GLOBAL LOW DIMENSIONAL SEISMIC CHAOS 
IN THE HELLENIC REGION

A. C. IlioPoulos and G. P. Pavlos

Democritus University of Thrace, Faculty of Engineering,
Department of Electrical and Computer Engineering,
Kimmeria University Campus, 67100 Xanthi, Greece

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In this study, we present results concerning seismogenesis in the Hellenic region (land and sea of Greece), applying nonlinear analysis to an earthquake time series. The model of the dripping faucet is used as a physical interpretation of the seismic process and the construction of inter-event seismic time series. Geometrical and dynamical characteristics estimated in the reconstructed state space support the low dimensional, chaotic character of the global seismic process in the Hellenic region. The method of stochastic surrogate data was employed to the exclusion of “pseudo chaos” caused by the nonlinear distortion of a purely stochastic process. These results are in agreement with general theoretical models concerning distributed driven threshold dynamics applied to the case of seismic processes. Moreover, the observed global character of low dimensionality and chaoticity over such a complex system of faults supports the hypothesis that seismogenesis is characterized by spatiotemporal intermittent chaos throughout the Hellenic region.

Keywords: Hellenic seismogenesis; nonlinear time series analysis; low dimensional deterministic chaos; inter-event times; distributed driven threshold dynamics.

1. Introduction

Seismogenesis is a complex phenomenon which may be explained by a great variety of nonlinear models and theoretical concepts. According to Rundle et al. [1997a], models of the fault systems related to seismogenesis can be grouped into three general categories:

(a) Dynamical models concerning the transfer of stress between sites on the fault, which involve deterministic evolution. These models include multiple slider block models based on the pioneering work of Burridge and Knopoff [1967] and their variations reviewed by Turcotte [1999]. An extension of these models to Cellular Automata methods for distributed systems is provided by the sandpile SOC models [Bak et al., 1988], the forest-fire and cascade models [Turcotte, 1999] and the Olami, Feder and Christensen (OFC) models [Caruso et al., 2006].

(b) Probabilistic models in which various statistical features of earthquakes are computed from the observed data. These models include the groups of epidemic-type aftershock (ETAS) model introduced by Kagan and Knopoff [1987] and Ogata [1988], the hierarchical models of seismicity [Shnirman & Blanter, 1999], the percolation type models [Lomnitz-Adler, 1985] as well as the coagulation type probabilistic models [Czechowski, 2003].

(c) Models of far from equilibrium statistical physics and self-organized spinodal models, including phase transitions and processes of spinodal, nucleation, driven threshold dynamics [Rundle et al., 2003].
Almost all of the above theoretical models of seismicity are in good agreement with observations and can be used to deduce the main characteristics of seismic phenomenology [Ben-Zion, 2008]. This phenomenology includes, among others, power law distributions and scale invariance laws such as:

(i) The Gutenberg-Richter distribution \( (\sim 1/E^{1+\beta}) \) of earthquake energies.

(ii) The Omori law \( (\sim t^0) \) for foreshock’s or aftershock’s rate.

(iii) The productivity law \( (\sim E^\alpha) \) giving the number of earthquakes triggered by an event of energy \( E \).

(iv) The unified scaling law of earthquakes spatio-temporal clustering and recurrence times \( (P(\tau) \sim \tau^{-\alpha} f(\tau L_0^d/\Delta^3)) \) [Bak et al., 2002; Corral, 2007].

The existence of power law distributions led Bak et al. [1988] to explain earthquakes as a Self Organized Critical (SOC) process, as well as Kagan [1994] to consider seismicity as the “turbulence of the solid” earth crust. These concepts showed that earthquakes can be understood via the general theory of statistical physics for dynamical processes of far from equilibrium phase transitions applied to a distributed faults system [Chang & Cheng-Chin, 2008; Tsallis, 2009].

The SOC process has already been connected in the past to the unpredictability of earthquakes since the SOC dynamics is related to the “edge of chaos” phenomenon characterized by strong randomness and high-dimensionality [Sykes et al., 1999]. SOC models have often been considered as alternatives to the low dimensional chaos interpretation of many realistic physical systems [Pavlos et al., 2007, 2008].

On the other hand, chaos includes low dimensional determinism, which is in contrast to complete unpredictability or randomness. In this case, the deterministic predictability is certainly lost over long enough time scales, but long-term, intermediate and short term prediction could be related to a chaotic seismic-cycle process [Erson, 2001; Ben-Zion et al., 2004].

The modern theory of nonlinear dynamics of far from equilibrium distributed systems suggests that SOC and low dimensional chaos may indeed provide different explanations for the same physical system under different conditions or at different critical points [Pavlos et al., 2008].

In this study, we present reconstructed geometrical and dynamical characteristics of the Hellenic seismogenesis obtained by nonlinear analysis of inter-event seismic time series. Our results provide strong evidence for global low dimensional chaos in the Hellenic region, suggesting that a proper analysis of earthquakes requires their understanding through nonequilibrium thermodynamics and classical statistical physics processes. This is in continuation of certain previous studies [Pavlos et al., 1994; Pavlos et al., 2007; Iliopoulos et al., 2008] which also supported the concept of low dimensional seismic chaos in the Hellenic region. Particularly, in Secs. 2 and 3, we describe the methodology and physical meaning of the nonlinear algorithm used for the analysis of the data and in Sec. 4 we present the results of the nonlinear analysis concerning geometrical and dynamical characteristics. Finally, in Sec. 5 we summarize the results of our data analysis and in Sec. 6 we discuss a theoretical justification of the obtained results using modern theoretical concepts.

2. Theoretical Framework

2.1. The seismic physical system

A fault zone may be treated as a complex system with interacting elements, tectonic plates and fault segments. Generally, the lithosphere presents a hierarchy of volumes, or “blocks”, which move relative to each other. Tectonic plates are the major blocks which are divided into smaller ones. They are separated by less rigid boundary zones thinner than the corresponding blocks, while each zone also represents a hierarchical structure [Keilis-Borok, 1990].

On the other hand, the solid outer crust of the earth (of approximately 20 km thickness), rests on a tectonic shell divided into a small number of moving plates. The relative velocities of plates are of the order of a few centimetres per year, due to the high temperature-pressure phase changes and the consequent powerful convective flow in the Earth’s mantle [Knopoff, 2000].

Due to solid-solid friction, enormous elastic strains are developed on the Earth’s crust. When the accumulated stress exceeds the frictional force, slips can occur between the crust and the tectonic plate or between the local faults and the stored elastic energy can be released during earthquakes in the form of bursts [Chakrabarti, 2007]. The fault interaction at a local or regional scale is supported by rich phenomenological evidence. The possibility of interaction between seismogenic structures at long distances was supported by Melini et al. [2002].
The application of nonlinear analysis to seismic data in the Hellenic region also supports this concept as will be explained in the present paper (see also [Pavlos et al., 2007]).

Earthquakes reveal the internal dynamics of the lithospheric fault system including the upper and lower crust, as well as the upper mantle [Kerner & Simons, 2005]. The release of heat from the earth’s core causes convection in the mantle, which constitutes the driving force behind plate tectonic motions influenced by the coupling between two systems: the mantle and the lithospheric crust. From this point of view, the earthquake dynamics can be modeled as an input–output dynamics [Pavlos et al., 2007]. The results of the nonlinear analysis of seismic data presented in this paper can support such a concept, too.

2.2. Seismic dripping

According to Pavlos et al. [1994], seismic events can be considered as the drop formation in a dripping faucet experiment, as shown in Fig. 1. The mechanical analogue of the dripping faucet model given by Shaw [1984] consists of a variable mass hanging on a spring. Shaw’s dripping faucet was modified by

\[
\begin{align*}
\frac{dP}{dt} &= Gm - kD \\
\frac{dD}{dt} &= P \\
\frac{dm}{dt} &= m(t)
\end{align*}
\]

Fig. 1. (a) Drop formation in the dripping faucet. (b) The mechanical analogue model of the dripping faucet process. (c) Modeling dissipative loading-unloading processes of magnetospheric substorms, as an input–output threshold system. (d) Inter-event time intervals (\(\Delta t_n\)) between one drop and the next one. (e) Inter-event time intervals (\(\Delta t_n\)) between successive earthquakes.
Baker et al. [1990] to describe dissipative loading-unloading processes of magnetospheric substorms, as an input-output threshold system [Fig. 1(c)]. As we describe below, the earthquake fault networks are also an input-output driven threshold system and can be modeled by a process similar to the dripping faucet [Fig. 1(e)].

The dripping faucet model is well known for illustrating the appearance of chaotic behavior in nonlinear systems, where time intervals between successive drop detachments are used to reconstruct the dynamics of the system [Fig. 1(d)]. At low dripping rates the system is periodic, while above a critical dripping rate the system exhibits chaotic behavior characterized by qualitatively different types of strange attractors. Using inter-event time intervals between one drop and the next one, the reconstructed dynamics reveals low dimensional deterministic behavior in the reconstructed state space. The distribution function of the drip intervals depends upon the flow rate following the scenario of period doubling and the onset of chaotic behavior [Neda et al., 1995]. Concerning a seismic process, the loading rate, \( m(t) \), of mass in the mechanistic dripping faucet model of Shaw corresponds to the transfer of stress in the fault system by the mantle and plate tectonic dynamics (the external driver of the system), while mass unloading corresponds to earthquakes, as releases of the elastic strain energy stored along a fault. The dripping faucet similarly to the earthquake process can be understood as a local, driven, threshold process.

2.3. Power laws, scaling and dimensionality in the seismic process

Generally, the temporal sequence of earthquakes can be described by a point process characterized by a power spectrum density \( S(f) \) decaying as \( f^{-\beta} \) at low frequencies. If the point process is Poissonian, occurrence times are uncorrelated and \( \beta \approx 0 \), while the probability density function follows an exponential law, \( P(t) \approx e^{-\lambda t} \), where \( \lambda \) is the mean rate of the process. On the other hand, when \( \beta \neq 0 \), a self-similar point process exists, which has a decreasing power-law inter-event probability density \( P(t) \approx t^{-(1+\beta)} \) [Telesca et al., 2002].

The SOC theory of Bak et al. [1988] was proposed to explain power laws in the seismic process. The SOC theory implies that the Earth’s crust subjected to the pressure from tectonic plate motion may be seen as a dissipative system with infinitely many degrees of freedom, able to produce avalanche events. Yang et al. [2004] argued that earthquakes are unlikely examples of SOC, showing that the first return time probability \( P_n(T) \) exhibits power law distribution. This is fundamentally inconsistent with SOC models, which describe processes with Poissonian density functions. In this way, these authors cast doubt on the applicability of SOC to earthquake prediction.

Corral [2005] suggested that while the Bak-Tang-Wiesenfeld model indeed displays a Poissonian distribution of recurrence times, other models of SOC can reveal different recurrence time distributions. In this direction, Woodard et al. [2007] showed that correlated dynamics and long time memory effects can be present in self-organized critical systems.

Following this observation, Corral [2005] and Woodard et al. [2007] argued that the existence of temporal correlations cannot contradict the notion of a SOC process. However, although SOC processes can sustain temporal correlations, they cannot reveal low dimensionality. According to Bak et al. [1988], the SOC system lives in a “critical state”, which represents an attractor for the dynamics. However, this is very different from a low dimensional chaotic attractor, since in the case of a SOC attractor the number of degrees of freedom is proportional to the size of the system. Also, concerning high dimensional SOC attractors, uncertainty grows with time according to a power law, while the uncertainty in a low dimensional chaotic attractor increases exponentially.

This implies the existence of zero Lyapunov exponents for the case of SOC, while it is well known that low dimensional chaos requires at least one positive Lyapunov exponent. From this point of view, the SOC dynamics evolves on the border of chaos, a behavior called weak chaos [Bak & Chen, 1991]. Moreover, critical points and changes in the attractor properties of nonequilibrium low dimensional dynamical systems can be analyzed only by fine tuning the control parameters. In contrast, for SOC systems, no fine tuning is needed as these systems drive themselves to a critical state (attractor) with a wide range of length and time scales.

On the other hand, SOC has been pointed out as the principal characteristic of seismogenesis in many models and simulations of earthquake dynamics. More specifically, the OFC (Olami–Feder–Christensen) type of models reveal SOC features,
foreshock and aftershock clustering, aftershock diffusion and foreshock migration, scaling laws for the average distance between aftershocks and the mainshocks epicenter, foreshock magnitude distribution, distribution of foreshocks and aftershocks per mainshock, in accordance with real data [Helmstetter et al., 2004]. In the same direction, the epidemic type aftershock sequence (ETAS) model, which is a branching model of earthquake interactions can be considered as mean-field approximation of SOC or OFC models [Sornette, 2004].

The most widely accepted view is that seismogenesis includes two separate processes, one for mainshocks, which often follows a Poissonian distribution and an independent one generating aftershocks [Corral, 2007]. Furthermore, according to a generalized Poisson (GP) model, earthquakes are grouped into temporal clusters of events, which are uncorrelated, while intra-cluster earthquakes are correlated according to Omori’s law. Mega et al. [2003] showed the existence of intercluster correlations, which indicates a geophysical process generating clusters with memory by using diffusion entropy analysis. This is in agreement with SOC theory which views the geophysical system as a whole, irrespective of tectonic features and places all events, which corresponds to scale free critical dynamics in the crust of the Earth. However, in a series of studies, these same models used to simulate earthquakes as a SOC process were also able to reveal low dimensional chaos [Nakouskina et al., 1992; Viera & Lichtenberg, 1996; Viera, 1999; Montagne et al., 2004; Anghel, 2004; Pavlos et al., 2007]. Thus, the obvious question is whether SOC theory is the only one consistent with the above power law processes, or, as we claim, low dimensional chaos must also be used to provide a more complete framework for understanding seismogenesis in widely distributed domains, such as the Hellenic region.

3. The Methodology of Nonlinear Analysis of Seismic Data

In this section we present the algorithms and methods we shall use in this paper to analyze seismic data. The main purpose of experimental time series analysis is to extract significant information for the underlying dynamics of the observed signal, as well as to develop effective methods for modeling and prediction. Classical time series analysis confronts these problems using linear or nonlinear input–output methods [Priestley, 1988]. On the other hand, modern nonlinear chaotic analysis includes:

(a) Estimation of geometrical and dynamical characteristics of the system in its state space.
(b) Testing techniques for the discrimination between low dimensional nonlinear deterministic and linear stochastic process, as well as the existence of input–output deterministic dynamics.
(c) Forecasting algorithms (for a review of the chaotic analysis algorithm see [Kantz & Schreiber, 1997; Pavlos et al., 1999]).

3.1. The reconstructed state space

Modern analysis of seismic processes is based on the reconstructed dynamics for autonomous and purely deterministic systems with $n$ dynamical variables, according to which a delay reconstruction map $\Phi$ is considered which maps the state $X$ into the $m$-dimensional delay vectors

$$\Phi(x) = [h(x), h(f(x)), \ldots, h(f^{m-1}(x))],$$

$$x(t) \in \mathbb{R}^n.$$  (1)

This map is an embedding when $m \geq 2n + 1$, where $f$ describes the dynamical flow underlying the observed signal and $n$ is the dimension of the manifold of the system phase space dynamics [Takens, 1981; Broomhead & King, 1986]. The embedding $\Phi$ is a diffeomorphism which maps the orbits of the original state space in the reconstructed space, preserving their orientation and dynamical and geometrical characteristics, such as Lyapunov exponents and dimension of attractors, respectively.

In the reconstructed state space, we introduce also the SVD (Singular Value Decomposition) analysis and use it to: (i) filter the time series and (ii) decompose the series in its SVD reconstructed components which can be used for the detection of the underlying dynamics. Singular value analysis is
applied to the trajectory matrix $X$ estimated by the reconstructed state space [Broomehead & King, 1986; Pavlos et al., 1999]. The SVD analysis permits the reconstruction of the original flow, in terms of $n$ eigenvectors $V_i$, known as SVD reconstructed components corresponding to the spectrum of the singular values $\{\sigma_i\}$. The $d$ largest ones of them correspond to the $V_i$ eigenvectors, $i = 1, 2, \ldots, d$, that are sufficient for an accurate description of the underlying dynamics.

According to the above concepts, the methods of nonlinear analysis can now be applied in the reconstructed state space, as described below.

3.2. Geometrical characteristics of the reconstructed dynamics

The geometrical characteristics of the reconstructed dynamics include:

(a) The correlation dimension

$$D_m = \lim_{r \to \infty} \frac{d \ln C_m(r)}{d \ln(r)}$$

of the dynamical trajectories of the system in state space, where $C_m(r)$ is the correlation integral of the trajectory and $D_m$ is its slope. The low value saturation $D = \lim_{m \to \infty} D_m$ of the slopes of the correlation integrals is related to the number $d$ of fundamental coordinates of the internal dynamics. For the estimation of the correlation integral we used the algorithm of Grassberger and Procaccia [1983].

(b) The spectrum of the distinct nonzero eigenvalues $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_n \geq 0$ of the structure matrix constructed by the trajectory matrix estimated in the reconstructed state space according to SVD theory. For signals perturbed by white noise, we use the distinct eigenvalues $\{\sigma_i\}$, which lie above the noise plateau.

We have also used the method of Theiler [1991] to exclude time correlated states in the correlation integral estimation, thus discriminating between the dynamical character of the correlation integral scaling and the low value saturation of slopes characterizing self-affinity (or crinkliness) of trajectories in a Brownian process. When the dynamics possesses a finite (small) number of degrees of freedom, we observe saturation to low values $D$ of the slopes $D_m$ obtained in (2) for a sufficiently large embedding $m$. The dimension of the attractor of the dynamics is then at least the smallest integer larger than $D$ or at most $2D + 1$, according to Takens’ theorem.

3.3. Dynamical characteristics of the underlying dynamics

As is well-known, a nonlinear dissipative system can reveal rich dynamics, including simple attractors, such as limit points, limit cycles, quasi-periodic tori or strange (chaotic) attractors. The existence of a strange attractor underlying the observed signal can then be tested by estimating the dynamical characteristics, such as:

(a) The spectrum of Lyapunov exponents which measure the rate of convergence or divergence of nearby trajectories in all directions of the phase space. The Lyapunov spectrum is found by the derivative matrix of the dynamics in the reconstructed state space. We construct the spectrum of Lyapunov exponents ordered as the $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_d$ by the relation

$$\lambda_j = \sum_{i=1}^{N} \log(\|A_i e_j\|)$$

where $A_i$ is the local approximation of the derivative matrix $DF$ at the reconstructed trajectory points $x_i(t_i), i = 1, 2, \ldots, N$ of the $d$-dimensional reconstructed state space and $e_j$ is a corresponding set of orthogonal vectors at the points $x_i(t_i)$. We follow Wolf et al. [1985] for the independent estimation of the largest ($L_{\text{max}}$) Lyapunov exponent according to $L_{\text{max}} = \lim_{t \to \infty} \frac{1}{t} \log(d(t)/d(0))$, $t \to \infty$ and $d(0) \to 0$ where $d(t)$ measures the separation between neighboring points in the reconstructed phase space.

(b) The mutual information and the autocorrelation function.

The autocorrelation function $C(\tau)$ is related to the power spectrum of the signal, according to the Wiener–Khintchine theorem and decays rapidly in time when the underlying dynamics evolves on a dynamical fractal subset of the dynamical state space. However, the autocorrelation function cannot discriminate deterministic chaos from colored noise with identical power spectrum, as it is only a linear estimator of the observed dynamics. A non-linear estimator of the dynamical evolution in the state space is the mutual information, which can reveal the fact that the observed dynamics agrees with Shannon’s concept of an ergodic information
The mutual information $I_{SQ}$ between two observables, $S$ and $Q$ in the underlying dynamics is given by the relation

$$I_{SQ} = H(S) - H\left(\frac{S}{Q}\right)$$

$$= H(S) + H(Q) - H(Q, S) \quad (4)$$

where $H(S)$ is the amount of average information gained from a measurement of $S$ and $H(S/Q)$ is the amount of information of $S$ given that $Q$ is known. If this relation is applied to time series, it leads to:

$$I(\tau) = -\sum_{x(i)} P(x(i)) \log_2 P(x(i))$$

$$- \sum_{x(i-\tau)} P(x(i-\tau)) \log_2 P(x(i-\tau))$$

$$+ \sum_{x(i)} \sum_{x(i-\tau)} P(x(i), x(i-\tau))$$

$$\times \log_2 P(x(i), x(i-\tau)) \quad (5)$$

If the samples $\{Q \equiv x(i)\}$ and $\{S \equiv x(i+\tau)\}$ are statistically independent then the mutual information will vanish for this value of $\tau$. Thus, knowledge of the second sample cannot be gained by knowing the first. On the other hand, if the first sample uniquely determines the second sample then $I(\tau) = I_{max}$, which is true when $\tau = 0$. In this paper, we follow the work of Fraser and Swinney [1986] to estimate the mutual information of an experimental time series and use it as a discriminating statistic between the surrogate data and the seismic time series as well as an indicator of the delay time $\tau$.

For the estimation of the best reconstruction time $\tau$ we use the first local minimum of the auto-correlation coefficient and the mutual information, as well as higher values of $\tau$ for which the estimated geometrical and dynamical characteristics of the reconstructed attractor remain invariable.

### 3.4. Modeling and prediction

The observed time series $X(t_i) \equiv X(i), i = 1, 2, \ldots, N$, resulting from the temporal evolution of an orbit in the reconstructed phase space can be used for modeling and prediction of the seismic process. The mirrored dynamics in the reconstructed phase space $X(i+1) = F(X(i))$ can be used for the estimation of the predictor $F^T, T$ time steps ahead according to the relation

$$\hat{X}(i+T) = F^T(X(i)). \quad (6)$$

The map $F^T(X(i))$ may be approximated by different functional forms of global, local or semi-local type [Lillekjendlie et al., 1994]. To verify the performance of this approach, two measures of the modeling or prediction error are computed. The first one is the normalized root mean square error (NRMSE), given by the relation:

$$\text{NRMSE} = \sqrt{\frac{\sum_{k=1}^{M} [x(k) - \bar{x}(k)]^2}{\sum_{k=1}^{M} [x(k) - \bar{x}]^2}} \quad (7)$$

where $M$ is the amount of the predicted values and $\bar{x}$ the mean value of the original data. If NRMSE $\cong 0$ perfect model performance is achieved, while if NRMSE $> 1$ the modeling or prediction is lower than the one obtained using the mean value as a model. Another measure is the correlation coefficient (CC), which characterizes the actual data. An estimate of CC is obtained from the ratio of the covariance over the root of the product of the variance of the two data sets:

$$\text{CC} = \frac{\sum_{k=1}^{M} [x(k) - \bar{x}][x'(k) - \bar{x}']}{\sqrt{\sum_{k=1}^{M} [x(k) - \bar{x}]^2} \sqrt{\sum_{k=1}^{M} [x'(k) - \bar{x}']^2}} \quad (8)$$

where $\bar{x}$ and $\bar{x'}$ are the mean values of real and predicted values, correspondingly. It is known that CC takes values in $[-1, 1]$. When CC $= 1$ best correlations are obtained, i.e. the performance of the model is excellent, while for values of CC close to zero or negative, the performance is very poor [Pavlos et al., 2003].

In the case of linear models which describe deterministic systems, nearby trajectories evolve similarly, at least for a short time if the system is chaotic. Thus, on the constructed attractor at any point $X(i)$, one can locally approximate $F^T$ and use it to estimate $X(i+T)$ taking into account the $k$ nearest neighbors of $X(i)$. The local approximation of $F^T$ can be established with a linear map of the form $\hat{x}(i+T) = a_o + a^tX(i)$ [Farmer & Sidorowich, 1987; Casdagli et al., 1992].
A simple nonlinear approximation of $F^T$, on the other hand, may be achieved by a polynomial which involves linear and nonlinear terms or linear plus nonlinear terms of degree $q$. A polynomial of degree $q = 2$ in $m$ delay variables is given by the Volterra–

$$X(i + T) = a_0 + a_1 x(i) + a_2 x(i - \tau) + \cdots + a_m x(i - (m - 1) \tau) + \cdots + a_{m+1} x(i)^2 + a_{m+2} x(i) x(i - \tau) + \cdots + a_M x(i - (m - 1) \tau)^2$$

where $M = (m + q)!/m!$. Of course, a polynomial of low degree cannot model complex dynamics and for purely chaotic systems it is more likely to be insufficient. However, in practice, the evident dynamics (linear or nonlinear) is contained in the largest terms, as higher order nonlinearities may be obscured by noise [Barahona & Poon, 1996]. In our case, we use $q = 2$ because we are interested only in investigating the existence of nonlinearity in the data.

### 3.5. Surrogate data analysis

The method of surrogate data is used to distinguish between linearity and nonlinearity as well as between chaoticity and pure stochasticity, since a linear stochastic signal can mimic a nonlinear chaotic process after a static nonlinear distortion [Theiler et al., 1992a, 1992b]. Surrogate data are constructed according to Schreiber and Schmitz [1996] to mimic the original data, regarding their autocorrelation and amplitude distribution.

In particular, the procedure starts with a white noise signal, in which the Fourier amplitudes are replaced by the corresponding amplitudes of the original data. In the second step, the rank order of the derived stochastic signal is used to reorder the original time series. By doing this, the amplitude distribution is preserved, but the matching of the two power spectra achieved at the first step is altered. The two steps are subsequently repeated several times until the change in the matching of the power spectra is sufficiently small. Surrogate data thus provide the most general type of nonlinear stochastic signals that can approach the geometrical or dynamical characteristics of the original data. They can be used for the rejection of every null hypothesis that identifies the observed low dimensional chaos as a purely nonchaotic stochastic linear process. For an extensive description of the nonlinear analysis algorithm see also [Pavlos et al., 1999].

In order to distinguish a nonlinear deterministic process from a linear stochastic one, we use as discriminating statistic a quantity $Q$ derived from a method sensitive to nonlinearity, for example the correlation dimension, the maximum Lyapunov exponent, the mutual information, etc. The discriminating statistic $Q$ is then calculated for the original and the surrogate data and the null hypothesis is verified or rejected depending on the “number of sigmas”

$$S = \text{Integer part} \left[ \frac{\mu_{\text{obs}} - \mu_{\text{sur}}}{\sigma_{\text{sur}}} \right]$$

where $\mu_{\text{sur}}$ and $\sigma_{\text{sur}}$ are the mean value and standard deviation of $Q$ taken from the surrogate data and $\mu_{\text{obs}}$ is the mean value of $Q$ derived from the original data. For a single time series, $\mu_{\text{obs}}$ is the single $Q$ value [Theiler et al., 1992a]. The significance of the statistics is a dimensionless quantity and we report it in terms of units of $S$ “sigmas”. When $S$ takes values higher than 2–3 then the probability that the observed time series does not belong to the same family with its surrogate data is higher than 0.95–0.99, correspondingly.

### 4. Estimation of Geometrical and Dynamical Characteristics

In this section, we present results concerning the geometrical and dynamical characteristics of the underlying dynamics of the seismic time series created following a conjecture introduced by Pavlos et al. [1994], i.e. earthquakes fall into the category of a dripping faucet. Figure 2 summarizes the main physical concepts underlying the nonlinear analysis of seismic data and represents the main components of the earthquake process including the fault dynamics described by the flow $F(x, z, w)$ in its state space, the external energy input $x(t)$ caused by the plate tectonics, the stochastic input of noise caused by microscopic randomness and the output signal process according to the embedding theory.

Figure 3(a) shows the inter-event time series of earthquake events in the Hellenic region of magnitude over 3.3 Richter. This time series contains $N = 11\,837$ data points and was constructed using the bulletins of the National Observatory of Athens during the period 1968–1993. Figures 3(b) and 3(c) show the first ($V_1$) SVD component and
the reconstructed time series, using the components $V_{2-s}$, following $V_1$. Figure 3(d) shows the autocorrelation coefficients for the original seismic time series, the first SVD component $V_1$ and the $V_{2-s}$ of the reconstructed time series. The autocorrelation functions reveal an abrupt decay during the first ten time steps for the original and the reconstructed $V_{2-s}$ time series and during the first 10-20 time steps for the first SVD component. We can see that the $V_1$ component reveals lower rate of decorrelation than the original and the SVD reconstructed $V_{2-s}$ time series, a result that indicates the existence of two different dynamical components, one corresponding to the $V_1$ SVD component and the other to the $V_{2-s}$ reconstructed time series.

Figures 3(e)–3(h) are similar to Figs. 3(a)–3(d) but represent surrogate samples of time series used for testing the null hypothesis of purely linear stochastic process. The similarity of the corresponding surrogates with the original time series is due to the robustness of the method we used for the construction of the surrogates to mimic the original data in their linear characteristics (see Sec. 2.5).

4.2. Geometrical characteristics

Figure 5 presents the geometrical characteristics of the seismic time series and its SVD components described by the slopes of the correlation integrals and the singular value spectrum. Slopes of the correlation integrals estimated for surrogate data are presented also. Figure 5(a) shows the slope $D$ of the correlation integral at its scaling region for the seismic time series and its surrogate data as function of delay time ($\tau$) that was used for the reconstruction of an $m = 8$-dimensional state space. The Theiler parameter ($w$) for the exclusion of time correlated states becomes equal to $\tau$, for every value of $\tau$, larger than the correlation time of the signal. The slope value $D_m$ at the scaling region of the correlation integral is constant and coincides with the scaling exponent of the correlation integral according to the relation $C(\tau, m) \sim \tau^{D_m}$. The scaling exponent of the surrogate data also remains constant and strictly higher than the scaling exponent of seismic data, as the delay increases from $\tau = 2$-25 time steps. At the time step $\tau = 25$
Fig. 3. (a) The inter-event time series of earthquakes which occurred in the Hellenic region. (b) Time series corresponding to \( V_1 \) SVD component. (c) Time series corresponding to \( V_{2-8} \) reconstructed time series. (d) The autocorrelation coefficient as a function of lag time \( k \) for the original time series and its SVD components. (e)-(h) Similar to (a)-(d) but for a surrogate sample time series.
Fig. 4. (a)–(f) The logarithm of the structure function estimated for the Lorenz model time series, for the inter-event times, for the $V_1$ and $V_2$ SVD components of the earthquake inter-event times, for a surrogate sample series and a white noise series.
the scaling exponents of seismic and surrogate data become equal. This profile of scaling exponents is in agreement with the hypothesis of existence of a low dimensional strange attractor for the seismic process as the geometrical structure of the trajectory in the reconstructed space depends on the delay time while the trajectory structure of the stochastic (surrogate) data is independent of ∇.

Figures 5(b)–5(d) present the slopes of the correlation integrals of seismic and surrogate data for the original and its SVD components $V_1, V_2, \ldots$. The slopes were estimated in $m$-dimensional reconstructed state space using embedding dimensions $m = 7–10$ for the original inter-event time series as well as for their SVD components $V_1$ and $V_2, \ldots$. As we observe in these figures, the seismic time series and its SVD components reveal a low value saturation of the slopes ($D \approx 2–3$), indicating 3–7 dynamical degrees of freedom, according to the embedding theory, while this is not true for the surrogate data.

![Graphs showing correlation dimension and slopes of correlation integrals](https://www.worldscientific.com/doi/abs/10.1142/9789814583372_0023)
The null hypothesis of the seismic process as a linear stochastic one was further tested using a rich sample of surrogate data (30) for the discriminating statistic of slopes. The significance of the statistics is shown in Fig. 5(e), has values higher than two sigmas. So the null hypothesis in this case can be rejected with confidence more than 95–99%.

Figure 5(f) shows the normalized singular values spectra estimated for the inter-event seismic time series as well as for its three main SVD components, $V_1$, $V_2$, $V_3$, which capture the significant physical information of the original signal. For the estimation of the singular values spectra we used a fixed window length $\tau_m = 8.0\tau = 8$. The lower and upper limits for $\tau_c$ are given by the relation $\tau_c < \tau_c < 4\tau_c$, where $\tau_c$ is the decorrelation time estimated by the autocorrelation function [Albano et al., 1988]. As can be seen from this figure, the first singular value $\sigma_1$ of the inter-event time series is much larger than the next, $\sigma_i, i \geq 2$. After the second value, the normalized singular values are suppressed to the noise level. This singular value profile indicates the existence of one prominent dynamical component corresponding to the first singular value $\sigma_1$. However, the estimation of the normalized singular values spectra for the first three SVD components $V_1, V_2, V_3$, reveals two different spectra profiles. The spectrum of the $V_1$ SVD component is similar to inter-event time series spectrum with one prominent singular value $\sigma_1$ while the next values are suppressed to zero level. In contrast, the spectra of the next SVD components $(V_2, V_3)$ reveal a normal pattern with 3–4 significant nonzero singular values.

As we have shown in previous studies [Athanasiu & Pavlos, 2001; Pavlos et al., 2007] this phenomenon can be observed when the original signal includes two distinct underlying physical processes related to the first ($V_1$) and next SVD signals correspondingly. This result is also in agreement with the observed differentiation of the autocorrelation profile of $V_1$ and $V_2, V_3$ SVD components, shown in Fig. 3(d).

4.3. Dynamical characteristics

4.3.1. Mutual information

Figures 6(a)–6(c) show the mutual information as a function of lag time for the seismic time series, the surrogate data and their SVD components, estimated according to Frazer & Swinney [1986]. Mutual information is seen to decay to zero very similarly for the seismic data and their surrogates. However, the decay rate is much slower for the first SVD component, $V_1$, of the seismic and surrogate data, compared with the original time series and its $V_{2–8}$ SVD components. The mutual information of the original seismic time series and its $V_{2–8}$ components falls to zero after the first five lag times, while the $V_1$ SVD component does so after the $\sim 10–20$ first lag times.

The nonlinear dynamics of the seismic process can be supported by estimating the significance of statistics shown in Figs. 6(d)–6(f), concerning mutual information. In the case of the original signal, the significance of the statistics remains higher than 2 sigmas during the first $\sim 10$ lag times, except for two values of $\tau$ such as $\tau = 2.6$ for which there is no significant difference from the surrogates, indicating the presence of noise. However, filtering the data using only the SVD components,
Fig. 6. (a) Mutual information estimates for the earthquake time series and its 30 surrogates as a function of the lag time. (b)–(c) Similar to (a) but for $V_1$ component and $V_{2-8}$ reconstructed series, respectively. (d) The significance of the discriminating statistic of the mutual information as a function of the lag time shown in (a). (e)–(f) Similar to (d) but for $V_1$ component and $V_{2-8}$ reconstructed series, respectively.
the significance of the statistics, shown in Figs. 6(a) and 6(f), attains values higher than ~5–10 sigmas during the ~50 first lag times. These results support the hypothesis of dynamical nonlinearity and nonstochasticity of the seismic process with high confidence, as well as the existence of two distinct processes corresponding to $V_1$ SVD component and to the $V_{2-s}$ reconstructed series.

4.3.2 Largest Lyapunov exponent

The largest Lyapunov exponent was estimated according to the algorithm of Wolf et al. [1985] for the seismic time series and its SVD components as well as for a sample of 30 surrogate data. Figure 7(a) shows the ($L_{\text{max}}$) Lyapunov exponent estimated for the original inter-event time series and the SVD components of the reconstructed signal. The $L_{\text{max}}$ values where found to be $\sim 0,75 \text{bits/event}$ for the $(V_1)$ SVD component, $\sim 1,1 \text{bits/event}$ for the SVD reconstructed $(V_{2-s})$ seismic signal and $\sim 1,5 \text{bits/event}$ for the original seismic signal. These positive values were found to be significantly lower than surrogate linear stochastic data as we observe in Figs. 7(b)–7(d). The discrimination between the seismic data and their surrogate for the $L_{\text{max}}$ statistics is possible as the significance of the statistical test remains higher than $\sim 2-3$ sigmas, see Fig. 7(e). This result clearly permits the rejection of the null hypothesis with high confidence. The low dimensional nonlinearity of the dynamics shown previously and the positive value of the $L_{\text{max}}$ coefficient, indicate clearly the chaotic character of seismic process underlying the original data, as well as its SVD components.

4.3.3 Spectrum of Lyapunov exponents

It is known that Wolf’s et al. [1985] algorithm is not very robust as it yields a finite exponent for stochastic data also, where the true exponent is infinite [Kantz & Schreiber, 1997]. In order to further strengthen our results concerning the estimation of the Maximum Lyapunov Exponent we used another class of algorithms [Sano & Sawada, 1985; Eckmann et al., 1986]. In particular, we constructed the spectrum of Lyapunov exponents as a function of the embedding dimension ($m$). The Lyapunov spectrum is of high importance as it measures the rate of convergence or divergence of close trajectories in all $d$ directions of the phase space, giving further evidence for the possible low dimensional and chaotic character of the attractor in the reconstructed phase space. In particular, positive exponents correspond to instability, negative exponents to convergence, while at least one zero exponent must exist for the expansion along the trajectory.

Figures 8(a)–8(c) present the Lyapunov exponent spectrum for the original seismic time series, its $(V_1)$ SVD component and the $(V_{2-s})$ reconstructed seismic signal. Figures 8(d)–8(f) are similar to Figs. 8(a)–8(c) but correspond to surrogate data of the inter-event time series, the $V_1$ component and the $(V_{2-s})$ reconstructed series, correspondingly. The first Lyapunov exponent was found positive $(L_1 > 0)$, the second was found to be zero $(L_2 = 0)$ and the others were found to be negative $(L_j < 0)$ for the three cases of seismic data. This result is in agreement with the hypothesis of a low dimensional seismic strange attractor [Tsonis, 1992]. On the other hand, the surrogate data reveal 2–3 positive Lyapunov exponents, indicating a high degree of chaos [Argyris et al., 1998].

4.4. Modeling and prediction

In the following section we present crucial results concerning the cross correlation coefficients (CC) and the normalized root mean square differences (NRMSE) according to the prediction and modeling methods described previously.

4.4.1. Local linear prediction (LLP)

Figure 9(a) presents the correlation coefficient between the real and the predicted values estimated for the original inter-event seismic time series, its first $(V_1)$ SVD component and the $(V_{2-s})$ reconstructed seismic signal. As we can see the estimated (CC) coefficient decays as the prediction step increases, indicating the deterministic character of inter-event seismic time series, as well as the $V_1$ SVD component and the $(V_{2-s})$ reconstructed series. Moreover, the $V_1$ component is much more predictable than the inter-event time series and the $(V_{2-s})$ reconstructed series, a result that indicates the difference between the two SVD components.

Figure 9(b) is similar to Fig. 9(a) and shows the inter-event seismic time series and its surrogate data. The stochastic character of surrogate data is in agreement with their weaker predictability as their (CC) coefficients remain at lower values than in the case of seismic data. Figure 9(c) presents the logarithm of NRMSE of the modeled
Fig. 7. (a) The largest Lyapunov exponent ($\lambda_{\text{max}}$) as a function of the seismic events, for the seismic time series and its SVD components. (b) The largest Lyapunov exponent for 30 surrogate data and the earthquake inter-event time series as a function of events estimated for delay time $\tau = 5$ and embedding dimension $m = 6$. (c)–(d) Similar to (b) but for the $V_1$-component and the $V_2-8$ reconstructed time series of the earthquake time series with embedding dimension $m = 6$ and corresponding delay time $\tau = 5$, respectively. (e) The significance of the discriminating statistic of the largest Lyapunov exponent as a function of the events, for the seismic signal and its SVD components.
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Figure 8. (a) The spectrum of the first five Lyapunov exponents estimated for the earthquake time series, as a function of the embedding dimension $m$, using delay time $\tau = 5$. (b)–(c) Similar to (a) but for the $V_1$-component and reconstructed, $V_{2-8}$, time series of the earthquake time series, using delay time $\tau = 16$ and $\tau = 1$, correspondingly. (d)–(f) Similar to (a)–(c) but for the surrogate time series of the seismic time series, for the $V_1$ SVD component and the $V_{2-8}$ reconstructed series, correspondingly.
or predicted values from the actual values as function of the first 100, 1000 and 10 000 training values of the inter-event seismic time series and its SVD component $V_1$ and the $V_2 - 8$ reconstructed seismic signal.

Observe the decrease of NRMSE values, as the training set width of seismic data increases. Obviously the performance of the local linear prediction method is clearly improved as we increase the training data set. This is in accordance with the deterministic and chaotic character of seismic data [Farmer & Sidorowich, 1987; Smith, 1992].

Moreover, Fig. 9(d) is similar to Fig. 9(c) but in this case we compare the seismic interevent time series with surrogate data. As we can see, the predictability of surrogate data cannot be improved by increasing the training data set, a result that discriminates the seismic inter-event time series from the stochastic surrogate.

4.4.2. Global polynomial fitting

The modeling with global polynomials provides efficient discrimination between the original and
surrogate data, especially when we define the discrimination statistics to be the difference of the modeling error as we pass from linear to nonlinear polynomial terms. We use the NRMSE to quantify the modeling error and consider 1 step ahead mappings from all the polynomials of the Volterra–Wiener series. In Figs. 10(a)–10(c) the NRMSE for 30 surrogate data and the original seismic data, with their SVD components, is shown as a function of the polynomial terms of the Volterra–Wiener series by using memory $m = 6$ and degree $q = 2$ in Eq. (9), described in Sec. 3.4.

The first seven polynomial terms are linear and the rest are nonlinear interactions. Although, we observe similar decay profile of the NRMSE in the linear region of the seismic and surrogate data modeling, in the nonlinear region we observe an abrupt and strong decay of the NRMSE only for the seismic data and its SVD components. Namely, there exists a noticeable nonlinearity for the seismic time series and its SVD components indicated by the reduction of the error as we pass from the linear to the nonlinear fitting. This feature is almost absent from the surrogate data.

![Graphs showing the NRMSE](image)

Fig. 10. (a) Normal root mean squares error (NRMSE) estimated by using polynomial fitting for the earthquake time series and its surrogate data, as a function of the polynomial terms, of the Volterra–Wiener model estimated for parameters $m = 6$, $\tau = 5$, $q = 2$ and 1 steps ahead mappings. The first seven terms are linear and the next terms are nonlinear. (b) Similar to (a) but for the $V_1$ component using parameters $m = 6$, $\tau = 8$, $q = 2$ and 1 steps ahead mappings. (c) Similar to (a) but for the $V_{2-8}$ reconstructed time series using parameters $m = 6$, $\tau = 5$, $q = 2$ and 1 steps ahead mappings. (d)–(f) Significance of the reduction error statistic corresponding to the difference of the polynomial fitting of the first seven linear terms from the polynomial fitting containing all the possible linear–nonlinear terms, for the three time series and its surrogate data shown in (a)–(c).
The significance of the NRMSE discriminating statistics, as function of the polynomial terms, is shown in Figs. 10(d)–10(f) for the original seismic data and the $V_1, V_{2-8}$ SVD components. The significance of the statistics corresponding to the inter-event time series, shown in Fig. 10(d), reveals abrupt growth at the position of the first nonlinear term at values larger than 2 sigmas. Using the SVD components, as it can be seen in Figs. 9(c)–9(f), the significance of the statistics attains values greater than 8 sigmas, further revealing the nonlinearity in the data. These results of polynomial modeling clearly provide further evidence supporting the nonlinear character of the underlying seismic dynamics, with high confidence. Also, the discrimination of $V_1$ and $V_{2-8}$ component is evident as the nonlinear character of the $V_{2-8}$ is clearly stronger than the $V_1$ case.

5. Summary of Data Analysis and Results

In this study, we have applied an extensive methodology to an inter-event seismic time series concerning the Hellenic region, which included estimation of structure functions, geometrical and dynamical characteristics as well as surrogate data for testing the null hypothesis regarding linear stochasticity. Our results are summarized as follows:

(a) The correlation dimension estimated by the slopes of the correlation integrals, constructed for the trajectory in the reconstructed state space, was found to be lower than $m = 4$ for the original seismic time series and its SVD components, indicating 3–7 dynamical degrees of freedom.

(b) The apparent chaotic character of the seismic dynamics was found also in every case of the analyzed seismic signals, due to the existence of positive Lyapunov exponents estimated for the reconstructed trajectories in the state space of seismic dynamics.

(c) The number of independent variables of the seismic dynamics estimated by using the singular value spectrum was found to be $\sim 4$. This result is in agreement with the corresponding result estimated by using the correlation dimension method. In this case, the saturation value of the slopes was found to be lower than 4.

(d) The spectrum of the Lyapunov exponents for the original time series and the SVD components has a typical strange attractor profile.

(e) The null hypothesis of nonlinear stochasticity, which might be due to static nonlinear distortion of a white noise process efficient to mimic the seismic dynamics, is rejected with 95–99% confidence in every case of discrimination statistics that was used in relation with the geometrical or dynamical characteristics.

(f) The SVD analysis shows the existence of two distinct, low dimensional and chaotic, seismic processes related to the $V_1$ SVD component and the second to the higher $V_{2-8}$ reconstructed signal. This was revealed by studying the autocorrelation coefficients, as well as the geometrical and dynamical characteristics of the experimental signals. Moreover, the first eigenvalue of the
V_1 signal suppresses the next ones to the noise floor but the higher SVD components reveal normal profile of singular value spectrum. This character also indicates the physical differentiation of the two SVD components. Finally, the SVD analysis used as a filter further revealed the nonlinear character of the original data.

(g) Nonlinearity was found to be stronger for the \( V_{2..8} \) reconstructed seismic signal than for the \( V_1 \) component.

(h) The predictability of the \( V_1 \) component was found to be higher than for the original or the \( V_{2..8} \) reconstructed seismic signal.

6. Discussion and Theoretical Interpretation

In the previous sections of this study, we presented strong evidence for low dimensional chaos concerning the global phenomenon of seismogenesis in the Hellenic region. These results encourage one to look for a deeper theoretical understanding of the seismic processes. In such a theory, low dimensional scaling laws, until now, have been almost exclusively explained using SOC dynamics. However, the hypothesis of low dimensional chaotic seismic dynamics can also be connected to scaling laws. This can be achieved through the general theory of nonlinear distributed dynamics related to seismogenesis. In such a theory, low dimensional chaos can be connected to a variety of earthquake’s complex phenomena, for example space-time clustering, migration of activity along fault systems, Gutenberg-Richter and Omori’s (foreshock and aftershock) scaling laws.

Seismic scaling laws, until now, have been almost exclusively explained using SOC dynamics. However, according to Rundle et al. [1978b] the earthquake fault dynamics can be related to a Markovian process [Petrosky & Prigogine, 1996].

Furthermore, the earthquake fault dynamics described by Eqs. (12) and (13) belongs to the general type of nonlinear dynamics of stochastic systems for which renormalization group theory, extended for far from equilibrium dynamics, can be applied. In general, far from equilibrium nonlinear stochastic dynamics can be described by a set of generalized Langevin equations [Chang, 1992]:

\[
\frac{\partial \phi(x,t)}{\partial t} = f(x, \phi, t) + n(x, t)
\]

where \( f_i \) \((i = 1, 2, \ldots, n)\) are nonrandom forces corresponding to the functional derivative of the free energy functional, \( x_\mu \) \((\mu = 1, 2, \ldots, d)\) are the spatial coordinates, \( t \) is the time, and \( \phi(x, t) \) represents the stochastic variables which describe the fault dynamics and \( n(x, t) \) are random force fields or noises. According to Chang et al. [1978] the behavior of a nonlinear stochastic system far from equilibrium can be described by the density functional \( P \), defined by

\[
P(\phi(x, t)) = \int D(x) \exp \left\{ -i \cdot \int L(\phi, \phi, x) dx \right\} dt
\]
$L(\dot{\phi}, \phi, x)$ being the stochastic Lagrangian of the system, which describes the full dynamics of the stochastic system.

Moreover, the far from equilibrium renormalization group theory applied to the stochastic Lagrangian $L$ gives the singular points (fixed points) in the affine space of the stochastic distributed system [Chang, 1999]. At the fixed points the system reveals the character of criticality, as near criticality the correlations among the fluctuations of the random dynamic field are extremely long-ranged and there exist many correlation scales. Also, close to dynamic criticality certain linear combinations of the parameters, characterizing the stochastic Lagrangian of the system, correlate with each other in the form of power laws and the stochastic system can be described by a small number of relevant parameters characterizing the truncated system of equations with low or high dimensionality.

According to these theoretical results, the stochastic system can exhibit low dimensional chaos or high dimensional SOC like behavior, including spatiotemporal fractal structures with power law profiles. The power laws are connected to the near criticality phase transition process which creates spatial and temporal correlations as well as strong or weak reduction (self-organization) of the infinite dimensionality corresponding to a spatially distributed system. According to Lyra and Tsallis [1998], the power laws are not caused by the SOC process, but by the nonextensive statistics observed at far from equilibrium process with long range correlations.

From this point of view, a SOC or low dimensional chaos interpretation depends upon the kind of the critical fixed (singular) point in the functional solution space of the system. When the stochastic system is externally driven or perturbed, it can be moved from a particular state of criticality to another characterized by a different fixed point and different dimensionality or scaling laws. Thus, SOC theory could be a special kind of critical dynamics of an externally driven stochastic system [Lu, 1995]. Furthermore, according to Chang [1999] as well as [Veira, 1999], SOC and low dimensional chaos can coexist in the same dynamical system as a process manifested by different kinds of fixed (critical) points in its solution space. Due to this fact, seismogenesis can be described by a high dimensional SOC process or low dimensional chaos or another more general dynamical process depending upon the observed fault system and the local region of the earth’s crust.

As the dynamical system evolves in time (autonomously or under external forcing), the state of the system described by the values of the dynamical parameters in the stochastic Lagrangian $L$, changes as well. The change of the critical state of the system can reveal different dynamical scenarios, as it evolves from one critical state to another, after external tuning. Also, it is possible to reveal local instabilities by creating metastable states which evolve to states of lower energy. This is a local symmetry breaking phenomenon and leads to a local phase transition process. Such local instabilities are connected to avalanche or nucleation dynamics, which can be present in systems that are at mean-field or near-mean-field state, with the possibility of spinodal decomposition process [Klein et al., 2007; Schweiger et al., 2007]. According to the above theoretical approach foreshocks, mainshocks, aftershocks, as well as the plurality of scaling laws or the spatiotemporal correlations of seismogenesis are caused by the same physical mechanism, the manifestations of which can be either low dimensional seismic chaos, seismic SOC or another dynamical scenario.

Moreover, the theory of far from equilibrium critical dynamics can be related to the equilibrium phase transition theory, as both include local metastable states characterized by spinodal lines and spinodal phase transitions. For the case of an earthquake system of faults, the fundamental elastic interactions result in the formation of a mean field regime [Tiampo et al., 2007], where repeated earthquakes reveal that the system is residing permanently in the neighborhood of a spinodal line [Rundle et al., 2003]. The spatial and temporal correlations become large as the fault system approaches a spinodal critical point [Ben-Zion, 2008].

According to this, the driven threshold seismic dynamics arises when a cell subjected to persistent external forcing fails as the failure threshold is reached. In this picture, earthquakes correspond to local avalanches of failed cells, while the avalanches of driven threshold dynamics is caused by a nucleation process known as spinodal nucleation [Klein et al., 2007]. In contrast to the classical nucleation theory, in the case of spinodal nucleation related to the long range interactions of the faults system, the nucleating droplets are noncompact and are ramified [Klein & Unger, 1983]. This nonclassical spinodal nucleation dynamics of the faults system
can be related to the critical dynamics of a nonlinear stochastic field system governed by generalized Langevin equations.

In the case of the Hellenic earthquake system we assume that the plate tectonic forces (which are long-range elastic interactions) drive the whole system of faults into a state of metastable equilibrium, where mean field conditions prevail leading the system near the spinodal line. The dynamics is thus low dimensional and chaotic according to the previous theoretical concepts and the experimental results of this study. After all, we conclude that the low dimensional deterministic and chaotic character of the earthquake process in the global Hellenic fault system proposed in this study constitutes a natural paradigm for the application of modern theoretical concepts concerning the far from equilibrium critical dynamics. In this direction, it is of high interest to extend our future studies in the spatiotemporal features of the seismic process, as well as to search for dynamically different seismic patterns at discrete seismic regions of the Earth’s crust.

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References


