Torque Vectoring for an Electric Vehicle
Using an LPV Drive Controller and a Torque and Slip Limiter

Gerd Kaiser, Qin Liu, Christian Hoffmann, Matthias Korte and Herbert Werner

Abstract—This paper proposes a torque vectoring strategy for the propulsion of an electric vehicle with two independent electric machines at the front wheels. The torque vectoring controller includes a vehicle dynamics controller and a motor torque and wheel slip limiter. The nonlinear vehicle dynamics controller is designed as Linear Parameter-Varying (LPV) gain-scheduled controller for tracking the longitudinal velocity and the yaw rate of the vehicle. A linearly interpolated Torque and Slip Limiter (TSL) is derived to cope with saturation of the electric motors and wheel slip limitations. The TSL is based on an extension of a linear time-invariant anti-windup scheme to fit the proposed LPV controller, and uses available wheel slip information to prevent wheel spinning or blocking. A nonlinear 14-degree-of-freedom vehicle model with an advanced Dugoff tire model has been calibrated with real measurement data. This model is used to simulate the closed-loop vehicle behavior. Simulation results show good vehicle dynamics and safety properties.

I. INTRODUCTION

In future fewer vehicles will be equipped with pure combustion engines and the electrification of the drive train in hybrid electric vehicles will continue and will result in pure electric propulsion systems [1]. With electric motors it is easier to decentralize the drive train, which results in individually driven (two or four) wheels. There are new safety and efficiency requirements which will be addressed for example in the European project eFuture [2]. A basic setup of the vehicle prototype is shown in figure 1.

One way of controlling electric vehicles is torque vectoring. This has been addressed in [3], [4], [5], [6] and [7]. Each of these approaches has however certain drawbacks, as discussed in the next Section. The basic idea of torque vectoring is that driver requests (steering angle, brake and acceleration pedal position) are processed and distributed as torque commands to the wheels of the vehicle. In a limited range the longitudinal and lateral dynamics can be treated as decoupled and controlled separately. With approaching the limits of the propulsion system or the tire forces, decoupling is not possible anymore. In addition to the vehicle dynamics the controller has to take into account the limited wheel forces [8]. Finally a torque vectoring controller should be tunable for a trade-off between longitudinal and lateral driver requests in case of saturation. Here a torque-vectoring scheme is proposed that is based on an LPV and a torque and slip limiter and tested in simulation studies, using the prototype of the European project eFuture. This car is a compact size, battery electric vehicle with two electric motors at the front left and right wheels.

This paper is organized as follows: A control strategy for torque vectoring is proposed in Section II. A suitable vehicle model is developed in Section III-A. In Section III-B a linear parameter-varying controller is designed to control the vehicle dynamics. The limitations due to motor and tire saturation are discussed in Section IV where a torque slip limiter is derived to cope with the physical restrictions. In Section V the controller behavior is discussed and simulation results on a nonlinear 14 degrees of freedom vehicle model are shown. Conclusions are given in Section VI.

II. CONTROL STRUCTURE

Torque vectoring controllers can be designed with different structures. One possibility is to use a drive controller [4], [5], [7] and in parallel a wheel slip limiting controller as in Fig. 2. The drive controller is designed to calculate a desired yaw moment and desired longitudinal force. These two requests will be combined in order to calculate torque requests for the powered wheels. If the wheels start to spin or block, the requested motor torques will be reduced by an opposite torque request from the slip limiter. The main problem of this design is the missing interconnection between the wheel slip limitation block and the drive controller. It is not clear whether the longitudinal velocity or the yaw rate request is processed and which request will be degraded if wheel limitations or motor saturations appear.

A second idea is to use a cascaded structure as shown in Fig. 3 with an upper and lower controller, see [3], [6]. The
upper controller receives the longitudinal and lateral requests and calculates the desired wheel slips. The desired wheel slips will be limited and compared with the actual wheel slips to calculate the desired driving torque at each wheel. The drawback in this design is the strong dependence of the wheel slip; especially small wheel slip values are difficult to observe. The handling of the electric motor saturation is also difficult.

In this paper an interconnected controller structure as in Fig. 4 is proposed. The controller for the longitudinal and lateral requests is extended with a motor Torque and wheel Slip Limiter (TSL). The inputs to the vehicle model are the longitudinal forces \( F \) which act on the chassis (with \( i \) indicating the position front/rear, left/right in the vehicle). The outputs are the longitudinal velocity \( v \) and the yaw rate \( \dot{\psi} \). The error signals \( e_v \) and \( e_{\dot{\psi}} \) are the inputs of the controller. The outputs of the controller are the longitudinal force requests \( F_{\text{des},i} \) for the left and right wheel and are applied to the electric motors. An additional input to the plant and the controller is the steering angle \( \delta \) of the front wheels, because this angle will change the lateral behaviour of the vehicle and is compensated by a feedforward controller.

The TSL is active if the electric motor reaches its saturation limit or if a wheel starts to slip. The limiter has to perform slips to calculate the desired driving torque at each wheel. The drawback in this design is the strong dependence of the wheel slip; especially small wheel slip values are difficult to observe. The handling of the electric motor saturation is also difficult.

Remark 1: For generating desired values for longitudinal velocity and yaw rate, a modified single track model as in [7] or [10] is used.

### III. Controller Design: Modeling and LPV Synthesis

The task of the torque vectoring controller is to improve the vehicle behavior while driving. The vehicle model is discussed in Section III-A, before the controller is derived in Section III-B.

**Remark 1:** For generating desired values for longitudinal velocity and yaw rate, a modified single track model as in [7] or [10] is used.

#### A. Vehicle model

In this paper, the prototype of the European project eFuture [2] is considered. It is equipped with two electric motors for driving the front wheels independently. In [6] and [11], advantages of tracking the yaw rate and the side slip angle are discussed. But with the given actuators, only tracking of a combination of these properties is possible. Therefore, the design proposed here focuses on tracking the yaw rate \( \psi \). As the second tracking variable, the longitudinal velocity is chosen.

**Remark 2:** When including an additional actuator e.g. for modifying the steering angle, it is possible to decouple the yaw rate and side slip angle.

To control the longitudinal velocity \( v \) and the yaw rate \( \psi \), a nonlinear Single-Track Model [11] with three states is used. The model is accurate if the wheel slip satisfies \( |\lambda| < 0.1 \) and the wheel side slip angle \( |\alpha| < 0.08 \).

The inputs to the vehicle model are the longitudinal forces \( F_{FL} \) and \( F_{FR} \), which act on the vehicle chassis at the location of the front wheels. Additionally the steering angle \( \delta \) of the front wheels influences the vehicle. The steering is here considered as a measured disturbance. The vehicle states are the velocity in longitudinal direction \( v \), the velocity in lateral direction \( v \) and the yaw rate \( \dot{\psi} \). The measured outputs are the longitudinal velocity and the yaw rate. In reality, the vehicle velocity in longitudinal direction can not be measured but there are techniques [12], [13] to observe this quantity accurately. The vehicle dynamics can be described as

\[
\dot{v}_x = v_y \dot{\psi} + \frac{1}{M} (F_{FL} + F_{FR}) \tag{1}
\]

\[
\dot{v}_y = -v_x \dot{\psi} - \frac{C_{y,f} + C_{y,r}}{M v_x} v_y + \frac{l_x C_{y,r} - l_f C_{y,f}}{M v_x} \dot{\psi} + \frac{C_{y,f}}{M} \delta \tag{2}
\]

\[
\ddot{\psi} = \frac{l_x C_{y,r} - l_f C_{y,f}}{I_x v_x} v_y - \frac{I_y^2 C_{y,f} + I_r^2 C_{y,r}}{I_z v_x} \dot{\psi} + \frac{l_f C_{y,f}}{I_z} \delta + \frac{u_{\text{ff}}}{2 I_z} (F_{FR} - F_{FL}) \tag{3}
\]

The physical parameters are defined in Table I.
TABLE I

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<td>1.240</td>
<td>distance front axle to center of gravity [m]</td>
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<td>( l_r )</td>
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<td>mass of the vehicle [kg]</td>
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<tr>
<td>( I_z )</td>
<td>1800</td>
<td>moment of inertia around vertical axis [kg m²]</td>
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</table>

B. LPV controller synthesis

A polytopic Linear Parameter-Varying (LPV) controller is proposed to control the nonlinear plant. During the last decade LPV control has been increasingly used for applications in the field of flight control [14], [15] and robotics [16]. The advantage of LPV control is that well-known linear design strategies, like \( H_\infty \) design, can be extended to nonlinear systems. The controller is based on a constant quadratic Lyapunov function, which guarantees stability and performance in the whole parameter space. The controller is tuned with standard shaping filters. In order to design a gain-scheduled controller, the plant is rewritten as an LPV model [17]

\[
G(\theta) := \begin{cases}
\dot{x} = A(\theta)x + B(\theta)u \\
y = C(\theta)x + D(\theta)u
\end{cases}
\]

(4)

where \( x \in \mathbb{R}^n \) is the state vector, \( u \in \mathbb{R}^m \) the input and \( y \in \mathbb{R}^l \) the output vector. The mappings \( A(\theta), B(\theta), C(\theta) \) and \( D(\theta) \) are functions of \( \theta \in \mathbb{R}^p \), where

\[
\theta(t) = \theta_0(\rho(t))
\]

(5)

is a vector of scheduling parameters and \( \rho : \mathbb{R}^k \to \mathbb{R}^p \) an analytic mapping of measurable scheduling signals \( \rho(t) \in \mathbb{R}^k \) onto the admissible scheduling parameter set

\[
P \subset \mathbb{R}^p : \theta \in P, \forall t > 0,
\]

(6)

which is assumed to be compact. If the scheduling signals and therefore the scheduling parameters depend on system states or inputs, i.e. \( \theta = \theta(x, u) \), the LPV model is called quasi-LPV. If the model (4) is affine in \( \theta \), a polytopic representation

\[
\begin{bmatrix}
A(\theta) & B(\theta) \\
C(\theta) & D(\theta)
\end{bmatrix} = \sum_{i=1}^{p} \theta_i \begin{bmatrix}
A_i & B_i \\
C_i & D_i
\end{bmatrix}
\]

(7)

can be used, where the • model matrices represent vertices in the parameter space. The model input matrix \( B \) is partitioned as \( [B_u, B_d]^T \) to represent the effect of control input and disturbance, respectively. For controller synthesis the generalized plant \( P \) in Fig. 5 is used. To shape the closed-loop behaviour, a sensitivity filter \( W_s \) and a control sensitivity filter \( W_c \) are applied. To turn the model (1) - (3) into polytopic LPV form, define

\[
\theta_1 = \frac{1}{v_x}, \quad \theta_2 = \frac{1}{v_y}, \quad \theta_3 = \frac{1}{\psi}
\]

(8)

\[
x = \begin{bmatrix} v_x, v_y, \psi \end{bmatrix}^T, \quad u = [F_{FL}, F_{FR}]^T, \quad d = \delta, \quad y = \begin{bmatrix} v_x, \psi \end{bmatrix}^T
\]

A state space model in LPV form can then be written as

\[
A(\theta) = \begin{bmatrix}
0 & \theta_2 & -\frac{C_{\gamma_1}}{M} \theta_1 \\
-\theta_2 & 0 & \frac{C_{\gamma_1} - C_{\gamma_2}}{M} \theta_1 \\
\frac{C_{\gamma_2}}{M} \theta_1 & -\frac{C_{\gamma_1}}{M} \theta_1 & 0
\end{bmatrix}
\]

\[
B_u = \begin{bmatrix}
\frac{1}{M} & 0 \\
0 & \frac{1}{M} \\
-\frac{w_f}{2 I_z} & \frac{w_f}{2 I_z}
\end{bmatrix}, \quad B_d = \begin{bmatrix}
0 \\
0 \\
\frac{C_f}{M}
\end{bmatrix}
\]

\[
C = \begin{bmatrix}
1 & 0 & 0 \\
0 & 0 & 1
\end{bmatrix}, \quad D = \begin{bmatrix}
0 & 0 \\
0 & 0
\end{bmatrix}
\]

To achieve reference tracking and disturbance rejection, a two-degree-of-freedom controller is used [4]. The controller \( K(\theta) \) has the form

\[
K(\theta) := \begin{cases}
\dot{x}_k = A_k(\theta)x_k + B_{k,e}(\theta)e + B_{k,d} \delta \\
u = C_k(\theta)x_k + D_{k,e}(\theta)e + D_{k,d} \delta
\end{cases}
\]

(9)

For every vertex of the polytopic parameter set a controller is calculated, see Fig. 6. An LPV controller that guarantees stability and performance in the whole parameter set is then obtained by interpolating between the vertex controllers; for more details see [14]. It turns out that an extra LMI constraint on the spectral radius of the controller \( A_k \) matrix has to be imposed to avoid fast controller poles, following the
procedure in [18]. The controller is designed to be strictly proper \((D_k = 0)\) to improve the numerical condition of the problem. A mixed sensitivity loop shaping approach [19] is employed to tune the controller according to the control objectives. The scheduling parameters are within \(\theta_1 \in \left[ \frac{1}{50}, \frac{1}{15} \right] \text{ and } \theta_2 \in [-2, 2] \text{ rad s}^{-1}\). The shaping filters \(W_s\) and \(W_k\) are chosen as
\[
W_s = \text{diag} \left( \frac{200}{50s + 1}, \frac{33.33}{31s + 1} \right)
\]
\[
W_k = \text{diag} \left( \frac{3.3s + 200}{0.167s + 1000}, \frac{3.3s + 200}{0.167s + 1000} \right)
\]

IV. TORQUE AND SLIP LIMITER

The function of the motor Torque and wheel Slip Limiter (TSL) is proposed in this Section. The TSL handles actuator limits which can degrade the performance of the closed-loop system or even drive it into unstable operation [20]. The general structure resembles an anti-windup compensator, but the TSL is not related to a single actuator saturation; furthermore an internal state of the actuator is also limited. In this design the actuator is treated as a black box, including the electric motor (with inverter, controller, etc) and the wheel of the vehicle. The input to the actuator is a force request and the output is a longitudinal force which acts on the vehicle chassis and will be explained in Section IV-A. The TSL has two ways of changing the dynamics of the closed-looped system: Reduce the output of the controller, or modify the input of the controller, see Section IV-B.

A. TSL input and output

The input \(\Delta F_{\text{eMot}}\) of the TSL is the difference between the requested longitudinal force \(F_{\text{req}}\) and the created force from the electric motor \(F_{\text{eMot}} = T_{\text{meas}}/r\), were \(r\) is the tire radius and \(T_{\text{meas}}\) the measured motor torque. The motor torque is limited by the maximal motor torque \(T_{\text{eMot}}\), the maximal motor power \(P_{\text{eMot}}\) and the slew rate of the motor \(T_{\text{eMot}}/\lambda\). These limitations are listed in Table II.

The tire force limit [8] is nonlinear and not measurable, but can be estimated using the wheel slip \(\lambda\). The force generation is linear to the wheel slip until reaching the limit \(\lambda_{\text{lim}}\). In [21] this limit depends on the driving and road condition as well as safety considerations and varies from 0.1 to 0.4. This value is here assumed fixed as \(\lambda_{\text{lim}} = 0.1\), but in a real implementation this value will be dynamically modified using a multi-dimensional lookup table. The force difference can be calculated as
\[
\Delta F_{\lambda} = \begin{cases} 
0 & \text{for } |\lambda| \leq \lambda_{\text{lim}} \\
C_x(\lambda - \lambda_{\text{lim}}) & \text{for } \lambda > \lambda_{\text{lim}} \\
C_x(\lambda + \lambda_{\text{lim}}) & \text{for } \lambda < -\lambda_{\text{lim}} 
\end{cases}
\]
where \(C_x\) is the longitudinal tire stiffness.

Remark 3: The scheme (10) is not only useful for suppressing controller windup, but can also be used to limit the wheel slip \(\lambda_{\text{lim}}\). This is an extension of the classical anti-windup compensator.

The force difference \(\Delta F\) is calculated as the maximum of the electric motor saturation \(\Delta F_{\text{eMot}}\) and the slip dependent limitation \(\Delta F_{\lambda}\). The first output of the torque and slip limiter modifies the output of the controller and is defined as \(y_u = \Gamma_u \Delta F_{u}\). The second output changes the controller input and is defined by \(y_y = \Gamma_y \Delta F_{y}\).

B. TSL synthesis

A LTI anti-windup approach [22] is employed and extended to construct an LPV gain-scheduled drive controller. Due to implementation constraints, a low-order design is chosen with filter dynamics \(W_u\) and \(W_y\). By tuning \(W_u\) and \(W_y\) a suitable trade-off between longitudinal and lateral tracking can be achieved. To adapt the TSL to the LPV framework an anti-windup compensator is calculated for every vertex. These compensators are interpolated to realize the TSL. The scheduling signals are the parameters \(\theta_1\) and \(\theta_2\) from the polytopic controller design in Section III-B. The dynamic parts \(W_u\) and \(W_y\) are manually fixed; the stationary gains \(\Gamma = [\Gamma_u, \Gamma_y]^T\) are calculated as
\[
\begin{bmatrix} \Gamma_u(\theta) \\ \Gamma_y(\theta) \end{bmatrix} = \sum_{i=1}^{r} \alpha_i \begin{bmatrix} \Gamma_{u,i} \\ \Gamma_{y,i} \end{bmatrix}, \quad \alpha_i \geq 0, \quad \sum_{i=1}^{r} \alpha_i = 1 \tag{11}
\]

The static anti-windup gains \(\Gamma_{u,i}\) and \(\Gamma_{y,i}\) can be calculated by solving
\[
Q_i A_i^T + A_i Q_i - \Phi_i - 2U_i I = 0, \quad \Phi_i - \gamma_i I - \gamma_i I < 0, \tag{12}
\]
with \(\Phi_i = B_{u,i} U_i + B_{L,i} - Q_i C_{i,i}^T\) and where
\[
\bar{A}_i := \begin{bmatrix}
A_i & B_i C_{k,i} & B_i C_u & 0 \\
0 & A_{k,i} & 0 & -B_{k,i} C_y \\
0 & 0 & A_u & 0 \\
0 & 0 & 0 & A_y
\end{bmatrix}
\]
Note that $D$ and $D_k$ are zero in this design. Also only the feedback related parts of the controller and plant are considered for the anti-windup design. For more details see [22]. The filters $W_u$ and $W_y$ are converted into state space representation with matrices $[A_u, B_u, C_u, 0]$ and $[A_y, B_y, C_y, 0]$. If LMI (12) is satisfied with a minimal $\gamma_i$, the optimal, static gain is calculated by $\Gamma_i = [\Gamma_{u,i}, \Gamma_{y,i}]^T = \bar{L}_i \bar{U}_i^{-1}$.

V. SIMULATION RESULTS

For simulation a nonlinear 14-degrees-of-freedom vehicle model is used. This model uses six degrees of freedom for the movement of the center of gravity, four degrees for the movement and four degrees for the movement of the wheels. The tire characteristics are calculated with a modified Dugoff model [23] and additionally restoring moments [8] at the wheels. The model is calibrated with real measurement data to represent the eFuture Prototype [2]. Two differently tuned torque vectoring control schemes are compared in simulation. The LPV controller from Section III-B is fixed. The first (referred as TSL$_1$) and second (TSL$_2$) configuration are each equipped with the TSL from Section IV. The filters are selected as

$$W_{u,1} = W_{u,2} = \theta_1 \text{diag} \begin{pmatrix} 0 & 0 \\ 0.01s + 1 & 0.01s + 1 \end{pmatrix}$$  \hspace{1cm} (13)

$$W_{y,1} = \theta_1 \text{diag} \begin{pmatrix} 10 & 1 \\ 0.01s + 1 & 0.01s + 1 \end{pmatrix}$$  \hspace{1cm} (14)

$$W_{y,2} = \theta_1 \text{diag} \begin{pmatrix} 1.5 & 8 \\ 0.01s + 1 & 0.01s + 1 \end{pmatrix}$$  \hspace{1cm} (15)

Multiplying the filters with the scheduling parameter $\theta_1$ improves the feasibility of the solution. This is not surprising because the vehicle model (1) - (3) has a strong dependence on the longitudinal velocity of the vehicle.

The driving scenario is based on the norm ISO 7401 but modified to show the limitations of the electric motors and wheel slip. The surface adhesion is set to $\mu = 0.3$ (wet road) and reference step-inputs with 60 degrees at the steering wheel are used. After 1 second the steering wheel angle is changed to 60 degrees and after 2.8 seconds back to 0 degree. After 4 seconds the desired longitudinal velocity is changed from 60 to 55 kph. With these steps the longitudinal and lateral behaviour is separately tested. The combined behaviour is tested after 8.5 seconds. The desired velocity is increased to 65 kph and the steering angle is set to 60 degrees. Fig. 8(b) shows the longitudinal velocity. The controllers achieve an acceptable overshoot of 1 kph for the longitudinal steps. The major difference between the controllers is seen between 9 and 12 seconds when the combined requests are given. TSL$_1$ archives the desired velocity 2 seconds faster. The sequence of steering steps is displayed in Fig. 8(a) and the yaw rate response is given in Fig. 8(c). TSL$_1$ and TSL$_2$ can follow the reference with an acceptable overshoot. When the longitudinal and lateral requests are applied simultaneously, TSL$_2$ can track the desired yaw rate more accurately. TSL$_1$ starts to track the yaw rate, if the longitudinal request is nearly fulfilled. Fig. 8(d) shows the wheel slip of the front left wheel. At around 5 seconds the slip is oscillating around -0.1. This behaviour is related to the desired wheel slip limitation.

Fig. 8(e) displays the torque output of the front left motor. There are torque oscillations around 5 seconds which are related to the wheel slip limitation. Also the limitations due to the slew rate of the motors are visible, e.g. between 4 and 4.4 seconds or 5.2 and 5.5 seconds. The major difference in the motor torques is between 9 and 12.2 seconds. TSL$_1$ reaches the longitudinal velocity faster and applies the maximum possible torque. TSL$_2$ tracks the yaw rate and requires a torque difference between left and right, which means here to reduce the torque of the front left motor.

VI. CONCLUSIONS

A new control structure for torque vectoring is proposed. Polytopic LPV controller synthesis is used to control the longitudinal and lateral nonlinear vehicle behaviour. Wheel slip and electric motor limitations are taken into account by a two step, gain-scheduled torque and slip limiter. The TSL is tuneable for a trade-off between tracking the longitudinal velocity and the yaw rate. However the model and the controller are not valid for standstill, so the function torque vectoring is switched off for low velocities. Simulation results are promising and will be further validated in a real time simulator and finally in the prototype of the eFuture project. The synthesis approach employing a constant Lyapunov function guarantees stability in the complete parameter space, however different LPV synthesis concepts based on parameter-dependent Lyapunov functions will be tested in the future. The main contribution of this paper is the development of a base torque vectoring controller concept which can be implemented in a real vehicle.

### TABLE II

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Limit</th>
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<tbody>
<tr>
<td>$I_{eMot}$</td>
<td>775 [Nm]</td>
<td>maximal torque of one electric motor</td>
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<tr>
<td>$\Gamma_{eMot}$</td>
<td>1000 [kW]</td>
<td>maximal power of one electric motor</td>
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<td>$P_{eMot}$</td>
<td>20 [kW]</td>
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<td>$C_x$</td>
<td>60000 [Nm]</td>
<td>longitudinal tire stiffness</td>
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<tr>
<td>$\mu$</td>
<td>0.3</td>
<td>adhesion coefficient between road and wheel</td>
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<td>$\lambda_{lim}$</td>
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<td>desired wheel slip limit</td>
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The filters are selected as $W_u$ and $W_y$ are converted into state space representation with matrices $[A_u, B_u, C_u, 0]$ and $[A_y, B_y, C_y, 0]$. If LMI (12) is satisfied with a minimal $\gamma_i$, the optimal, static gain is calculated by $\Gamma_i = [\Gamma_{u,i}, \Gamma_{y,i}]^T = \bar{L}_i \bar{U}_i^{-1}$.
Fig. 8. Simulation of various step requests

(1) Steering wheel input
(2) Longitudinal velocity
(3) Yaw rate
(4) Wheel slip FR
(5) Torque electric motor FL

References