Performance Comparison of Energy-Efficient Power Control for CDMA Code Acquisition and Detection

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Abstract—This paper investigates the performance of distributed power control techniques in the uplink of a flat-fading CDMA wireless network, based on noncooperative game theory. Two energy-efficient criteria, tailored for data detection and code synchronization, are compared in terms of achieved utility at the Nash equilibrium, showing that the optimal power allocation devised for data detection is not the best solution during synchronization, and analogously the optimal allocation for code acquisition does not provide the best performance during data detection. Closed-form expressions for the optimal powers at the Nash equilibrium are derived, and a unified formulation is proposed to allow a more elaborated power allocation method to be adopted, in which the terminals, using the synchronization-oriented approach, can revert to the data-detection optimal criterion after synchronization is over.

I. INTRODUCTION

Optimal radio resource allocation has emerged as a key issue in the network design since the early days of CDMA wireless communications [1]. In addition to classical optimization techniques, in the last decade game theory [2] has been broadly and profitably used to investigate the resource competition by modeling the network as an economic system, in which any action of a terminal affects the performance of others as well.

Among the main areas of application of game theory to wireless communications (bandwidth allocation, routing, sequence adaptation, just to mention a few), there is a substantial literature in the field of power control techniques for infrastructure CDMA networks (e.g., [3]–[5]), mostly focused on data detection. On the other hand, in addition to data detection, every uplink receiver in the base station of a CDMA network must perform the fundamental function of initial signal synchronization to lock onto the terminal’s signature code. The conjecture we try to (dis)prove here is that the optimum decentralized power allocation devised for data detection leads also to a resource allocation strategy that is good for such synchronization function. To the best of our knowledge, similar approaches focusing on synchronization performance have not been investigated in the literature. This paper elaborates on the formulation proposed in [6], in which the optimal resource allocation during code synchronization is tackled as a distributed game based on maximizing the code detection probability per energy consumed. The performance achieved by this approach at the Nash equilibrium is compared to that provided by game-theoretic techniques tailored for data detection, that aim at maximizing the number of bits correctly delivered to the receiver per energy consumed.

The remainder of the paper is structured as follows. The detection- and synchronization-oriented formulations are given in Sect. II, whereas Sect. III provides a unified analysis of the Nash equilibrium. Sect. IV compares the performance of the two approaches in terms of energy efficiency, and some conclusions are drawn in Sect. V.

II. ENERGY-EFFICIENT GAME-THEORETIC FORMULATION

A. Detection-oriented approach

In the uplink of a multiaccess CDMA infrastructure network, we focus on the issue of data detection, assuming for simplicity the presence of $K$ equi-format users with binary signaling and a common spreading factor $M$ (i.e., the transmission bit rate $R_b = 1/T_b$ is common among all users, where $T_b = MT_c$ is the bit time and $T_c$ is the chip time). The transmission takes place over a frequency-flat and slow-fading additive white Gaussian noise (AWGN) channel. Since wireless data networks are often populated by many battery-powered mobile terminals, a primary goal is the maximization of the number of transmitted bits per energy unit rather than the pure maximization of the link throughput. This goal can be achieved through application of a noncooperative game [2] wherein the users are allowed to choose their transmit powers according to a utility-maximization criterion, where the utility is defined as the ratio of throughput to transmit power.

Let $G_d = [\mathcal{K}, \{P_k\}, \{u_k(p)\}]$ be the noncooperative power control game, where $\mathcal{K} = \{1, \ldots, K\}$ is the index set for the terminal users of the multiuser wireless network; $P_k = [0, \bar{p}_k]$ is the strategy set, with $\bar{p}_k$ denoting the maximum power constraint; and $u_k(p)$ is the payoff function for user $k$. A good method to assess the tradeoff between obtaining high signal-to-interference-plus-noise ratio (SINR) levels and consuming low energy is through the number of bits that can be transmitted per joule of energy consumed [3], [4]. This can be quantified by defining the utility function of user $k$ to be the ratio of its throughput $T_k$ to its transmit power $p_k$, i.e.,

$$u_k(p) = u_k([p_k, p_{\mathcal{K}}]) = \frac{T_k}{p_k} = \frac{D}{L} R_b f(\gamma_k) \frac{1}{p_k}, \quad (1)$$

where $\gamma_k$ is the signal-to-interference-plus-noise ratio at user $k$, and $D$ and $L$ are constants.
where \( \mathbf{p} = [p_1, \ldots, p_K] = [p_k, \mathbf{p}_k] \) represents the vector of transmit powers, with \( K \) denoting the number of users, and \( \mathbf{p}_k \) representing the vector of elements of \( \mathbf{p} \) other than the \( k \)-th element; \( D \) is the number of information bits per packet; \( L \) is the total number of bits per packet; \( \gamma_k \) is the SINR for user \( k \), defined as the ratio between the energy per bit collected at its receiver \( E_k^{(r)} \) and the received power spectral densities (PSDs) due to the multiple access interference (MAI) \( I_{0,k} \) and to the additive white Gaussian noise (AWGN) \( N_0 \):

\[
\gamma_k = \frac{E_k^{(r)}}{I_{0,k} + N_0} = \frac{M \cdot h_k^2 p_k}{\sum_{l \neq k} h_l^2 p_l + \sigma^2}, \tag{2}
\]

with \( h_l \) and \( \sigma^2 = N_0/T_c \) denoting user \( l \)'s path gain and the AWGN power, respectively; and \( f(\gamma_k) \) is the efficiency function which measures the packet success rate (PSR), i.e., the probability that a packet is received without an error. We assume that a packet is retransmitted if it has one or more bit errors, i.e., no forward error correction (FEC) techniques are considered. The shape of \( f(\gamma_k) \) depends on the details of the physical layer, including modulation, coding, and packet size. A useful approximation for the PSR for moderate to large values of \( L \) is given by the efficiency function \( f(\gamma_k) = (1 - e^{-\gamma_k^2})^L \), that will be used for the numerical results of this paper. However, the following analysis is valid for any efficiency function that is increasing, S-shaped (sigmoidal), continuously differentiable, with \( f(0) = 0, f(+\infty) = 1 \), and \( f'(0) = df(\gamma_k)/d\gamma_k|_{\gamma_k=0} = 0 \), which hold in most practical cases [7]. Note that the utility function (1), which has units of b/J, or simply J\(^{-1}\), represents the total number of correct data bits that are delivered to the destination, and captures the tradeoff between data rate and battery life.

B. Synchronization-oriented approach

Let us consider the same wireless network described in Sect. II-A, focusing now on a game-theoretic resource allocation scheme specifically suited to the problem of code synchronization. In this context, the maximization of the achieved throughput per energy consumed is replaced by the maximization of the probability of code acquisition per energy consumed to capture the tradeoff between improving synchronization performance and saving as much energy as possible. To this aim, we can measure the energy efficiency of the \( k \)-th uplink transmitter-receiver by defining the utility function to be the ratio of the probability of correct code synchronization \( P_D \) to the transmitted energy per bit \( E_k^{(t)} = p_k/R_b \) [6]:

\[
u_k(\mathbf{p}, \gamma_k) = u_k([p_k, \mathbf{p}_k], \gamma_k) = \frac{R_b \cdot P_D(\gamma_k, \lambda_k; \rho_k)}{p_k}, \tag{3}\]

where \( R_b, \gamma_k \) and \( \mathbf{p} \) are defined as in Sect. II-A; \( \lambda_k \in [0, 1] \) is the \( k \)-th receiver’s detection threshold; and \( P_D(\gamma_k, \lambda_k; \rho_k) \) is the probability of detection of the correct code alignment, which is a function of i) the threshold \( \lambda_k \); ii) the SINR of user \( k \)'s \( \gamma_k \); and iii) the synchronization strategy \( \rho_k \), which depends on the way the \( k \)-th receiver at the common concentration point combines the outputs of the serial search acquisition [8] to provide the sufficient statistics \( w_k \). To decide whether the \( k \)-th receiver is in the in-sync or in the out-of-sync status, \( w_k \) is compared with \( \lambda_k \). If the test fails \( (w_k < \lambda_k) \), then a new tentative code shift is selected for the local pseudo-noise sequence generation. If the test succeeds \( (w_k > \lambda_k) \), then the receiver assumes correct code alignment. Since false code locks are extremely detrimental for the receiver in terms of increased time for correct synchronization and subsequent data detection, a key performance indicator is given by the probability of false alarm \( P_{FA}(\gamma_k, \lambda_k; p_k) \), to be maintained as low as possible while maximizing (3). Note that, similarly to (1), the utility function (3) has units of J\(^{-1}\).

In this context, we can formulate a noncooperative game in which every pair transmitter-receiver seeks to maximize its own utility by choosing an optimum pair of transmit power (at its transmit side) and detection threshold (at its receive side). Let thus \( G_s = ([K], \{A_k\}, \{u_k\}) \) be such game, in which \( K = \{1, \ldots, K\} \) is the index set for the pairs transmitter-receiver (the players); \( A_k = \mathcal{P}_k \times \Lambda_k \) is the strategy set for player \( k \), where \( \mathcal{P}_k \) is the transmit power set, and \( \Lambda_k \) is the threshold set; and \( u_k \) is the payoff function for the \( k \)-th player, defined as in (3). The power strategy set is \( \mathcal{P}_k = [0, p_k] \), and the threshold strategy set is \( \Lambda_k = [0, 1] \) for all receivers \( k \in K \). Note that, although \( E_k^{(t)} \) is a function of \( p_k \) only, \( P_D \) depends on the achieved SINR at the receiver in addition to the threshold. Using (2), we can verify that \( \gamma_k \) depends not only on \( p_k \), but also on all others’ powers \( p_j \). Hence, maximizing (3) is a multidimensional problem. To limit the occurrence of spurious wrong detections, we can place a maximum tolerable probability of false alarm \( \overline{P}_{FA,k} \). In some sense, \( \overline{P}_{FA,k} \) represents user \( k \)'s QoS requirement, that depends on the desired accuracy to be achieved. This implies that the strategy set becomes the subset \( A_k \subset \mathcal{P}_k \times \Lambda_k \) that provides \( P_{FA}(\gamma_k, \lambda_k; p_k) \leq \overline{P}_{FA,k} \) [6].

III. Analysis of the Nash equilibrium

The solution that is most widely used for noncooperative game theoretic problems is the Nash equilibrium (NE) [2]. An NE is a set of strategies such that no player can unilaterally improve its own utility. Formally, a vector \( \mathbf{a}^* = [a_1^*, \ldots, a_K^*] = [a_k^*, \mathbf{a}_k^*] \) is an NE of a game \( \mathcal{G} \), with \( a_k^* \) denoting the pair \( (p_k^*, \lambda_k^*) \) if \( \mathcal{G} = \mathcal{G}_s \), and the power \( p_k^* \) if \( \mathcal{G} = \mathcal{G}_d \); for, if all \( k \in K \), \( u_k(a_k^*, \mathbf{a}_k^*) \geq u_k(a_k, \mathbf{a}_k^*) \) for all strategies \( a_k \in A_k \), with \( a_k \) denoting the pair \( (p_k, \lambda_k) \) if \( \mathcal{G} = \mathcal{G}_s \), and the power \( p_k \) if \( \mathcal{G} = \mathcal{G}_d \). The NE is of particular interest in the context of distributed algorithms, in that it offers a predictable, stable outcome of a game where multiple agents with conflicting interests compete through self-optimization and reach a point which no player wishes to deviate from. However, such point does not necessarily exist.

Theorem 1 ([4], [6]): The game \( \mathcal{G} \) (either \( \mathcal{G}_d \) or \( \mathcal{G}_s \)) admits a unique NE if

\[
\Phi = \sum_{k=1}^{K} \varphi_k < 1, \tag{4}\]
where

\[ \varphi_k = (M/\gamma_k^* + 1)^{-1} > 0, \] (5)

and \( \gamma_k^* \) is the SINR that maximizes the utility \( u_k \), defined as in (1) if \( G = G_d \), and as in (3) if \( G = G_s \). The unique NE of the game \( G \) is achieved when user \( k \)’s transmit power is

\[ p_k^* = \frac{\varphi_k}{k_k} \cdot \frac{\sigma^2}{1 - \Phi}. \] (6)

The proof, omitted for the sake of brevity, is based upon the properties of quasi-concavity [4] for the utility \( u_k \), and of a standard function [9] for user \( k \)’s best response to \( a_k^* \) [6].

The only difference between games \( G_d \) and \( G_s \) occurs in computing \( \gamma_k^* \), which regulates the quantity \( \varphi_k \). In the case \( G = G_d \), \( \gamma_k^* \) is such that

\[ f'(\gamma_k^*) \cdot \gamma_k^* = f(\gamma_k^*), \] (7)

where \( f'(\gamma_k^*) = df(\gamma_k)/d\gamma_k |_{\gamma_k = \gamma_k^*} \). In the case \( G = G_s \), \( \gamma_k^* \) is equal to

\[ \gamma_k^* = \begin{cases} \bar{\gamma}_k, & \gamma_k \leq g(\gamma_k; 2\gamma_k, \rho_k), \\ \bar{\gamma}_k, & \gamma_k > g(\gamma_k; 2\gamma_k, \rho_k), \end{cases} \] (8)

where \( \gamma_k^* \) is the SINR that provides \( P_{FA}(\gamma_k^*, \lambda_k = 1; \rho_k) = \Phi_{FA,k} \), and the SINR level \( \hat{\gamma}_k \) is the solution of \( \gamma_k = g(\hat{\gamma}_k; 2\hat{\gamma}_k, \rho_k) \), with

\[ g(\gamma_k; 2\gamma_k, \rho_k) \triangleq \frac{P_D(\gamma_k; \Delta_k(\gamma_k) \rho_k)}{P_D(\gamma_k; \Delta_k(\gamma_k) \rho_k)} \] (9)

with \( P_D'(\gamma_k, \Delta_k(\gamma_k); \rho_k) = dP_D(\gamma_k, \Delta_k(\gamma_k); \rho_k)/d\gamma_k |_{\gamma_k = \gamma_k^*} \), and the detection threshold \( \Delta_k(\gamma_k) \triangleq \Delta(\gamma_k; 2\gamma_k, \rho_k) \) is such that \( P_{FA}(\gamma_k, \lambda_k = \Delta_k(\gamma_k); \rho_k) = \Phi_{FA,k} \), which is also a function of the receiver type \( \rho_k \), as better characterized in the remainder of the paper (see also [6], [10] for further details).

Furthermore, in the case \( G = G_s \), the detection threshold at the NE is chosen according to

\[ \lambda_k^* = \bar{\lambda}_k(\gamma_k^*). \] (10)

When the condition (4) holds, each transmitter in the network, acting as a rational self-optimizing player, can regulate its transmit power so as to achieve the optimal level (6) following a best-response criterion [2]. Using the analysis developed in [4] and [9], this approach can be used to provide an iterative algorithm, which mimics a dynamic game to guarantee a good tradeoff between fairness of the network and efficiency of the resource allocation scheme. At the \((n+1)\)th step of the algorithm, user \( k \) updates its (optimal) transmit power \( p_k^*(n+1) \) according to

\[ p_k^*(n+1) = p_k^*(n) \cdot \frac{\gamma_k^*}{\gamma_k(n)}, \] (11)

where \( p_k^*(n) \) is the transmit power selected by user \( k \) at the \( n \)th step, and \( \gamma_k(n) \) is the SINR experienced by user \( k \) at the \( n \)th step, that can be fed back by the access point using a return channel with a very modest data rate requirement. Note that \( \gamma_k(n) \) is the only parameter that is locally unavailable to user \( k \), since every transmitter \( k \) can compute its own \( \gamma_k^* \) independently of each other before the iterative updating mechanism starts, following (7) if \( G = G_d \) and (8) if \( G = G_s \).

This is expedient not only to derive a decentralized allocation scheme, but also to allow users already in-sync — and thus following the data-detection optimum criterion (1) — to coexist with other terminals still in the acquisition phase — and thus following (3). In other words, Theorem 1 also applies to a game \( G \), in which some players use (1) as their own utility function, while the others consider (3), provided that condition (4) holds, with \( \varphi_k \) computed again as in (5). The quantity \( \varphi_k \) can be in fact thought of as the size that user \( k \) occupies in the space of the available resources (the strategy space \( A_k \)). We can show that \( \gamma_k^* \) (and thus \( p_k^* \)) increases as \( \varphi_k \) increases. Hence, the larger \( \varphi_k \), the larger amount of such resources “consumed” [11], [12], which offers a qualitative interpretation of the constraint (4).

**IV. PERFORMANCE COMPARISON**

The analysis of the NE presented in Sect. III is convenient to provide a unified framework to measure the energy efficiency of both the detection- and the synchronization-oriented approaches described in Sects. II-A and II-B, respectively. In this section, we aim at comparing the performance achieved at the NE by the two approaches, so as to highlight the need for separate resource allocation criteria for code acquisition and data detection phases.

For the sake of notation, we will omit the subscript \( k \) from now on (apart from denoting with \( \chi \) the other users in the network), and replace it with either ‘d’ or ‘s’, if the considered user adopts the detection- and the synchronization-oriented formulation, respectively. More specifically, \( \gamma_d^* \) represents the optimal SINR at the NE computed following (7), whereas \( \gamma_s^* \) is derived using (8). Similarly, \( \varphi_d \) and \( \varphi_s \) are used to emphasize that, although (5) remains the same for both approaches, it is a function of the selected \( \gamma^* = \{\gamma_d^*, \gamma_s^*\} \). Finally, \( u_d^*(a^*, a_s^*) \) represents the achieved utility that follows from the formulation (1) when the user adopts a strategy \( a^* \) at the NE (that can be either \( \gamma_d^* \) or \( \gamma_s^* \), and thus \( a_d^* = p_d^* \) and \( a_s^* = p_s^* \), respectively), and the others select \( a_k^* = p_k^* \), no matter what their approaches are. Analogously, \( u_d^*(a^*, a_s^*) \) measures the utility that follows from (3), where \( a^* \) can be either \( (p_d^*, \Delta_d(\gamma_d^*)) \) or \( (p_s^*, \Delta_s(\gamma_s^*)) \), and \( a_k^* = p_k^* \) irrespectively of their approaches. As an example, \( u_d^*(a_d^*, a_s^*) \) measures the energy efficiency in terms of probability of code detection per consumed energy at the transmitter — i.e., using the utility function (3) — when a user adopts the detection-oriented criterion instead of the synchronization-oriented one.

Let us then combine (6) with (1) and (3) to measure the energy efficiency achieved in the two approaches. To provide a fair comparison, we assume \( \Phi \) to be the same in the two scenarios, as well as the channel attenuation \( h \). Hence, the quantity

\[ \chi \triangleq R_b h^2 \cdot \frac{1 - \Phi}{\sigma^2}, \] (12)
is assumed to be constant. In the detection-oriented case,
\[
u^*_d([a^*, a^*_k]) = \lambda \cdot \frac{D}{L} \cdot \frac{f(\gamma^*)}{\varphi},
\]
(13)

Similarly, in the synchronization-oriented approach,
\[
u^*_s([a^*, a^*_k]) = \lambda \cdot \frac{P_D(\gamma^*, \lambda^*; \rho)}{\varphi},
\]
(14)
in which \(\lambda^* \triangleq \min(1, \Delta(\gamma^*))\) to minimize the probability of false alarm. Note that, although the minimum does not appear in (10), since \(\gamma_d^* \geq \gamma\) by construction, here we must account for detection-oriented solutions that may provide \(\gamma_d^* < \gamma\), thus yielding thresholds greater than 1. When it happens, the QoS requirement \(P_{FA,s}\) cannot be met, although \(\lambda_d^* = 1\) can still be used, with \(P_D(\gamma_d^*, \lambda_d^*; \rho) = 0.5\).

It is interesting to evaluate the relevant parameters at the NE provided by the two approaches, using some real-world parameters. We make the following assumptions to the system model. At the receiver, during the code synchronization stage, we use a coherent code detector, in which the phase offset \(\theta_k\) of the \(k\)-th uplink channel is assumed to be known at the receiver by means of perfect phase estimation. Similar results can be obtained assuming \(\theta_k\) to be unknown (non-coherent detection), since the performance of both classes is very close for practical working conditions [10]. When using a coherent synchronization strategy (denoted by \(\rho = 0\) for the sake of notation in [10]),
\[
P_{FA}(\gamma, \lambda; \rho = 0) = Q\left(\lambda \sqrt{2\gamma}\right),
\]
(15)
\[
P_D(\gamma, \lambda; \rho = 0) = 1 - Q\left((1 - \lambda) \sqrt{2\gamma}\right),
\]
(16)
where \(Q(x) = \frac{1}{2} \int_x^{+\infty} \exp(-t^2/2) \, dt\) is the complementary cumulative distribution function (ccdf) of a standard random variable. This allows us to specify the functions \(\gamma\) and \(\Delta(\gamma)\), expedient to compute the strategies at the NE:
\[
\gamma = \frac{1}{2} \left[Q^{-1}(P_{FA,s})\right]^2
\]
(17)
\[
\Delta(\gamma) = \sqrt{2/\gamma}
\]
(18)
where \(Q^{-1}(\cdot)\) is the inverse \(Q\)-function. Concerning the data-detection phase, we assume packets with no overhead (i.e., \(L = D\)), using \(f(\gamma_k) = (1 - e^{-\gamma_k/2})^L\) to model the PSR.

Fig. 1 shows the SINR \(\gamma^*\) at the NE, for different values of the maximum probability of false alarm \(P_{FA,s}\), in a network using \(M = 1024\) as the common spreading factor. The dashed line corresponds to the SINR (8), thus following the synchronization-oriented criterion. Solid lines report the SINR levels at the NE when the terminal adopts the detection-oriented criterion (7). Only three different values for \(L\) are considered for the sake of presentation: \(L = 10\) (square markers), \(L = 100\) (circles), and \(L = 1000\) (triangles). As can be seen, the SINR level \(\gamma^*_d\) changes accordingly to the QoS constraint \(P_{FA,s}\), whereas \(\gamma^*_s\) is insensitive to the constraint. Note that the dashed curve intersects each solid line for exactly one values of \(P_{FA,s}\), that decreases as \(L\) increases (the case with no intersection occurs only for extremely large \(L\)). This means that, for this QoS constraint, the detection-oriented solution \(\gamma^*_d\) coincides with \(\gamma^*_s\). The next plots will show how this reflects upon the energy efficiency of the two solutions. Note also that \(\gamma^*_d\) is not shown for those values of \(P_{FA,s}\) that yield \(\lambda_d^* = 1\), to emphasize that such QoS requirement \(P_{FA,s}\) cannot be met and to perform a fair comparison.

Fig. 2 shows the probability of detection \(P_D\) at the NE as a function of \(P_{FA,s}\). Clearly, when \(\gamma^*_d = \gamma^*_s\), \(P_D\) is the same. When \(\gamma^*_d < \gamma^*_s\), the probability \(P_D\) achieved by the detection-oriented solution is lower than that provided by the synchronization-optimal one. On the other hand, when \(\gamma^*_d > \gamma^*_s\), \(P_D\) is greater. However, the latter case comes at the expense of an increased power consumption, as \(\gamma^*_d > \gamma^*_s\) shows. To quantify it in the practice, let us measure the normal-
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... approach, showing that it is highly desirable (in terms of energy efficiency) to devise a scheme for a mixed-population network, wherein some in-sync terminals coexist with out-of-sync ones. The analysis derived here also provides a unified framework that allows both classes of terminals to coexist. Future work is needed to elaborate on this formulation to assess the case of a mixed-population network.

V. CONCLUSION

This paper investigated the performance of two game-theoretic resource allocation schemes based on energy-efficient criteria for CDMA wireless networks. The first approach, specifically suited for data detection, aims at maximizing the throughput per transmitted power, so as to capture the tradeoff between delivering information data and saving energy. We showed that this criterion does not provide the best performance in the initial signal synchronization, during which the most significant utility function is represented by the ratio of the probability of code alignment detection to the transmitted energy per bit, under a constraint on the maximum probability of spurious code locks. To this aim, we compared the utility achieved by the detection-oriented formulation at the Nash equilibrium with that provided by the synchronization-oriented approach, showing that it is highly desirable (in terms of energy efficiency) to devise a scheme for a mixed-population network, wherein some in-sync terminals coexist with out-of-sync ones. The analysis derived here also provides a unified framework that allows both classes of terminals to coexist. Future work is needed to elaborate on this formulation to assess the case of a mixed-population network.

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