Hairpin vortices in turbulent channel flow

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Abstract

The phenomenon of birth and development of hairpin vortices in the wall region of turbulent channel flow is investigated numerically. The system of the Navier-Stokes equations is integrated by using a mixed spectral-finite difference numerical technique following the procedure of the Direct Numerical Simulation of turbulence (DNS), with accuracy requirements such that all the essential turbulent scales are resolved. The value of the Reynolds number based on wall-shear velocity and channel half height is $Re_t = 200$. A specially-assembled computing machine is used for the calculations, that includes one Intel® i7 Core™ quad-core processor, and two Nvidia® C-1060 Tesla™ devices. The swirling-strength criterion for vortex eduction is applied to the fluctuating portion of the computed velocity-field numerical database, and the resulting hairpin vortical structures are visualized as they evolve with time in the computing domain.

Keywords: Direct numerical simulation of turbulence; swirling-strength criterion for vortex eduction; hairpin vortices

1. Introduction

Wall-bounded turbulence is an issue that has attracted scientists since several years. The turbulent flow in a plane channel in particular has been analyzed with DNS (the method of the Direct Numerical Simulation of turbulence) among others, by Kim et al. [1], Lyons et al. [2], Kasagi et al. [3], Antonia et al. [4], Rutledge and Sleicher [5], Moser et al. [6], Abe et al. [7], Iwamoto et al. [8], Del Alamo and Jiménez [9], Del Alamo et al. [10], Tanahashi et al. [11], Iwamoto et al. [12], Hoyas and Jiménez [13], Hu et al. [14], at different values of the Reynolds number.

In the aforementioned works the system of the governing equations is mainly solved in the framework of the fractional-step method, in conjunction with Runge-Kutta algorithms for time marching. In most cases, the unsteady three-dimensional Navier-Stokes equations in rotational form are integrated in space by using either the fully spectral Fourier-Chebychev numerical technique originally introduced by Kim et al. [1] or minor variants of the latter, or also fully spectral techniques as introduced by other authors.

Inspite of the remarkable amount of work that has been accomplished on this subject, there are no definite conclusions on the character of the phenomena that occur in the near-wall region of wall-bounded flow, in particular as concerns the formation and the evolution in time of characteristic turbulent-flow structures.

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In the present work, the aforementioned issues are addressed with particular attention to the hairpin-like vortical structures that form near the solid walls of turbulent channel flow.

2. Numerical simulations

For the scopes of present work, the Navier-Stokes equations are integrated by means of the mixed spectral (Fourier) - finite difference method originally introduced by Alfonsi et al. [15], where a grid-stretching law of hyperbolic-tangent type is incorporated along the direction orthogonal to the solid walls. The Reynolds number based on wall-shear velocity and channel half-height is $Re_\tau = 200$.

More in particular, the system of the unsteady Navier-Stokes equations for incompressible fluids (due to the value of the Reynolds number of the calculations) in three dimensions and nondimensional, conservative form, is considered:

$$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_j} \left( u_i u_j \right) = -\frac{\partial p}{\partial x_i} + \frac{1}{Re_\tau} \frac{\partial^2 u_i}{\partial x_j \partial x_j}$$  \hspace{1cm} (1a)$$

$$\frac{\partial u_i}{\partial x_i} = 0$$  \hspace{1cm} (1b)$$

where $u_i(u, v, w)$ are the velocity components in the cartesian coordinate system $x_i(x,y,z)$, and $p$ is the pressure.

Variables and operators are nondimensionalized by the channel half-height $\delta$ for lengths, the wall-shear velocity $u_\tau = \sqrt{\tau_w/\rho}$ for velocities ($\tau_w$ is the mean shear stress at the wall), $\rho u_\tau^2$ for pressure and $\delta/u_\tau$ for time, being $Re_\tau = (u_\tau \delta/\nu) = 200$ the value of the friction Reynolds number, $\rho$ the fluid density and $\nu$ the fluid kinematic viscosity.

The fields are admitted to be periodic in the streamwise ($x$) and spanwise ($z$) directions, and equations (1) are Fourier transformed accordingly. The nonlinear terms in the momentum equation are evaluated pseudospectrally, by anti-transforming the velocities back in physical space to perform the products (FFTs are used). The $2/3$ $s$ dealiasing procedure is applied to avoid errors in transforming the results back to Fourier space.

In order to have a better spatial resolution near the walls, a grid-stretching law of hyperbolic-tangent type is introduced for the grid points along $y$, the direction orthogonal to the solid walls:

$$y_{j}^{str} = Py_j + (1-P) \left[ 1 - \frac{\tanh[Q(1-y_j)]}{\tanh Q} \right]$$  \hspace{1cm} (2)$$

where the $y_{j}^{str}$ are the stretched points, $y_j$ indicates the uniform distribution, and $P, Q$ are two parameters that characterize the distribution ($P = 1.9$, $Q = 1.9$). The partial derivatives along $y$ are calculated according to distribution (2), using appropriate second-order finite-difference expressions.

For time advancement a third-order Runge-Kutta algorithm has been implemented, where the time marching is accomplished with the fractional-step method. No-slip boundary conditions at the walls and cyclic conditions in the streamwise and spanwise directions have been applied to the velocity. Different numerical properties of the mixed-spectral finite difference algorithm have been extensively tested and validated, and the results can be found in Alfonsi et al. [15] and Passoni et al. [16].

The calculations have been executed on a computational grid that included 256 grid points along the streamwise direction ($x$), 181 grid points along the vertical direction ($y$), and 256 points along the spanwise direction ($z$), while the nondimensional time step was $\Delta t^+ = 10^{-4}$. The nondimensional values of the grid spacing were $\Delta x^+ = 9.82$, $\Delta y^+ = 0.25$ (at the wall), $\Delta y^+ = 3.87$ (at the channel center), and $\Delta z^+ = 4.91$. 
As concerns DNS resolution, these values have to be compared with those of the nondimensional Kolmogorov length microscale $\eta^+ = 1.89$ and with the nondimensional Kolmogorov time microscale $\tau_{\eta}^+ = 3.59$. It can be easily verified that, in both time and length (near the solid walls), the Kolmogorov microscales are well resolved in the calculations.

With the aforementioned calculation parameters, the initial transient of the flow in the channel has been first simulated until the turbulent statistically-steady state has been reached. Then the latter state has been simulated for $50 \times 10^3$ nondimensional time steps $\Delta t^+$. One-hundred nondimensional flow-field instants of the turbulent statistically-steady state have been recorded, one every $500 \Delta t^+$.

3. Computing system and code implementation

A hybrid CPU/GPU computing system has been assembled for the execution of the calculations. The system includes (see also at Fig. 1):

- 1 motherboard Asus P6T7 WS SuperComputer;
- 1 Intel Core i7 (quad-core) processor with 12 GB of DDR3 memory for optimal sequential execution;
- 2 Nvidia Tesla C-1060 devices (T-10 architecture) with 4 GB GDDR3 memory and 240 cores at 1.3 GHz for massively-parallel execution;
- 1 Nvidia GeForce GTX 285 with 1 GB GDDR3 for visualization;
- 1 storage system, based on 5 Western Digital VelociRaptor 300 GB SATA Hard Drives (10000 rpm) and 1 Seagate Barracuda 1 TB SATA Hard Drive (7200 rpm);
- 1 power supply Cooler Master Real Power Pro (1250 W).

The Tesla C-1060 is based on the CUDA technology (Compute Unified Device Architecture) by Nvidiia. It consists of 30 many-core processors, each containing 8 single-precision floating-point units, 1 double-precision
floating-point unit, and 16 KB shared memory for threads cooperation. The maximum total number of active hardware threads is \(30.720 \times 10^3\). The theoretical peak performance reaches 933 GFlop/s.

In the CUDA programming model, the GPU is seen as a compute device that executes a portion of an application (the kernel) that can be isolated as a function, thus working independently on different data. A kernel is compiled on the device (the Tesla C-1060) and is launched as a grid of thread blocks on the many-core processors. The CPU (host) manages the launch of the kernels, the allocation/deallocation of the device data structures, and the I/O functions.

Using the CUDA paradigm, the Navier-Stokes solver has been configured as follows:

- a pre-processing step for the plane-channel domain discretization and initial/boundary conditions implementation (CPU);
- the numerical integration of Navier-Stokes equations, in particular subdivided in a kernel for the convective and the diffusive terms, and a kernel for the Poisson problem (GPU);
- a post-processing step for statistical analysis and vortical-structure eduction (CPU).

In Fig. 2 a scheme of the computational process is represented.
4. Code performances

For the evaluation of the performances of the computational code on the above-described computing system, the speedup parameter has been mainly used. In order to gather the execution times, a sequential (CPU) version of the code has been run on, respectively, 1, 2 and 4 cores of the quad-core Intel i7 processor. Then, the hybrid CPU/GPU version of the code has been run on the system, involving one i7 core and one C-1060 device (only one Tesla C-1060 device has been used in these tests, of the two available on the system).

The results are summarized in Table 1, as related to one full time step of the calculations (that includes the three substeps of the Runge-Kutta time-marching procedure). Note that the Total Time reported at the fifth column of Table 1, also includes the time for the solver "velocity updating" phase, and all the necessary data-transfer operations (see also at Fig. 2).

Table 1. Execution times for one time step of the calculation process.

<table>
<thead>
<tr>
<th>Processing Unit(s)</th>
<th>Convective Term (s)</th>
<th>Diffusive Term (s)</th>
<th>Poisson Problem (s)</th>
<th>Total Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 CPU</td>
<td>4.923</td>
<td>0.342</td>
<td>0.801</td>
<td>7.647</td>
</tr>
<tr>
<td>2 CPU</td>
<td>2.592</td>
<td>0.174</td>
<td>0.420</td>
<td>4.070</td>
</tr>
<tr>
<td>4 CPU</td>
<td>1.545</td>
<td>0.096</td>
<td>0.204</td>
<td>2.430</td>
</tr>
<tr>
<td>1 CPU + 1 GPU</td>
<td>0.02628</td>
<td>0.02073</td>
<td>0.03600</td>
<td>0.30741</td>
</tr>
</tbody>
</table>

In Table 2 the performance of the code is reported in terms of speedup, as run on 1, 2, 4 CPUs and on the hybrid CPU/GPU configuration, as related to a full simulation-time step. It can be easily noticed that the speedup of the CPU/GPU code outperforms that of the 4-CPU code almost of an order of magnitude.

Table 2. Speedup of the calculations (full simulation-time step).

<table>
<thead>
<tr>
<th>Speedup 1 CPU</th>
<th>Speedup 2 CPU</th>
<th>Speedup 4 CPU</th>
<th>Speedup 1 CPU + 1 GPU</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.88</td>
<td>3.15</td>
<td>24.99</td>
</tr>
</tbody>
</table>

5. Vortex-eduction method

Of the different existing vortex-detection techniques (see, among others, Alfonsi [17] for an extensive presentation of vortex-detection methods and coherent structures of turbulence) for the scopes of present work, the swirling strength method, as introduced by Zhou et al. [18] has been adopted for vortex eduction.

Zhou et al. [18] devised the criterion of identifying vortices by visualizing isosurfaces of values of the imaginary part of the complex eigenvalue pair of the velocity-gradient tensor (see also Chakraborty et al. [19]).

When the discriminant of the characteristic equation of the velocity-gradient tensor ($D > 0$), the velocity-gradient tensor has one real eigenvalue ($\lambda_1$) and a pair of complex-conjugate eigenvalues ($\lambda_2, \lambda_3$), that can be written as:
\[ \lambda_1 = \lambda_r \]  
\[ \lambda_2 = \lambda_{cr} + i\lambda_{ci} \]  
\[ \lambda_3 = \lambda_{cr} - i\lambda_{ci} \]  

The swirling strength, given by \( \lambda_{ci} \), is a measure of the local swirling rate inside the vortex (while the strength of stretching or compression is given by \( \lambda_r \)). According to Zhou et al. [18], isosurfaces of the imaginary part of the complex eigenvalue pair of the velocity-gradient tensor can be used to visualize vortices. The method is frame independent and due to the fact that the eigenvalue is complex only in regions of local circular or spiralling streamlines, it automatically eliminates regions having vorticity but no local spiralling motion.

6. Results

After the application of the \( \lambda_{ci} \) vortex-detection technique to the fluctuating portion of the computed velocity-field database, a flow field appears as highly populated by turbulent structures adjacent to both the upper and the lower wall of the computing domain, where the majority of them has no definite form. Hairpin-like vortices are recognizable at the lower wall of the channel through the nondimensional instants \( t^* = 23 \) and \( t^* = 28 \), where some of these instants are shown in Figg. 3-6. In these Figures the nondimensional value of the swirling-strength parameter that characterizes the external surfaces of the vortical structures is:

\[ \lambda_{ci}^+ = \frac{\lambda_{ci}\delta}{Re \cdot u_r} = 0.0300 \]  

Fig. 3 shows a representation of the vortical field at the channel lower wall at \( t^* = 25 \). Two main persistent hairpin vortices are visible [denoted as (1) and (2)] both characterized by a remarkable elevation of their heads, while the legs are hidden in part by the presence of other structures nearby the solid wall. At \( t^* = 26 \) (Fig. 4) hairpins (1) and (2) continue to travel toward the downstream direction, showing a further elevation of the top of their heads (for an explanation of this phenomenon, one can refer to Alfonsi and Primavera [20]).

The phenomenon continues through \( t^* = 27 \) (Fig. 5) and \( t^* = 28 \) (Figure 6) where hairpins (1) and (2) become further deployed toward the downstream end of the channel, and their heads further raise. The continuous raising of the heads of vortices (1) and (2) with time denotes that there exist turbulent structures of the ejection type (\( Q_2 \) turbulent events in terms of Quadrant Analysis) that push the latter heads upwards, and that - at the same time - the strength of these upward-pushing structures is not that high as to cause the destruction of hairpins (1) and (2) (see [20]). Moreover, the fact that the legs of hairpin (1) and (2) stay close to the wall all along the temporal sequence considered, denotes that there exist turbulent structures of sweep type (\( Q_4 \) turbulent events in terms of Quadrant Analysis) that keep the legs of the hairpins stably at the wall, while letting the right portions of the hairpins rotate clockwise, and the left parts counterclockwise (see [20] for a more detailed description of the above phenomena). These are the main reasons why hairpins (1) and (2) are persistent, differently from other structures that are of ephemeral nature.

Figs. 7 and 8 show the vortical field respectively at \( t^* = 26 \) and \( t^* = 27 \), where a cut has been performed along an \( x-y \) plane located slightly rightwards from the heads of hairpins (1) and (2). In this case, a range of values of \( \lambda_{ci}^+ \) is represented in the visualizations, in particular \( 0.0300 \leq \lambda_{ci}^+ \leq 0.0375 \).

Interestingly, it can be noticed how the higher values of the swirling strength characterize the inner portions of the hairpin vortical structures (as expected) while the lower value actually defines their external surface. In particular, in Fig. 8 it is possible to note how the numerical simulation - due to the high resolution of the calculations - has been able of representing also the inner low-swirling-strength core of hairpin (1).
Fig. 3. Hairpins (1) and (2) at the lower wall of the channel at nondimensional time $t^+ = 25$.

Fig. 4. Hairpins (1) and (2) at the lower wall of the channel at nondimensional time $t^+ = 26$. 
Fig. 5. Hairpins (1) and (2) at the lower wall of the channel at nondimensional time $t^+ = 27$.

Fig. 6. Hairpins (1) and (2) at the lower wall of the channel at nondimensional time $t^+ = 28$. 
Fig. 7. A $x$-$y$ plane section of vortical field at nondimensional time $t^+ = 26$.

Fig. 8. A $x$-$y$ plane section of vortical field at nondimensional time $t^+ = 27$. 
7. Concluding remarks

An analysis is performed on hairpin vortices in the wall region of turbulent channel flow. The vortical structures are extracted from an accurate DNS database using the swirling strength criterion.

Flow visualizations of isosurfaces of given values of the swirling-strength parameter provided exhaustive representations of the vortical field in the computing domain, in which persistent hairpins can always be distinguished from more ephemeral vortical structures.

Details of the inner portions of hairpin vortices have also been clearly represented, showing that the Direct Numerical Simulation of turbulence (DNS) is a computational approach that is able to accurately simulate turbulence physics.

References