Abstract—This article presents a novel learning methodology based on the hybrid mechanism for training interval type-2 non-singleton type-2 Takagi-Sugeno-Kang fuzzy logic systems (FLS). Using input-output data pairs during the forward pass of the training and prediction processes, the interval type-2 non-singleton type-2 TSK FLS consequent parameters are tuned by using the recursive least squares (RLS) method. In the backward pass, the antecedent parameters are tuned by using the back-propagation (BP) method. As reported in the literature, the performance indexes of these hybrid models have proved to be better than the individual training mechanism when used alone. The proposed hybrid methodology was tested thru the modeling and prediction of the steel strip temperature at the descaler box entry as rolled in an industrial hot strip mill. Results show that the proposed method compensates better for uncertain measurements than previous-type-2 Takagi-Sugeno-Kang hybrid learning or back propagation developments.

Keywords: IT2 TSK fuzzy logic systems, ANFIS, hybrid learning, temperature prediction.

I. INTRODUCTION (Heading 1)

Interval type-2 (IT2) fuzzy logic systems (FLS) constitute an emerging technology. In [1] both, one-pass and back-propagation (BP) methods are presented as IT2 Mamdani FLS learning methods, but only BP is presented as learning mechanism for IT2 Takagi-Sugeno-Kang (TSK) FLS systems. When BP method is used in both Mamdani and TSK FLS, none of antecedent and consequent parameters of the IT2 FLS is fixed at starting of training process; they are tuned using exclusively steepest descent method. In [1] recursive least squares (RLS) and recursive filter (REFIL) algorithms are not presented as IT2 FLS learning methods.

The aim of this work is to present and discuss a RLS-BP based hybrid-learning mechanism for antecedent and consequent parameters tuning during training process for interval type-2 non-singleton type-2 TSK FLS. Here such hybrid system is abbreviated as IT2 TSK NSFLS2 ANFIS. The abbreviation IT2 TSK NSFLS2 will be used for interval type-2 non-singleton type-2 TSK FLS systems with BP learning only.

The hybrid algorithm for IT2 Mamdani FLS has been already presented elsewhere [2]-[6] with three combinations of the hybrid mechanism: RLS-BP, REFIL-BP and orthogonal least-squares-BP (OLS-BP). The hybrid mechanism for singleton IT2 TSK SFLS (IT2 SFLS ANFIS) has been presented in [7,8] with two combinations of the hybrid learning mechanism: RLS-BP and REFIL-BP; whilst the hybrid algorithm for interval type-1 non-singleton IT2 TSK NSFLS1 (IT2 NSFLS1 ANFIS) has been presented in [9,10] only with the hybrid mechanism RLS-BP. Works on type-2 non-singleton IT2 TSK FLS systems using the RLS-BP learning mechanisms have not been found in the literature.

In this work, the IT2 TSK NSFLS2 ANFIS system that uses the hybrid learning mechanism (RLS-BP) has been developed and implemented for temperature prediction of the transfer bar at hot strip mill (HSM) finishing scale breaker (SB) entry zone. The same data-set was used in previous work [2-10] in order to serve as comparison of functionality and stability between the hybrids mechanisms developed over this line of work. The intention of this paper is to show the implementation in a real industrial application of the (RLS-BP) hybrid mechanism, training a type-2 non-singleton type-2 TSK FLS system using the hybrid mechanism composed by RLS-BP methods.

II. PROPOSED METHODOLOGY

A. Overview

Most of the hot strip mill processes are highly uncertain, non-linear, time varying and non-stationary [2], [11], having very complex mathematical representations. The IT2 TSK NSFLS2 ANFIS system takes easily the random and systematic components of type A or B standard uncertainty [12] of industrial measurements. The non-linearities are handled by FLS as identifiers and universal approximators of nonlinear dynamic systems [13]-[17]. Stationary and non-stationary additive noise is modeled as a Gaussian function centered at the measurement value. In stationary additive noise, the standard deviation takes a single value, whereas in non-stationary additive noise the standard deviation varies over an interval of values [1].

The IT2 TSK SFLS ANFIS based on RLS-BP learning algorithm presented in [8, 9] and IT2 TSK NSFLS1 ANFIS based on RLS-BP presented in [10, 11] are used as a benchmark algorithm for parameter estimation or systems identification assessment [1]. However comparisons with BP only algorithms are also presented since this is a more standard method.
As mentioned, works on IT2 TSK NSFLS2 ANFIS approach has not been found in the literature.

B. The Hybrid RLS-BP Mechanism in IT2 TSK NSFLS2 Systems

Table 1 shows the activities of the one pass learning algorithm of BP-BP method. The IT2 TSK NSFLS2 (BP-BP) outputs are calculated during forward pass. During the backward pass, the error propagates backward and the antecedent and consequent parameters are estimated using only the BP method.

### TABLE I. ONE PASS IN NON-HYBRID LEARNING MECHANISM

<table>
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<tr>
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<th>Forward Pass</th>
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<tr>
<td>Antecedent</td>
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The IT2 TSK NSFLS2 ANFIS system is trained using the hybrid mechanism; it uses RLS during forward pass for tuning of consequent parameters as well as the BP method for tuning of antecedent parameters, as shown in Table 2. It has the same training mechanism as the Adaptive Neural Fuzzy Inference System which is a hybrid learning algorithm first introduced by J. S.-R. Jang, for TSK FLS, the ANFIS system [18, 19].

### TABLE II. TWO PASSES IN HYBRID LEARNING MECHANISM

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<td>Consequent</td>
<td>RLS</td>
<td>Fixed</td>
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<td>Parameters</td>
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The training method is presented as in [1]: Given N input-output training data pairs, the training algorithm for E training epochs, should minimize the error function:

\[
e^{(t)} = \frac{1}{2} \left[ f_{IT2-FLS}(x^{(t)}) - y^{(t)} \right]^2
\]

where \( e' \) is the error function at time \( t \), \( f_{IT2-FLS}(x^{(t)}) \) is the output of the IT2 FLS using the input vector \( x^{(t)} \) from the input-output data pairs, and \( y^{(t)} \) is the output from the input-output data pairs.

III. APPLICATION

A. Hot Strip Mill Basics

Because of the complexities and uncertainties involved in rolling operations, the development of mathematical theories has been largely restricted to two-dimensional models applicable to heat losing in flat rolling operations.

Fig. 1 shows a simplified diagram of a HSM, from the initial point of the process at the reheat furnace entry to its end at the coilers.

![Figure 1](image)

Besides the mechanical, electrical and electronic equipment, a big potential for ensuring good quality lies in the automation systems and the used control techniques. The most critical process in the HSM occurs in the Finishing Mill (FM). There are several mathematical model based systems for setting up the FM.

A model-based set-up system [20] calculates the FM working references needed to obtain gauge, width and temperature at the FM exit stands. It takes as inputs: FM exit target gage, target width and target temperature, steel grade, hardness ratio from slab chemistry, load distribution, gauge offset, temperature offset, roll diameters, load distribution, transfer bar gauge, transfer bar width and transfer bar temperature entry.

B. Design of the IT2 TSK NSFLS2 System

The architecture of the IT2 TSK NSFLS2 ANFIS is established such that its parameters are continuously optimized. The number of rule-antecedents is fixed to two, one for the Reversing Mill (RM) exit surface temperature (x1), and one for transfer bar head traveling time (x2). Each antecedent-input space is divided into three fuzzy sets (FSs); using all possible combination a total number of nine rules are formulated.

An IT2 TSK NSFLS2 is again characterized by IF-THEN rules, but its antecedent and consequent sets are now IT2 sets.

C. Input-Output Data Pairs

From an industrial HSM, noisy input-output pairs of three different product types were collected and used as training and checking data. The inputs are the noisy measured RM exit surface temperature, and the measured RM exit to SB entry transfer bar traveling time. The output is the noisy measured SB entry surface temperature.
D. Fuzzy Rule Base

The IT2 TSK NSFLS2 ANFIS fuzzy rule base consists of a set of IF-THEN rules that represents the model of the system. The IT2 TSK NSFLS2 ANFIS system has two inputs \( x_i \in X_1 \), \( x_2 \in X_2 \) and one output \( y \in Y \). The rule base has \( M = 9 \) rules of the form:

\[
R^i \colon \text{IF } x_i \text{ is } \tilde{F}_1^i \text{ and } x_2 \text{ is } \tilde{F}_2^i, \quad \text{THEN } Y^i = C^i_1 x_1 + C^i_2 x_2,
\]

where \( Y^i \) the output of the \( i \)th rule is a fuzzy type-1 set, and the parameters \( C^i_j \), with \( i = 1,2,3,..,9 \) and \( j = 0,1,2 \), are the consequent type-1 FSs.

E. Fuzzy Rule Base

The primary MFs for each input of the IT2 TSK NSFLS2 ANFIS are Gaussians of the form:

\[
\mu_{x_1}(x_1) = \exp \left[ -\frac{1}{2} \left( \frac{x_1 - \mu_{x_1}}{\sigma_{x_1}} \right)^2 \right]
\]

where: \( \sigma_{x_1} \in [\sigma_{x_11}, \sigma_{x_12}] \) \( k=1,2 \) (the number of type-2 non-singleton inputs), \( n=1,2 \) (the lower and upper bounds of the uncertain deviation) and \( \mu_{x_1}(x_1) \) centered at the measured input \( x_1 = x'^i_1 \). The uncertain standard deviation \( [\sigma_{x_11}, \sigma_{x_12}] \) of RM exit surface temperature measurement was initially set as \([11.0, 14.0]^{\circ}\text{C}\) and the uncertain standard deviation \( [\sigma_{x_21}, \sigma_{x_22}] \) of head-end traveling time measurement was initially set to \([1.41, 3.41]\)s.

F. Antecedent Membership Function

The primary MFs for each antecedent are FSs described by Gaussian with uncertain means:

\[
\mu_{x_1}(x_1) = \exp \left[ -\frac{1}{2} \left( \frac{x_1 - m^i_{x_1}}{\sigma^i_{x_1}} \right)^2 \right]
\]

where \( m^i_{x_1} \in [m^i_{x_11}, m^i_{x_12}] \) is the uncertain mean, with \( k=1,2 \) (the number of antecedents) and \( i = 1,2,..,9 \) (the number of M rules), and \( \sigma^i_{x_1} \) is the standard deviation. The means of the antecedent fuzzy sets are uniformly distributed over the entire input space.

Table 3 shows the calculated interval values of uncertainty of \( x_1 \) input, where \( [m^i_{x_11}, m^i_{x_12}] \) is the uncertain mean, and \( \sigma_i \) is the standard deviation.

<table>
<thead>
<tr>
<th>( m^i_{x_11} )</th>
<th>( m^i_{x_12} )</th>
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<td>950</td>
<td>952</td>
<td>60</td>
</tr>
<tr>
<td>1016</td>
<td>1018</td>
<td>60</td>
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<tr>
<td>1080</td>
<td>1082</td>
<td>60</td>
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</table>

G. Consequent Membership Function

Each consequent is an interval type-1 fuzzy set with the values determined by \( Y^i = [y^i_1, y^i_2] \), where

\[
y^i_1 = \sum_{j=1}^{p} c^i_j x_j + c^0_j - \sum_{j=1}^{p} |x_j| s_j^i - s^0
\]

and

\[
y^i_2 = \sum_{j=1}^{p} c^i_j x_j + c^0_j + \sum_{j=1}^{p} |x_j| s_j^i + s^0
\]

where \( c^i_j \) denotes the center (mean) of \( C^i_j \) and \( s^i_j \) denotes the spread of \( C^i_j \), with \( i = 1,2,3,..,9 \) and \( j = 0,1,2 \). Then \( y^i_1 \) and \( y^i_2 \) are the consequent parameters.

When only the input-output data training pairs \( \{x^{(1)}_1 : y^{(1)}\}, \ldots, \{x^{(H)}_1 : y^{(H)}\} \) are available and there is not data information about the consequents, the initial values for the centroid parameters \( c^i_j \) and \( s^i_j \) can be chosen arbitrarily in the output space [16]. In this work the initial values of \( c^i_j \) were set equal to 0.001 and the initial values of \( s^i_j \) equal to 0.0001, for \( i = 1,2,3,..,9 \) and \( j = 0,1,2 \).
IV. RESULTS

The hybrid IT2 TSK NSFLS2 ANFIS system was trained and used to predict the SB entry temperature at a HSM, applying the RM exit measured transfer bar surface temperature and RM exit to SB entry zone traveling time as inputs. We ran fifty epochs of training; one hundred and ten parameters were tuned using eighty seven, sixty-eight and twenty-eight input-output training data pairs per epoch, for type A, type B and type C products respectively.

The performance evaluation for the hybrid IT2 TSK NSFLS2 ANFIS system was based on root mean-squared error (RMSE) benchmarking criteria as in [1].

Fig. 2 shows the RMSEs of the three IT2 TSK FLS systems trained using only the BP-BP algorithm; all of them for fifty epochs of training for the case of products of type C.

Fig. 3 shows the RMSEs of the three IT2 TSK ANFIS systems trained using the proposed hybrid RLS-BP algorithm, for the case of products of type C. Observe that from epoch 1 to 4 the IT2 TSK NSFLS1 ANFIS has better performance than both the singleton IT2 TSK SFLS ANFIS and the IT2 TSK NSFLS2 ANFIS. At epoch 1, the RMSE of the IT2 TSK NSFLS2 ANFIS has a poor performance. At epoch 3, it reaches its minimum RMSE and is stable for the rest of training.

The proposed IT2 TSK NSFLS2 ANFIS system has the best performance and stability after only two epoch of training. It is important to emphasize that the three IT2 TSK ANFIS systems are very senstive to the learning parameter gain.

V. CONCLUSIONS

An IT2 TSK NSFLS2 ANFIS using the hybrid RLS-BP training method was tested and compared for predicting the surface temperature of the transfer bar at SB entry. The antecedent MFs and consequent centroid of the IT2 TSK NSFLS2 ANFIS absorbed the uncertainty introduced by all the factors: the antecedent and consequent initially values, the noisy temperature measurements, and the inaccurate traveling time estimation. The type-2 non-singleton fuzzy inputs are able to compensate the uncertain measurements, expanding the applicability of IT2 TSK NSFLS2 ANFIS systems.

It has been shown that the proposed IT2 TSK NSFLS2 ANFIS system can be applied in modeling of the steel coil temperature. It has also envisaged its application in any uncertain and non-linear system prediction and control, as in furnace temperature control, aerospace stability control, turbine trust control and especially in those applications where there are measurements uncertainty.

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REFERENCES


