held by the DLR-HIT II Hand, as shown in Fig. 11(c). A cube with dimension increased by a factor 2.5 was grasped by the Barrett Hand, as shown in Fig. 11(b). The resulting scaling factor, necessary for the virtual sphere mapping algorithm, that depends both on the object size and on the position of the contact points, for the Barrett hand was $k_{sc} = 2.85$. It is worth to recall that the scaling factor, defined in eq. (19) is determined by the configuration of the both robotic and paradigmatic hand, once the contact points are fixed.

According to the results presented in [10], when three contact points are considered and a hard finger contact model is assumed, the dimension of the internal force subspace, i.e. the dimension of grasp matrix $G$ nullspace, is three, then at least a set of four synergies is needed to produce a rigid body motion, as those defined in eq. (12). The combination of the first four synergies for the paradigmatic hand were considered to obtain a rigid body motion on the object, they were evaluated by calculating matrix $\Gamma$ in eq. (12). The paradigmatic hand rigid body motion was then mapped onto movements of the two robotic hands through the proposed virtual sphere mapping algorithm. The centroid of the cube was displaced of 6mm, along the direction defined by the rigid-body motion imposed by the paradigmatic hand. For each integration step, matrix $\Gamma = [\Gamma^T_1, \Gamma^T_2, \Gamma^T_{nsc}]$ was evaluated from eq. (12). In this example, $n_s = 4$, $\Gamma_{zcs} \in \mathbb{R}^4$ and $\Gamma_{nsc} \in \mathbb{R}^4$ so the rigid body motion produced by the paradigmatic hand was generated by activating the following synergies $\delta z = \Gamma_{zcs} \delta T$, with $\delta T = 0.04s$. The corresponding object displacement was $\delta u \approx \Gamma_{nsc} \delta T$. The trajectory of the object center can be then updated and the grasp is evaluated in the new configuration for the paradigmatic hand. For the robotic hands, the object displacements were evaluated according to eq. (10) as $\delta u \approx V_e \delta z$. The simulation was carried out in 1.07s. The trajectories of the cube grasped by the paradigmatic hand and of those manipulated by the robotic hands were then evaluated by numerical integration. For the Barrett hand, the average distance between the final point of the trajectories was 3mm, the mean value of the difference, during all the path, was 1mm. These values have been evaluated taking into account the scaling factor. In the DLR-HIT II hand case the trajectories was practically the same. The sensibly better performances of the DLR-HIT II hand are a consequence of its higher dexterity with respect to the other one.

In Fig. 12 the trajectories of the cubes manipulated by the three hand models are shown. In this plot the actual trajectories are displayed, without re-scaling them to take into account the scaling factor, for this reason the Barrett Hand trajectory is longer than the other two.
controller with different robotic hands, or simply substitute a device without changing the controller, thus realizing a sort of abstraction layer for robotic hand control based on synergies.

This mapping has been numerically evaluated in grasping and manipulation tasks. Work is in progress to validate the virtual sphere mapping also for the approaching phase of grasps. Simulation results are very interesting in terms of performances as shown in the previous section. However, this approach presents some drawbacks. The proposed mapping is based on a heuristic approach: we choose to reproduce a part of the hand motion, which practically corresponds to move and squeeze a spherical object. Although squeezing and moving an object explain a wide range of tasks, many other possibilities exist in manipulating objects which are not modelled with this mapping. In [43], for instance, an ellipsoid was considered as virtual object, and its three-dimensional radial deformation was included in the mapping. The algorithm can be modified to consider different shapes and different types of movements in the task space. Increasing the number of parameters in the virtual object and virtual displacement definition allows to reproduce more complex hand movements, but, at the same time, increases the mapping complexity and robustness.

For a given object to grasp, different grasping planning algorithms [6] can be used to choose the contact points and the hand configurations to be used as parameters of the virtual sphere mapping.

Concerning the number of contact points in the paradigmatic and robotic hand, the numerical simulations showed that they influence the performance of the mapping procedure, in terms of object deformation and displacement. For the DLR-HIT II Hand, for instance, the best results are obtained when three contact points are considered on the paradigmatic hand and four points are considered on the robotic one. This is probably due to the fact that the fourth contact point does not add significant information useful for the mapping, but, on the contrary, it adds a sort of noise in the virtual object displacement and deformation estimation, that cannot be reproduced by the robotic hand. Furthermore, the different synergies lead to different performance in terms of elastic energy variation and object motions, in particular, the mapping of synergies that, in the paradigmatic hand, do not produce a significant virtual object displacement or radial deformation, is more difficult. This is the case, for example for the DLR-HIT II Hand, of the second synergy and three contact points in the paradigmatic hand, in which the fingers approximately moves on the virtual object surface, without producing a significant object displacement or radial deformation.

The performance of the mapping algorithm, in terms of replicating the effect produced by the paradigmatic hand on the object, clearly depends on the kinematic structure of the robotic hand, in particular, on the number of fingers and joints per finger, and their arrangement. Theoretically there is not a lower bound to the number of fingers and joints to implement the proposed mapping. The minimum level of hand structure complexity can be defined on the basis of the required performance level.

Finally, in this paper we focused on the reproduction, in the task space, of the object motion and grasp forces, produced
by controlling the paradigmatic hand through synergies. The human hand is still probably the best grasping device, and for this reason we started from it to develop the described mapping. In particular, the synergy organization of hand joint was observed and mapped in this work, since it represents a simple but versatile way to control a complex kinematic structure, and then naturally is the base on which the mapping procedure was initially developed and tested. However, it is worth to underline that the proposed mapping is not limited to synergies and can be generalized to arbitrary hand movements.

VII. CONCLUSION AND FUTURE WORK

Designing synergy-based control strategies in the paradigmatic hand domain can dramatically reduce the dimensionality of the grasping and manipulation problems for robotic hands. However, an efficient mapping is needed to deal with robotic hands with dissimilar kinematics. A synergy based mapping approach can be useful also in uncertain conditions. The human hand synergies are able to adapt to a wide range of tasks, and to work in uncertain environments, with an almost infinitely wide set of objects. The synergies are defined to synthesize the common components of all these tasks and then they represent an optimal trade-off between simplicity and versatility. A synergy based mapping can take advantage of these properties and consequently can result to be a robust control solution also when the environment conditions and/or the planned tasks are uncertain.

We proposed a method for mapping synergies that, using a virtual object, allows to specify the mappings directly in the task space, thus, avoiding the problem of dissimilar kinematics between human—like hand and robotic hands. We compared our solution to the most used solutions existing in literature and we evinced that the proposed method is more efficient in terms of internal forces mapping and direction of motion. We tested the method in simulation with two robotic hands model with dissimilar kinematic structure. Further investigations on different robotic hands have been already planned. One of the main issue of our approach is that the mapping is not linear and that its implementation could need a high computational burden. The ongoing research is evaluating the conditions whereby some simplification can be applied to get constant or slowly varying mapping. As future work, moreover, an integration with grasping simulator like Grasp-it! [44] is expected in order to use its grasp planner to determine initial position of the human and the robotic hand.

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