Performance Analysis of a Post-Mission Multi-Reference RTK DGPS Positioning Approach

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Abstract

Global Positioning System (GPS) reference networks for real-time kinematic (RTK) positioning have been tested in several countries in recent years. The use of a multi-reference station approach which leverages these networks has been proposed to overcome the limitations of the standard single reference station differential carrier phase positioning method. Differential GPS (DGPS) positioning at the cm-level is provided by resolving the carrier phase ambiguities as integer values using short observation time spans. However, this process becomes increasingly difficult as the distance between a GPS rover and a reference station increases due to the decorrelation of GPS errors with distance. The baseline length over which the distance dependent errors are effectively eliminated is generally limited to 10 km or less, especially when the ionosphere is active. In order to perform cm-level on-the-fly (OTF) positioning over longer distances, a solution is to generate carrier phase corrections for a rover by means of a reference station network whereby the errors are modelled over a region. For real-time applications, the data from each reference station is transmitted to a central processing centre where the carrier phase corrections are generated and sent back to the individual reference stations for wireless transmission to rovers together with the usual code and carrier phase data used for standard RTK GPS. In this system, network information is used to assist the rover to calculate accurate and reliable navigation and positioning solutions. However, the rover does not communicate with the network, as this would require two-way communication system. This is not a limitation in post mission, in which case the rover and network can share information in batch mode to maximize the use of the observations and more accurately determine the rover position. This allows for not only for the network stations assisting the rover but also to take advantage of the rover's data. The post mission network processing approach is suitable for quasi-instantaneous positioning requiring only a few observations epochs. This paper presents the theory of one advanced post-mission multiple reference station method, namely MultiRefPM™, and the results of a test based on a regional test network established in the Campania region of Italy. The study is characterized by the ultimate of objective of analysing methodological aspects of the MultiRefPM™ method in general with a specific focus on the post mission procedure.

Introduction

The real-time MultiRef™ method [Lachapelle et al., 2000; Cannon et al., 2001] represents the results of a research program initiated in 1996 at the Department of Geomatics Engineering, University of Calgary. Based upon the use of multiple Global Positioning System (GPS) reference stations, this method is a powerful instrument for carrier-phase based real-time kinematic (RTK) GPS positioning. In order to provide a distribution service of the carrier phase corrections, MultiRef™ offers the possibility of utilizing several stations operating in real-time. The simultaneous use of multiple reference stations allows better modelling of residual measurement errors, however standard differential techniques that use a single reference station are utilized at the rover receiver to maintain simplicity.

Correlated errors are better modelled throughout a region covered by a network, which leads to an improved performance of the on-the-fly (OTF) algorithms for carrier-phase ambiguity resolution. This improvement is reflected in an improved positioning accuracy and on a reduction in the number of stations required to provide an RTK service. The service area coverage can be considerably increased using a low number of reference receivers. One of the major advantages that network-based methods have over single reference station methods is that fewer reference stations are required to maintain a high accuracy positioning service, whereby a user requires only the rover receiver, without the need to install a base station in the vicinity.

This paper presents the theory of an advanced post-mission multiple reference station method, namely MultiRefPM™, and the results of a test based on a regional test network established in the Campania region of Italy. The study is characterized by the ultimate of objective of analysing methodological aspects of the MultiRefPM™ method in general with a specific focus on the post mission procedure.

Network RTK GPS Positioning

The MultiRef™ approach uses least squares collocation to predict the differential correlated errors at any location within the network coverage area [Raquet, 1998]. The following collocation equations are used to estimate the corrections for the network reference stations and the rover locations, respectively:

$$\begin{aligned} \delta\hat{\mathbf{l}} &= C_{\delta l} B^T (B C_{\delta l} B^T)^{-1} (B\overline{\mathbf{\Phi}} - \lambda\Delta\nabla\mathbf{N}) \\ \delta\hat{\mathbf{l}}_r &= C_{\delta l_r,\delta l} B^T (B C_{\delta l} B^T)^{-1} (B\overline{\mathbf{\Phi}} - \lambda\Delta\nabla\mathbf{N}) \end{aligned} \quad (1)$$

where $\delta\hat{\mathbf{l}}$ is a vector of corrections, $C_{\delta l_r,\delta l}$ is the covariance matrix between the corrected observations and the network observations, B is the double differencing matrix, $C_{\delta l}$ is the variance-covariance matrix of the network observations, $\overline{\mathbf{\Phi}}$ is a vector of the network observations in metres, and $\lambda\Delta\nabla\mathbf{N}$ is a vector of the double difference ambiguities in metres. Equation (1) shows that the corrections are a function of the mathematical relationship between the baselines (B), the stochastic relationship between the baselines ($C_{\delta l}$) and the stochastic relationship between the baselines and the rover ($C_{\delta l_r,\delta l}$). When the corrections are applied, this information is combined with the stochastic relationship between the rover and a single reference station as well as the mathematical relationship due to the double differencing of the measurements.

Network RTK procedures estimate the error at the rover by a weighted-average of the surrounding reference station's measured error. Some systems use a plane to determine the weights of the surrounding stations [Wanninger et al., 1999; Vollath et al., 2000; Wübbena et al., 2001], while others use least squares to determine the weights [Raquet, 1996; Landau et al., 2002]. In either case, the errors measured at each reference station are transferred to the rover by mathematical and stochastic models. In planar interpolations, each reference station is weighted as a linear function of the distance to the rover only. Using least squares collocation allows for more complicated weighting schemes given by the covariance function. This could weight not only the influence of each reference station but also each of the satellite's error at each of the reference stations. The covariance function could be a function of receiver separation, angular separation between the satellites, measurement noise, ionosphere pierce point distance, or elevation and azimuth.

For actual real-time applications, the data from each reference station is transmitted to a central processing centre where the carrier phase corrections are generated. The corrections are then applied to the observations of one of the reference stations. The rover can then use these corrected reference station observations with standard RTK software.

Post Mission Network RTK - MultiRefPM™

Network RTK implementation consists of three main steps. In the first step, the errors at the reference stations are estimated using carrier-phase observations. The second step interpolates these errors to the rover receiver location whereas the third step is to transmit the corrections to the rover. This is usually carried out in real time by generating virtual reference station data that the rover can accept, using a single reference station data format.

Real-time Network kinematic positioning is limited by many factors, one of which is the communication network used between the network control centre and the rovers. Due to bandwidth limitations with multiple rovers and an attempt to allow for user privacy, real-time Network kinematic positioning attempts to operate a broadcast-only system, whereby the network corrections are broadcast to all rovers and there is no information communicated from the rover back to the network.

This one-way communication mode is not an issue in post mission whereby the rover and network can share information both ways. This allows for the network stations to assist the rover but also the rover to assist the network as additional information. In fact, multiple rovers within the same region can assist each other in achieving accurate position and velocity estimates. In this respect, the advantages of rover multiplicity are well known [Lachapelle et al, 1993; Luo & Lachapelle, 2003].

Network RTK systems use reference stations to precisely measure the correlated errors affecting the region. These errors can only be measured when all other parameters are precisely determined, namely the station position and carrier phase ambiguities. With this in mind, the better a station's position and ambiguities are known, the more accurately one can separate measurement errors and systematic biases. Reference stations are an obvious choice because their positions are known, but any receiver can be used to estimate measurement errors. For example, there is no reason why a rover which has been static for an hour or more could not be treated any differently than a static reference station.

In terms of error modelling, multiple rovers in an area can each give an indication of the local environmental error conditions. Combining all of this information into a coherent model allows for new network rovers, with less defined position and velocity estimates, to benefit from decreased measurement error.

The assistance of the rover to the network can be seen in the baseline configurations for the network. Ambiguity resolution performance is a function of baseline length because the correlated errors increase in magnitude as the baseline length increases. In a broadcast-only Network RTK system, baselines are formed between the various reference stations. Rovers within the network will, by definition, be between two or more reference stations. Therefore by connecting baselines to the rovers as well as the reference station, this will shorten the overall baseline lengths within the network thus giving a higher likelihood of resolving the baseline ambiguities.

If instead of applying a weighted average (prediction) approach the rover's data and estimated states are added to the network filter. The network filter was previously used solely to estimate and resolve the network ambiguities. The addition of the rover's information incorporates all the information used in the collocation approach shown in Equation 1. The difference is that the network is not only assists the rover but at the same time, the rover is assisting the network.

The design matrix of the integrated approach is

$$A = \begin{bmatrix} \frac{\partial \Delta\nabla\Phi_1}{\partial x} & \frac{\partial \Delta\nabla\Phi_1}{\partial y} & \frac{\partial \Delta\nabla\Phi_1}{\partial z} & \lambda & 0 & & 0 & 0 & \\ \frac{\partial \Delta\nabla\Phi_2}{\partial x} & \frac{\partial \Delta\nabla\Phi_2}{\partial y} & \frac{\partial \Delta\nabla\Phi_2}{\partial z} & 0 & \lambda & & 0 & 0 & \\ & & & & & \ddots & & & \\ 0 & 0 & 0 & 0 & 0 & & \lambda & 0 & \\ 0 & 0 & 0 & 0 & 0 & & 0 & \lambda & \\ & & & & & & & & \ddots \end{bmatrix} \tag{2}$$

where the first n rows correspond to the double difference observations between the rover and one of the reference stations and second m rows correspond to double difference observations between the reference stations. The first three columns correspond to the rover's position estimates and the following n + m columns correspond to the ambiguities of all of the double difference observations. The double difference observations between the rover and one of the reference stations is used to estimate the rover's position, indicated by the terms in the first three columns, while the observations between the reference stations do not, indicated by the zero terms in the first three columns.

The form of this design matrix can be extended to accept any number of reference stations and rovers. The processing results shown include all the observations, code and carrier phase, processed in a single Kalman filter.

In order to maintain the information from the correction based approach, mathematical and stochastic information must be preserved in the integrated approach. The mathematical correlation is due to baselines that share a common reference station (or rover station) which use the same observations in their double difference measurements. Stochastic correlation comes from the covariance function. The covariance function states the likelihood of two values being the same based on a physical process. For example, it is known that the ionosphere is a spatially correlated error source, therefore two stations close to each other are likely to have the same ionospheric error. This likelihood is represented in the stochastic correlation.

Ionospheric Modelling

Network RTK lends itself to resolving ambiguities over much longer baselines than would be possible in single baseline RTK. As the baseline length increases, the magnitude of the correlated errors increases. Currently, the largest correlated error source is the ionosphere, which if not modelled can significantly reduce ambiguity resolution performance. Stochastic ionosphere modelling has been used effectively in the past by many investigators to reduce the effect of the ionosphere [e.g., Liu, G., 2001; Odijk, 2002; Alves et al., 2002; Liu, J., 2003]. In this model a slant ionosphere error is estimated for each double difference measurement, and in the current implementation, this parameter is measured using both code and carrier phase measurements.

Unfortunately, estimating the ionosphere with code can be troublesome because of the high measurement noise and multipath relative to the carrier phase. If the ambiguities are reasonably determined then the carrier phase observations will drive the ionospheric estimates, however if the ambiguities are weakly determined, then the code noise will overflow into the ionosphere estimates. Unfortunately, noisy ionospheric estimates affect the ambiguity estimates, delaying convergence and ultimately, ambiguity resolution.

To reduce the effect of code noise on the ionospheric estimates a pseudo-observation is used. This observation constrains the value of the ionospheric estimate to a reasonable value in such a way that multiple code observations must be averaged before the ionospheric estimate can be measured by the code. This averaging reduces the code noise and smoothes the convergence of the ionospheric estimate, which in turn smoothes the convergence of the ambiguities.

The value of the ionosphere constraint can be an approximate ionosphere value from a global ionospheric map (GIM), or alternatively, the broadcast ionospheric model [Liu, G., 2001]. If an external model is not used then the ionosphere can be constrained to zero which is reasonable because the double difference ionospheric errors are zero mean.

The weight of the ionospheric constraint is variable depending on the true ionospheric conditions. If the ionosphere is very active then a low weight should be used and if there is no ionosphere error then the ionospheric constraint should be applied fully. This weight of the pseudo-ionospheric constraint is adaptively adjusted based on the ionospheric level measured by the reference stations and the rover, similar to the adaptive approach proposed by Alves et al. [2002].

Position and Velocity Estimation

In general, rover GPS receivers are moving, however, in the test results presented, static receivers are used because this approach yields the true receiver's trajectory with which the MultiRefPM™ method can be compared. However, kinematic receiver estimation is implemented whereby both the position and velocity of the rover are determined each epoch. Velocities are estimated using a first order Gauss-Markov process. In the case of the MultiRefPM™ approach, all of the network ambiguities are estimated in the same Kalman filter as the position and velocity estimates. When the ambiguities are fixed, the float solution is adjusted to take into account the new, fixed ambiguity information. This is done using a conditional decorrelation as shown below [e.g., Teunissen and Kleusberg, 1998; Odijk, 2002].

$$\begin{aligned} b_{\text{fixed}} &= b_{\text{float}} - C_{b_{\text{float}},a_{\text{float}}} C^{-1}_{a_{\text{float}}} (a_{\text{float}} - a_{\text{fixed}}) \\ C_{b_{\text{fixed}}} &= C_{b_{\text{float}}} - C_{b_{\text{float}},a_{\text{float}}} C^{-1}_{a_{\text{float}}} C_{a_{\text{float}},b_{\text{float}}} \end{aligned} \tag{3}$$

where b_{fixed} is a vector containing the estimates of all the floating parameters (position, velocity, ionosphere and unfixed ambiguities) adjusted given the fixed ambiguities, b_{float} is a vector of the floating parameters before adjusting for the fixed ambiguities, $C_{b_{\text{float}},a_{\text{float}}}$ is the covariance matrix between the floating parameters and the float ambiguities that have been fixed, $C^{-1}_{a_{\text{float}}}$ is the variance-covariance matrix of the float ambiguities that have been fixed, a_{float} is a vector of the float ambiguities that have been fixed, and a_{fixed} is a vector of the fixed ambiguities. Note that not only the values of the estimated parameters change. The variance-covariance matrix of the floating parameters is also adjusted to account for the fixed ambiguity information.

Field Test

A regional test network established in the Campania region of Italy in 2002 was used to evaluate the performance of the above procedure. Figure 1 shows the Campania Network [Pugliano, 2002, 2003] composed of twelve GPS receivers extending throughout the entire region, with an inter-receiver distance of about 50 km.

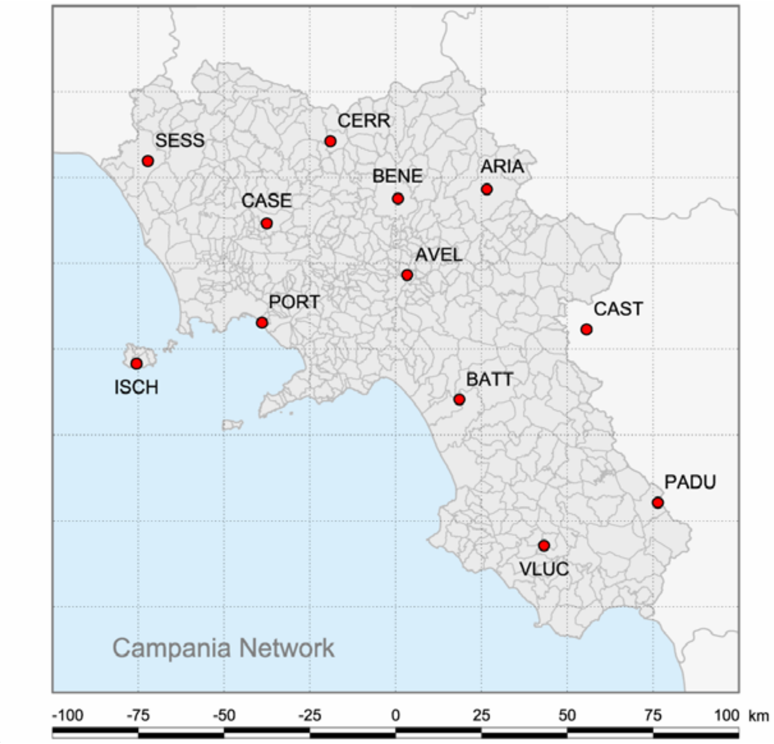

Figure 1: *Campania Network*

A 24-hour period observed on February 7, 2002, was selected for this study. The data was acquired concurrently from all twelve stations at a 1-Hz rate. This dataset was collected during an ionospherically active period. In fact the network has a wide variety of diurnal and day-to-day ionospheric effects. Figure 2 shows the ionospheric level for the PORT-CASE baseline (28 km) over a period of six days of measurements [Pugliano 2002]. The ionospheric bias in the middle of the day (08:00 16:00 local time) is very high, up to 10 ppm. The RMS of the ionosphere ranges from 2.1 ppm to 1.4 ppm.

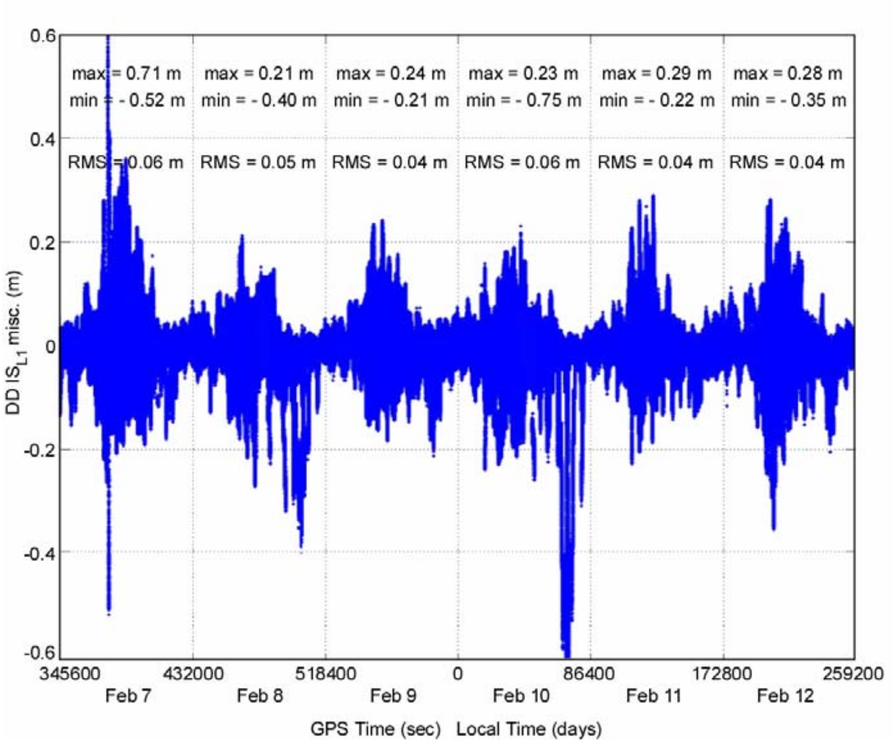

Figure 2: *Double difference ionospheric delay calculated over a 6 day period for the 28 km baseline (PORT-CASE), from February 7 to 12, 2002*

Measured Network Errors

In keeping with the approach of the MultiRef™ method, an initial calculation of the double difference measurement errors between the reference stations was made. To highlight these errors, the double-differencing process was applied to the measurement-minus-range carrier-phase observable $\overline{\Phi}$. Therefore, by applying the double-differencing process, the differential GPS errors are obtained as misclosure errors. Given the Bernese software-generated network coordinates and ambiguities between the reference stations, the fundamental formula which calculates the differential errors, expressed in metres, is given as

$$\Delta\nabla\delta l = \Delta\nabla\overline{\Phi} - \lambda\Delta\nabla N = \Delta\nabla d\rho - \Delta\nabla d_{ion} + \Delta\nabla d_{trop}^{r} + \Delta\nabla\varepsilon(\Phi) \qquad (4)$$

Double difference measurement errors were calculated on different baselines. Figure 3 shows the double difference errors for C/A code on L1 (C1), Y code on L2 (P2), L1 phase (L1), L2 phase (L2), widelane phase (WL), ionosphere free phase (IF), and geometry free (IS) for each observable on one specific baseline (ARIA to AVEL, 33 km), for data collected February 7.

The carrier phase errors are calculated using their fixed integer ambiguities. Widelane is the difference between L1 phase and L2 phase in cycles. The ionosphere free phase combination removes the effect of the ionosphere leaving only troposphere, orbit, multipath, and noise errors. The geometry-free combination is the difference between the carrier phase observations, with fixed ambiguities, of L1 and L2 in metresers. It is called geometry-free because this difference removes all geometry related errors (orbit, troposphere and reference station position error). It is also refered to as the ionosphere signal because almost all of the remaining error is due to the ionosphere. In this case, IS_{L1} shows the ionosphere error on the L1 frequency. Analyzing the graphs corresponding to the L1, L2, widelane (WL) and geometric-free (IS_{L1}) carrier phase observables, the correlation with the diurnal variation of ionospheric activity is evident.

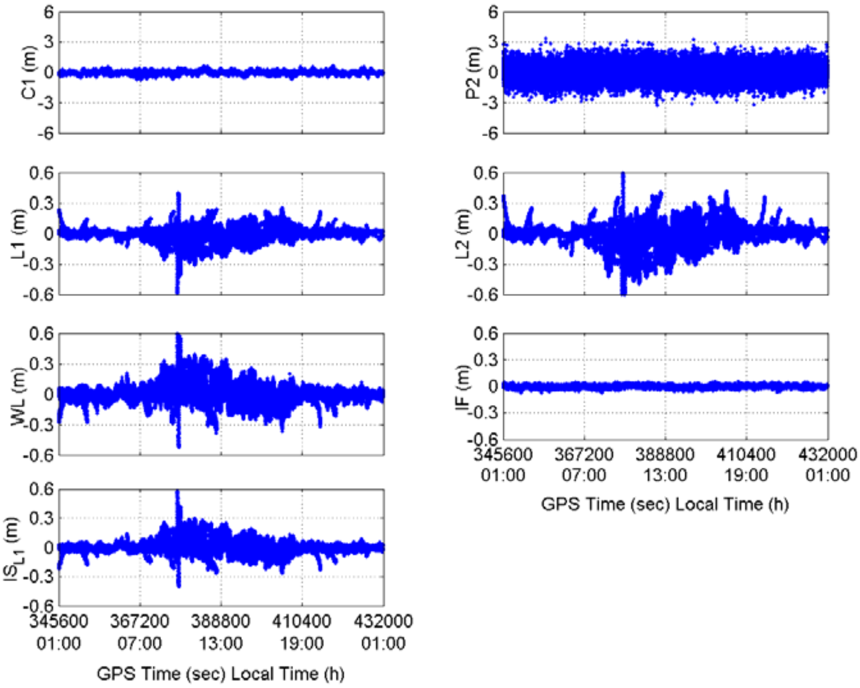

Figure 3: *Double difference errors for C/A code on L1 (C1), Y code on L2 (P2), L1 phase (L1), L2 phase (L2), widelane phase (WL), ionosphere free phase (IF), and geometry free phase (IS_{L1}) observables, 33 km ARIA-AVEL baseline, February 7, 2002*

Test Methodology

The primary objective of the processing and analysis was to evaluate the improvements made following the application of MultiRefPM™. The test methodology was developed as a function of the network geometric characteristics. In considering the different baseline lengths between the rover and the nearest reference station, the analysis is based on a comparison between the solutions obtained by a classic single reference station RTK positioning using raw data and the solutions obtained by MultiRefPM™.

Processing was conducted on six network configurations in order to experiment with the greatest variety of scenarios. Each configuration is comprised of measurements taken from selected stations among the twelve available stations. From time to time, a specific reference station was taken off the network to be used as a rover. In addition to providing different network geometries, the different configurations correspond to different baseline lengths between the rover and the nearest reference station.

In Figure 4, the six rovers tested are shown using different colours. All of the stations are used as network reference stations when not treated as a rover, with the exception of BENE. Table 1 summarizes the main characteristics of each scenario including the distance, in kilometres, between the rover and the closest reference station. This parameter, in particular, assumes great importance in the comparison between the MultiRefPM™ method and the traditional single reference station RTK technique.

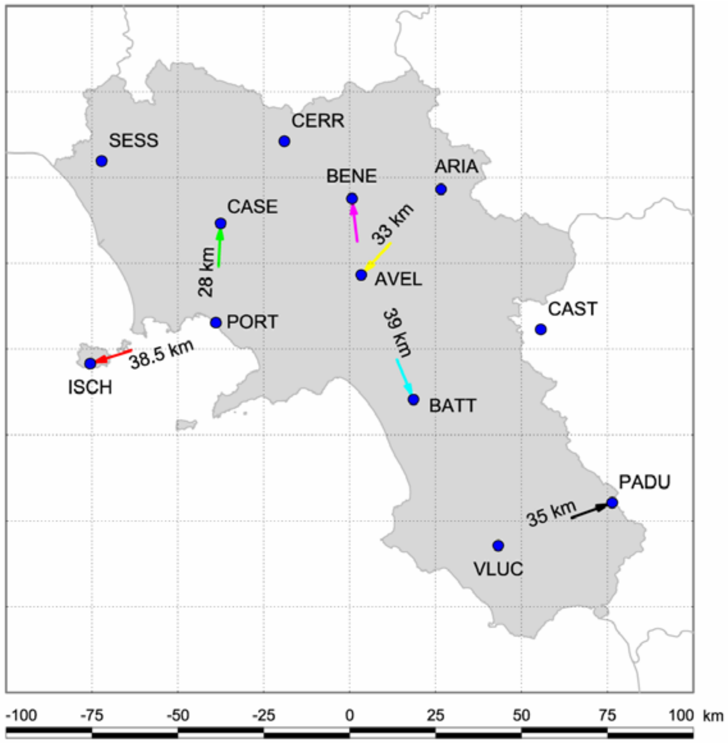

***Figure 4:** Network map for six scenarios*

***Table 1:** Characteristics of six test network scenarios*

Network	Rover	Nearest Reference Station	Baseline length (km)
AVEL-BENE	BENE	AVEL	22
PORT-CASE	CASE	PORT	28
ARIA-AVEL	AVEL	ARIA	33
VLUC-PADU	PADU	VLUC	35
PORT-ISCH	ISCH	PORT	38.5
AVEL-BATT	BATT	AVEL	39

Test results

The following analysis is conducted in the position domain. The objective is to analyze the extent to which the reduction of differential errors using the MultiRefPM™ approach reflects upon the accuracy of the rover position. More specifically, the evaluation of the results is based on the difference between the raw (single reference station RTK) and MultiRefPM™ solutions, epoch-by-epoch, and the known coordinates obtained initially using a Bernese batch method involving several days of data.

In reference to the six test network configurations, Figure 5 shows the position errors obtained at each epoch using raw data as well as corrected measurements. The MutliRefPM™ method shows a improved accuracy in all three components of the position domain. The comparison of the solutions clearly indicates the benefits of the MultiRefPM™ method.

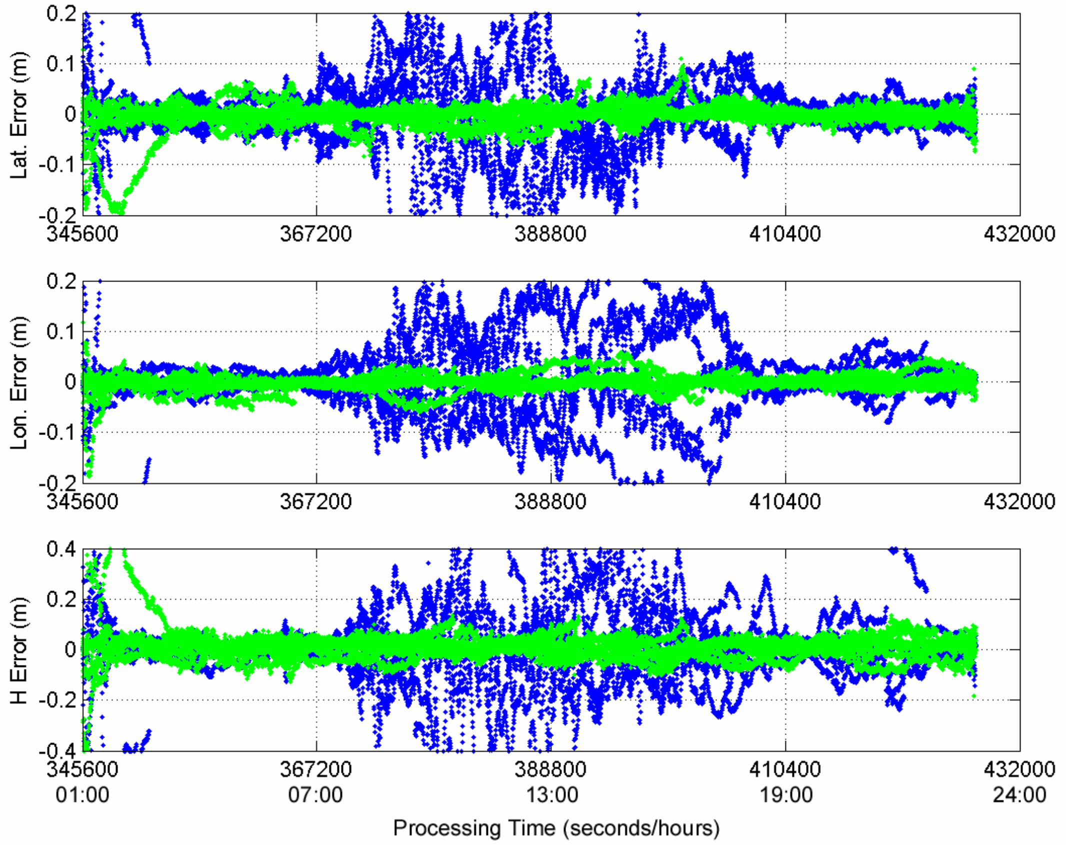

Figure 5: *Position errors for six baselines using raw (Blue) and MultiRePM™ (Green) methods, for February 7, 2002*

The 3D RMS position errors were also calculated and Figure 6 shows the results for three baselines. In comparison with the solution obtained by the classic single reference station method, the RMS errors are reduced from 12 to 3 cm, 13 to 5 cm and 27 to 3 cm, respectively. The error reduction is especially significant during the afternoon when the ionospheric effect is at its peak. Table 2 summarizes the results related to the comparisons conducted during the 24 hour observation period.

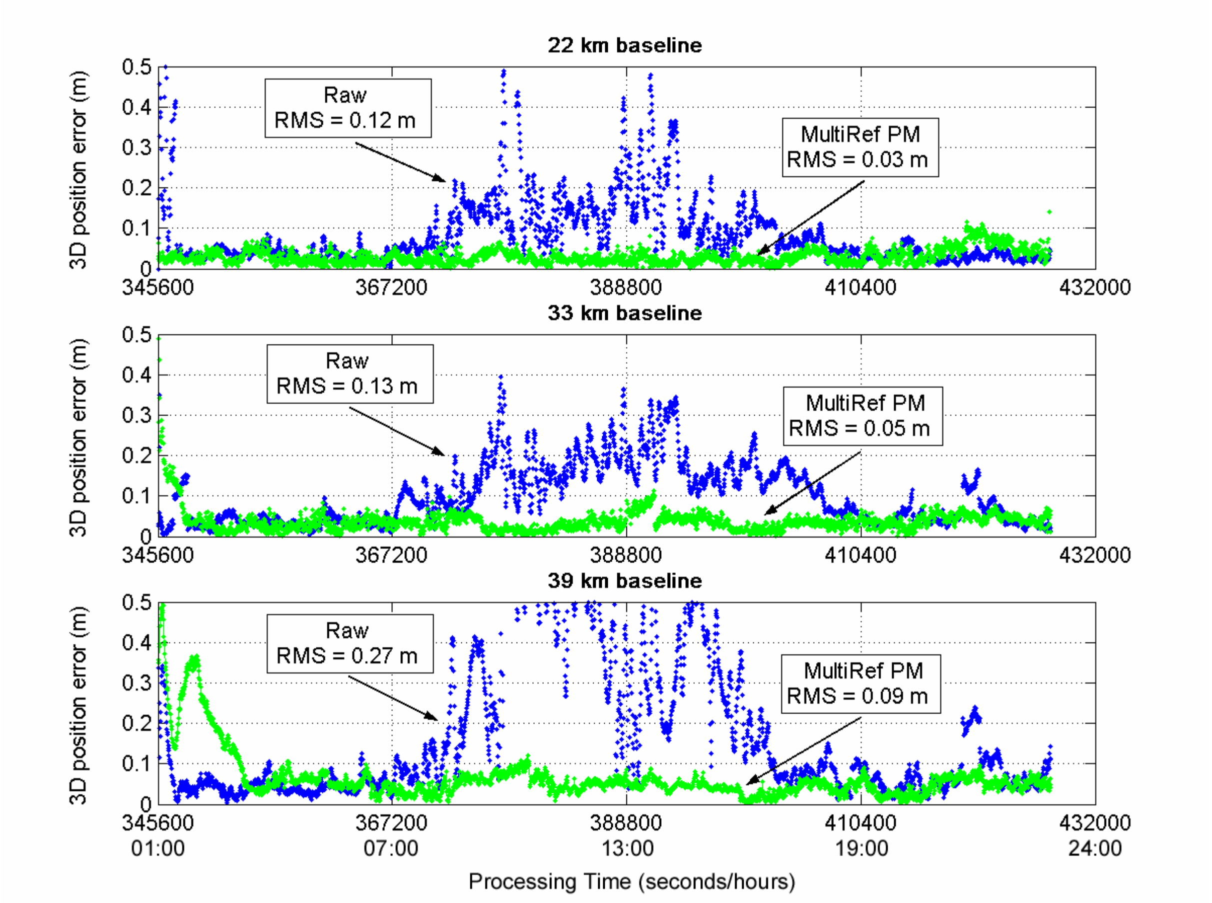

Figure 6: *3D position error for the 22 km (AVEL→BENE), 33 km (ARIA→AVEL) and 39 km (AVEL→BATT) baselines comparing raw and MultiRefPM™ solutions, February 7, 2002*

Table 2: *Raw and MultiRefPM™ 3D position errors RMS values and respective improvement for six scenarios, February 7, 2002*

Baseline	Length (km)	3D RMS position error (cm)		Percentage improvement
		Raw	MultiRef PM™	
AVEL→BENE	22	12.2	3.5	71%
PORT→CASE	28	12.7	5.8	55%
ARIA→AVEL	33	12.7	4.7	63%
VLUC→PADU	35	18.3	3.3	82%
PORT→ISCH	38.5	22.5	3.4	85%
AVEL→BATT	39	26.9	3.5	87%

These results are also shown in Figure 7 as a function of the length of the baseline. The trend in the RMS values for the single reference station case (raw) shows a strong correlation with the baseline lengths. The multiple reference station method is effective in modelling the spatially correlated errors as the corresponding correlation in this case is non-existent.

The PADU and ISCH stations are particularly interesting. In a real-time multiple reference station scenario, these two stations would be outside the network and would benefit little from it. With the MultiRefPM™ approach implemented, the rover (PADU or ISCH) becomes part of the network, and therefore contribute to achieving a high level of accuracy.

In addition to confirming the improvements ensuing from the utilization of the network approach, this analysis shows an enhanced uniformity among the six RMS values whereby improvements of up to 87% in terms of RMS were observed.

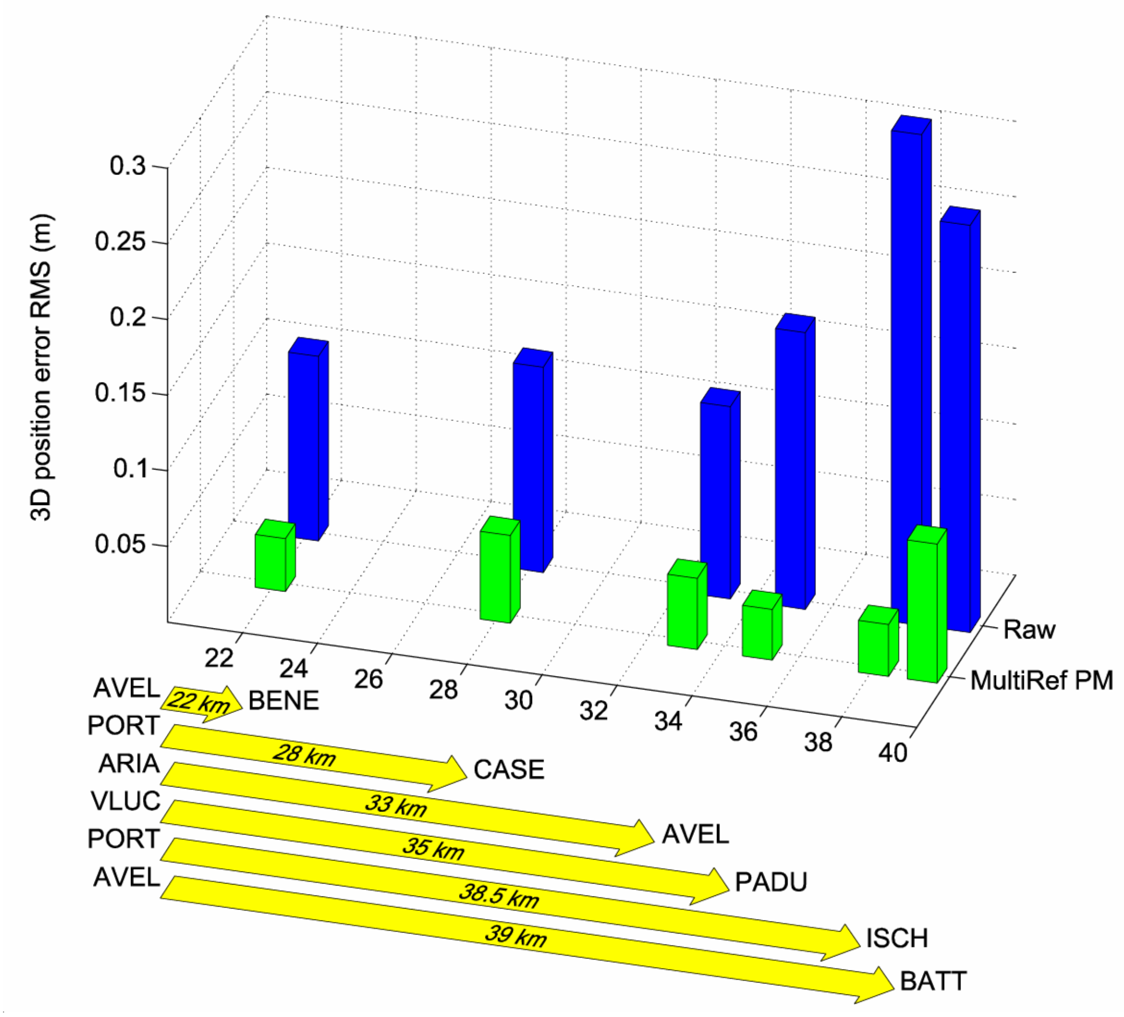

Figure 7: *Comparison of 3D RMS position errors for six scenarios*

Conclusions

In recent years, there has been constant development in the field of GPS positioning. The multi-reference station technique doubtlessly constitutes a further advancement which will have a significant effect on the utilization of GPS in the future. The primary importance of this advancement lies in the redefinition of the very concept of permanent station networks and is truly innovating the productive processing of GPS measurements. Therefore, there is a transition from the use of two receivers required for real time centimetre-accuracy to a new phase characterized by the possibility of guaranteeing the same level of accuracy to rovers equipped with only one receiver. The results obtained in this study have clearly revealed the improvements brought about by the application of the MultiRef Post Mission method in terms of accuracy making it particularly suitable for instantaneous positioning. One can sense the potential of this approach in the fields of surveying.

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