TECHNICAL NOTE

The Impact of Manufacturers’ Wholesale Prices on a Retailer’s Shelf-Space and Pricing Decisions*

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ABSTRACT

This article examines shelf-space allocation and pricing decisions in the marketing channel as the results of a static game played à la Stackelberg between two manufacturers of competing brands and one retailer. The competing manufacturers act as leaders that play a simultaneous and noncooperative game. They fix their transfer prices by taking into account the shelf-space allocation and pricemarkup decisions of their common exclusive dealer. The results indicate that the wholesale prices of brands are strongly linked to their share of the shelf. The main results of our numerical simulations may be summarized as follows: first, the lower the unit cost and/or the greater the price elasticity, the greater the shelf space allocated to that brand. Second, the higher the shelf-space elasticity, the lower are the wholesale prices and the profits of all channel members.

Subject Areas: Distribution Channels, Game Theory, Pricing, Shelf-Space Allocation, and Stackelberg Equilibrium.

INTRODUCTION

According to a 2001 Food Marketing Institute report, approximately 100,000 grocery products are available in the U.S. market, and, every year, thousands of new products are introduced. A typical supermarket has room for only 40,000 products. These numbers suggest that retailers actually face tough choices when allocating

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their limited shelf space to the different product categories and to brands within each category. Clearly, the profitability of carrying different items and customer satisfaction are the driving elements when searching for a solution. These two principles may actually be conflicting and thus, some user-friendly tools have long been developed to assist retailers in such decisions (e.g., APOLLO™, COSMOS®, SPACEMAN®, etc.). These tools are based on some common-sense rules. For instance, COSMOS is based on a rule developed by Buzzell, Salmon, and Vancil (1965) that removes the least profitable item and allocates its space to the most profitable one. Information on APOLLO is available at www.toolboxsolutions.com/apollo.asp and on SPACEMAN at us.acnielsen.com/products/ms.shtml.

Category assortment and the number of facings have a direct effect on consumers’ shopping behavior and thus on retailers’ profits. Corsten and Gruen (2003) reported that approximately 55% of consumers who do not find the brand for which they are looking either do not buy, delay their purchase, or check for the brand in a competing store. Inman and Winner (1998) found that between half and two-thirds of consumers’ brand-choice decisions are made at the point of purchase. These findings make product salience an important variable that influences demand and profit and strongly suggest that optimizing shelf-space allocation is a valuable exercise.

Retailers’ shelf-space-allocation decisions affect manufacturer performance because the availability of the brands and their shelf layout have major impact on brand selection. According to the same study by Corsten and Gruen (2003), 26% of consumers who do not find the sought-out brand buy a competing one. Further, Chandon, Hutchinson, and Young (2002) report that looking at a brand increases its consideration probability by between 30% and 120%. Such results clearly explain why manufacturers are continuously battling for higher shares of the shelf.

The shelf-space-allocation issue has been the subject of a number of studies, which can be schematically divided into two subsets. The first subset focuses on testing whether shelf space has an effect on sales and profitability and computed shelf-space elasticities by using data on sales and shelf space or by conducting experiments where shelf space is the controlled variable (e.g., Curhan, 1973; Drèze, Hoch, & Purk, 1994; Desmet & Renaudin, 1998). Although these studies give different estimates of shelf-space elasticities, they all recognize the positive impact of shelf space on sales and show that the productivity of the latter is subject to marginal decreasing return.

The second subset, closer to this study, developed mathematical models to provide optimal shelf-space-allocation policies (e.g., Anderson & Amato, 1974; Hansen & Heinsbroek, 1979; Corstjens & Doyle, 1981, 1983; Zufryden, 1986; Bultez & Naert, 1988; Urban, 1998; Yang, 2001). Typically in these models, the retailer’s profit has been the objective function to be optimized. Anderson and Amato (1974) and Hansen and Heinsbroek (1979) developed mathematical models that allow retailers to select the products to keep in the store and, at the same time, the shelf space to allocate to each product. Corstjens and Doyle (1981) criticized these models because they ignore the interactions that can exist between different products in the store, that is, substitution or complementary effects. These authors suggested a model that takes into account main and cross-space elasticities, different product profit margins, and inventory-management costs. Their results indicate
that retailers are better off when they allow more space to the products with higher margins and higher space elasticity when the products are substitutes. Corstjens and Doyle (1981) compared their results to those obtained when retailers use PROGALI or OBM, tools developed to assist retailers in shelf-space decisions, and when cross-elasticities are ignored in the demand function. These tools also give the highest share of shelf space to products with the highest elasticity, but do not perform as well as the model developed by the authors when it comes to comparing retailers’ profit.

In Corstjens and Doyle (1983), a dynamic dimension is introduced in the shelf-space-allocation problem. This dimension captures the potential of the products to be placed on the shelves, where the potential is measured by the sales growth rate. The results indicate that retailers must give the highest share of their shelf space to high-growth products and remove the oldest ones unless they are complementary to products with high potential.

A drawback in Corstjens and Doyle (1981, 1983) is that their models do not provide integer solutions as requested in such problems. Zufreyden (1986) suggested an integer-programming model with only main elasticity effects of shelf-space and nonspace variables. The author did not indicate any rule suggesting an optimal allocation of shelf space. However, the reported simulation results show that the higher the space elasticity, the higher the allocated shelf space. In their SHARP’s model, Bultez and Naert (1988) formulate the shelf-space-allocation problem as a standard mathematical programming one and provide an allocation rule that gives priority to items whose displays are the most profitable. More recently, Urban (1998) suggested a heuristic that allocates the shelf space by removing at each iteration the item in the assortment with the lowest contribution to profits. The procedure stops when profits start to decrease. Building on Corstjens and Doyle (1981), Bookbinder and Zarour (2001) proposed an optimization model that provides the percentage of space allocated to each item based on direct product profitability. Yang (2001) built an algorithm similar to the one used for solving the knapsack problem. Shelf space is allocated according to brand weight, measured by the ratio of sales profits per display area. The allocation is done after the satisfaction of the space availability constraint.

Although the above-cited studies provide valuable tools and interesting insights into the shelf-space-allocation problem, they neglect its impact on manufacturers and the role they can play in this decision process. We argue that a model incorporating both players is desirable for at least two reasons:

(i) The shelf-space-optimization models (as well as the user-friendly commercial tools) are using margins on items as given and recommending allocation rules based, in one way or another, on relative profitability. These margins depend on the wholesale prices, which are under manufacturers’ control. This indicates that the latter can actually influence the allocation process. This role must be recognized formally.

(ii) The profits of competing manufacturers are also interdependent through the shelf space. Indeed, previous research (see above) has highlighted the fact that shelf space has an impact on consumer behavior and, thus, on the demand for, and profitability of, different brands.
These two elements suggest that one should tackle this issue within the framework of conflict and cooperation in marketing channels. Indeed, retailers and manufacturers do not actually share the same point of view on what is the best shelf-space allocation. The marketing science literature has often adopted game theory as a natural analytical framework to deal with the issue of coordination, or lack of it, in marketing channels (e.g., McGuire & Staelin, 1983; Jeuland & Shugan, 1983a,b; Eliashberg & Steinberg, 1987, 1991; Ingene & Parry, 1995a,b; and Kumar, Loomba, & Hadjinicola, 2000, for the static case and Jørgensen & Zaccour, 2004, for the dynamic case). This literature has focused on mechanisms such as pricing, marketing expenditures, and profit sharing, which could coordinate the channel. Although this literature is abundant, it is only recently that a few studies have concentrated on the issue of shelf-space allocation (Wang & Gerchak, 2001; Martín-Herrán & Taboubi, 2005; Martín-Herrán, Taboubi, & Zaccour, 2005).

In Wang and Gerchak (2001), the shelf-space-allocation issue is examined as a coordinating tool in the marketing channel. Two kinds of channel structures (a channel composed of a single manufacturer selling its product through an exclusive dealer, a bilateral monopoly, and a channel with a monopolist manufacturer and two competing retailers) are studied in order to determine if a holding-cost subsidy, designed by the manufacturer to push retailers into allocating more shelf space to their products can maximize total channel profits. Despite the interesting insights given by this study, the fact that each retailer handles the product of a single manufacturer makes the issue of shelf-space allocation less interesting than in channel structures in which competing manufacturers battle to acquire the highest share of the space. Furthermore, in Wang and Gerchak (2001), retail and wholesale prices are exogenous.

Martín-Herrán and Taboubi (2005) and Martín-Herrán et al. (2005) examined the issue of shelf-space allocation in a dynamic game setting for a channel composed of two manufacturers and an exclusive retailer. In Martín-Herrán and Taboubi (2005), the information structure is Markovian and the manufacturers are assumed to act myopically, while in Martín-Herrán et al. (2005), shelf-space and advertising decisions are time-dependent and the hypothesis of myopia is removed. In Martín-Herrán and Taboubi (2005) and Martín-Herrán et al. (2005) the authors assume that the retail and wholesale prices are constant. In this article, we consider that both prices are decisions to be optimized by the players. Further, our model is static. Considering a channel with two manufacturers and one retailer, where manufacturers control their wholesale prices and the retailer chooses the shelf-space allocation and retail margins, we seek to answer the following questions:

(i) What is the equilibrium shelf-space-allocation rule?
(ii) How does it compare to those provided by models optimizing only retailer performance?
(iii) What are the driving forces that impact most on the equilibrium shelf-space allocation?

The remainder of the article consists of the following: First, we introduce the model. Second, we study the retailer’s pricing and shelf-space allocation problem and provide results for manufacturers’ wholesale-price decisions. Third, we give some numerical illustrations. Finally, we conclude.
THE MODEL

We consider a marketing channel composed of two competing manufacturers ($M_1$ and $M_2$) selling their brands through one representative retailer ($R$). This is the most parsimonious structure that would enable us to account for both the vertical (manufacturer–retailer) interaction in a channel and the battle between the manufacturers for shelf space. We naturally assume that the two brands belong to the same product category and thus are close substitutes. Put differently, we are supposing that the shelf space devoted to the category is already known and we focus on how it is shared between the brands. Manufacturer $M_k$, $k = 1, 2$, controls her wholesale price $w_k$, and the retailer, the shelf space allocated to each brand and its retail price.

We assume that the retailer practices a cost-based pricing policy; that is, the retailer offers brand $k$ at price $p_k$ given by

$$p_k = (1 + \theta)w_k, \quad k = 1, 2,$$

where $\theta$ is the markup to be determined optimally. In a survey of pricing practices in the industrial sector, Noble and Gruca (1999) reported that the most common pricing policy used by managers is cost-based (about 56% of respondents). This pricing policy is mainly used in situations where demand is very difficult to estimate. Note that although we assume, for simplicity, that both brands’ markups are the same, the retail margins and prices will nevertheless be different, unless the manufacturers’ wholesale prices are equal. We provide in the Appendix a short analysis of the case where the two markups would be different.

Let the total shelf space be normalized to 1. Denote by $S$ the share of shelf space allocated to brand 1 and thus by $1 - S$ the share reserved to the competing brand. The cost of shelf-space allocation is assumed constant, and because it does not, a priori, vary across brands, we set it at zero without any loss of generality. This is clearly a simplifying assumption for product categories and brands where the manufacturers have to pay retailers to have access to their shelf space.

Following Desmet and Renaudin (1998), Corstjens and Doyle (1981, 1983), and Zufryden (1986), we adopt the following sales function:

$$q_k = \alpha S^\gamma_k p_k^{-\mu_k} p_l^{\varepsilon_k}, \quad k, l = 1, 2, \quad l \neq k,$$

where $\alpha > 0, \mu_k \geq \varepsilon_k \geq 0, 0 < \gamma < 1$ and $S_1 = S$ and $S_2 = 1 - S$.

The above sales function à la Cobb-Douglas has a long tradition in economics. Indeed, the multiplicative form allows one to account for interaction between the variables and has the property that elasticities are constant. Furthermore, we have

$$q_k \geq 0; \quad \frac{\partial q_k}{\partial S_k} \geq 0; \quad \frac{\partial q_k}{\partial p_k} \leq 0; \quad \frac{\partial q_k}{\partial p_l} \geq 0;$$

that is, sales of brand $k$ are nonnegative, increasing in the shelf space allocated to that brand and in the competing brand’s retail price, and decreasing in its own retail price. In equation (2), the parameters $\mu_k$ and $\varepsilon_k$ are, respectively, the direct-price and cross-price elasticities of sales. The inequality $\mu_k \geq \varepsilon_k$ reflects the accepted assumption in economics, stating that own-price elasticity is higher in absolute value than cross-price elasticity. This assumption is made in oligopoly models.
(e.g., Friedman, 1977) and it has also been empirically validated in the context of price promotions models (e.g., Blattberg & Neslin, 1993). Further, the two brands have the same shelf-space elasticity, which is consistent with our assumption that they are close substitutes. The assumption on \( \gamma \) captures the property that the shelf space allocated to a brand is subject to marginal decreasing returns. Finally, \( \alpha \) is a scaling parameter, which is taken as the same for both brands.

To ease the computations, we further make the assumption that the difference between cross- and direct-price elasticities is independent of the brand, that is,

\[
\epsilon_k - \mu_k = \beta, \quad k = 1, 2.
\]

Note that this assumption is less restrictive than assuming symmetry between the brands (i.e., \( \epsilon_1 = \epsilon_2 \) and \( \mu_1 = \mu_2 \)). To interpret the above condition, we rewrite it as follows:

\[
\epsilon_k + \mu_l = \epsilon_l + \mu_k, \quad k, l = 1, 2, \quad k \neq l.
\]

Observe that the brands’ sales ratio is given by

\[
\frac{q_k}{q_l} = \frac{S_k^\gamma p_k^{-\mu_k} p_l^{\epsilon_k}}{S_l^\gamma p_l^{-\mu_l} p_k^{\epsilon_l}}, \quad k, l = 1, 2, \quad k \neq l.
\]

The expression \( \epsilon_k + \mu_l \) is the elasticity of the sales ratio \( \frac{q_k}{q_l} \) to the own-prices ratio \( \frac{p_k}{p_l} \) and \( \epsilon_l + \mu_k \) is the elasticity of the same sales ratio \( \frac{q_k}{q_l} \) to the cross-prices ratio \( \frac{p_l}{p_k} \). Indeed,

\[
- \left( \frac{\partial}{\partial \left( \frac{p_k}{p_l} \right)} \frac{p_k}{q_l} \right) = \epsilon_k + \mu_l,
\]

\[
- \left( \frac{\partial}{\partial \left( \frac{p_l}{p_k} \right)} \frac{p_l}{q_k} \right) = \epsilon_l + \mu_k.
\]

Thus, our condition amounts to saying that the effect of varying the price of this or that brand is the same in the absolute value. An equivalent interpretation of the condition in equation (3) is that both sales functions are homogeneous of the same degree \( \gamma + \beta \).

As in Zufreyden (1986), our sales function specification does not explicitly include the cross-shelf-space elasticities between products within the same product category. Note, however, that the constraint \( S_1 + S_2 = 1 \) implicitly introduces a relationship between direct- and cross-shelf-space elasticities. Indeed, observe that

\[
q_k = \alpha S_k^\gamma p_k^{-\mu_k} p_l^{\epsilon_k}, \quad k, l = 1, 2, \quad k \neq l,
\]

can be written,
\( q_k = \alpha (1 - S_l) p_k^{\mu_k} p_l^{\mu_l}, \quad k, l = 1, 2, \quad k \neq l. \)

It is then easy to see that cross-elasticity is given by,

\[ \frac{\partial q_k}{\partial S_l} = -\gamma S_l. \]

Thus, cross-elasticity is proportional to shelf-space ratio, with the proportionality factor being minus the direct elasticity. This is done mainly in order to include non-price variables in the sales function. Non-price variables, such as prices, have been shown to have a much higher effect on product sales than the shelf space (Zufreyden, 1986). Furthermore, Brown and Lee (1996) have found in their empirical study that cross-shelf-space elasticities are not statistically significant in explaining variations in brands’ sales.

Each player maximizes her own profit. The retailer’s optimization problem is given by

\[ \max_{\theta, S} J_R = \sum_{k=1}^{2} (p_k - w_k) q_k. \]

Replacing the expressions of the demand functions given in equation (2) and making use of equation (1), the retailer’s objective function can be rewritten as:

\[ J_R = \alpha \theta (1 + \theta)^\beta [S^\gamma w_1^{1-\mu_k} w_2^{\epsilon_{k1}} + (1 - S)^\gamma w_2^{1-\mu_k} w_1^{\epsilon_{k2}}]. \quad (4) \]

Similarly, the manufacturer \( M_k \) optimization problem is given by

\[ \max_{w_k} J_{M_k} = (w_k - c_k) q_k, \]

where \( c_k \) is the unit-constant-production cost. Using equations (2) and (1), the objective function becomes:

\[ J_{M_k} = (w_k - c_k) \alpha S_k^\gamma (1 + \theta)^\beta w_k^{\mu_k} w_l^{\epsilon_{k}}, \quad k, l = 1, 2, \quad k \neq l. \quad (5) \]

The game of pricing and shelf-space allocation is played à la Stackelberg with the manufacturers as leaders and the retailer as follower. In our context, a Stackelberg equilibrium is obtained by the following two-step procedure:

(i) First, we optimize for the follower to get her reaction functions (i.e., shelf-space allocation and markup) to leaders’ strategies (wholesale prices).

(ii) Second, we insert these functions into the manufacturers’ objectives and compute a Nash equilibrium.

**ANALYTICAL RESULTS**

Given the functional forms assumed here, the Stackelberg equilibrium cannot be solved analytically and we shall thus resort to numerical simulations to investigate its properties. Before doing so, we shall, however, state some analytical results as they relate to the retailer’s reaction functions to the manufacturers’ wholesale prices. We also establish a relationship between these wholesale prices and provide lower and upper bounds for them.
**Retailer’s Reaction Functions**

The following proposition characterizes the retailer’s reaction functions for shelf-space allocation and markup pricing policy at the equilibrium. Denote by \( \psi = \frac{1 - \mu_2 - \varepsilon_1}{\gamma - 1} = \frac{1 - \mu_1 - \varepsilon_2}{\gamma - 1} > 0 \).

**Proposition 1.**

(i) Assuming an interior solution, the retailer’s reaction function for shelf-space allocation is given by

\[
S^*(w_1, w_2) = \frac{w_2^\psi}{w_1^\psi + w_2^\psi}.
\]

(ii) If \( 1 + \beta < 0 \), then the retailer’s markup policy at the equilibrium is given by

\[
\theta^* = -\frac{1}{1 + \beta}.
\]

**Proof.** Assuming interior solutions, first-order optimality conditions for the retailer’s maximization problem read:

\[
\frac{\partial J_R}{\partial S} = \alpha \gamma \theta (1 + \theta)^\beta \left[ S^\gamma w_1^{1-\mu_1} w_2^{\varepsilon_1} - (1 - S)^\gamma w_2^{1-\mu_2} w_1^{\varepsilon_2} \right] = 0,
\]

\[
\frac{\partial J_R}{\partial \theta} = \alpha \left[ S^\gamma w_1^{1-\mu_1} w_2^{\varepsilon_1} + (1 - S)^\gamma w_2^{1-\mu_2} w_1^{\varepsilon_2} \right] (1 + \theta)^{\beta - 1} [1 + \theta + \theta \beta] = 0.
\]

For \( \theta > 0 \), condition (8) becomes,

\[
S^\gamma w_1^{1-\mu_1} w_2^{\varepsilon_1} - (1 - S)^\gamma w_2^{1-\mu_2} w_1^{\varepsilon_2} = 0,
\]

which gives, after straightforward manipulations, the result in equation (6).

For the expression in equation (9) to be equal to zero, we necessarily need to have

\[
(1 + \theta)^{\beta - 1} [1 + \theta + \theta \beta] = 0,
\]

which is equivalent to the result in equation (7). In order to have a positive markup price, we need to assume that \( 1 + \beta < 0 \) or, equivalently, that \( \mu_k - \varepsilon_k - 1 > 0 \).

**Remark 1.** It can be easily proved that the Hessian matrix associated with the retailer’s objective function \( J_R \) is negative definite at any point \((S, \theta)\) near to \((S^*, \theta^*)\). Therefore, the necessary optimality conditions are also sufficient.

Item (i) in Proposition 1 indicates that the shelf space allocated to brand 1, and similarly to brand 2, is a function of all the model’s parameters (direct- and cross-price elasticities and shelf-space elasticity) and, as it should be, it depends on wholesale prices set by the manufacturers. In retailer shelf-space-optimization models, the wholesale prices are assumed to be exogenous. Here, the wholesale price can be interpreted as a mechanism used by a manufacturer to obtain the desirable share.
of the shelf. Notice also that this reaction function satisfies $0 < S^*(w_1, w_2) < 1$, for all parameters and wholesale price values. Hence the solution is indeed interior.

The shelf space allocated to each brand is decreasing in its wholesale price and increasing in the competitive brand’s wholesale price. Indeed, the partial derivatives of $S^*(w_1, w_2)$ are given by

$$\frac{\partial S^*}{\partial w_1}(w_1, w_2) = -\frac{\psi w_1^{-1} w_2^\psi}{[w_1^\psi + w_2^\psi]^2} < 0,$$

$$\frac{\partial S^*}{\partial w_2}(w_1, w_2) = \frac{\psi w_2^{-1} w_1^\psi}{[w_1^\psi + w_2^\psi]^2} > 0.$$  \hspace{1cm} (10)

Item (ii) in Proposition 1 indicates that the markup set by the retailer is constant and depends only on the direct- and cross-price elasticities of sales. The markup increases when the price elasticity for the brand decreases and when its cross-price elasticity increases. This result is intuitive because it confirms that the retailer will increase its markup or retail margin for those brands whose sales are less sensitive to a variation in their own price, but also when the brand’s sales are very sensitive to the price variations of the competing brand.

Although the final equilibrium expression of the shelf space allocated by the retailer to each brand will be obtained after computing the wholesale price $w_k, k = 1, 2$, we can nevertheless characterize the ratio of shelf space.

**Proposition 2.** The equilibrium ratio of brands’ respective shelf space is equal to the ratio of brands’ revenues, that is,

$$\frac{S^*}{1 - S^*} = \frac{p_1 q_1}{p_2 q_2}.$$

**Proof.** Use equation (6) to compute

$$\frac{S^*}{1 - S^*} = \left(\frac{w_2^{1-\mu_2-\varepsilon_1}}{w_1^{1-\mu_1-\varepsilon_2}}\right)^\delta,$$

where $\delta = \frac{1}{\gamma - 1} < 0$.

Rewrite the above equation as

$$\left(\frac{S^*}{1 - S^*}\right)^\gamma = \frac{w_2^{1-\mu_2-\varepsilon_1}}{w_1^{1-\mu_1-\varepsilon_2}} \frac{S^*}{1 - S^*}.$$

The ratio of brands’ sales is given by

$$\frac{q_1}{q_2} = \frac{(S^*)^{\gamma} p_1^{-\mu_1} p_2^{\varepsilon_1}}{(1 - S^*)^{\gamma} p_2^{-\mu_2} p_1^{\varepsilon_2}} = \left(\frac{S^*}{1 - S^*}\right)^{\gamma} \frac{(1 + \theta)^{\beta} w_1^{-\mu_1} w_2^{\varepsilon_1}}{(1 + \theta)^{\beta} w_2^{-\mu_2} w_1^{\varepsilon_2}} = \left(\frac{S^*}{1 - S^*}\right)^{\gamma} w_1^{-(\varepsilon_2 + \mu_1)} w_2^{\varepsilon_1 + \mu_2}.$$  \hspace{1cm} (13)
Substituting \((\frac{S^*}{1-S^*})^\gamma\) by its value from equation (12) into equation (13), leads to

\[
\frac{q_1}{q_2} = \frac{w_2}{w_1} \frac{S^*}{1-S^*} \iff \frac{w_1q_1}{w_2q_2} = \frac{S^*}{1-S^*}.
\]

Recalling that \(p_k = (1 + \theta)w_k\), \(k = 1, 2\), the above equation is equivalent to

\[
\frac{p_1q_1}{p_2q_2} = \frac{S^*}{1-S^*}.
\]

In the absence of retail cost, revenues are equivalent to profit. Therefore, the allocation rule in equation (14) says that the brands’ relative shares of the shelf space must be equal to their relative profitability. This result extends to a game setting the rule found in the optimization-shelf-space literature, with the important difference being that here, retail and wholesale prices are endogenous and not given ex ante.

The results in the above two propositions provide answers to our first two questions (actually qualitative answers; the quantitative part requires solving the manufacturers’ game).

Manufacturers’ Wholesale Prices

The manufacturers take into account the retailer’s reaction functions and play a Nash game. Therefore, both manufacturers’ wholesale prices at equilibrium are obtained by replacing the retailer’s reaction functions into their profit functions and maximizing them.

Manufacturer \(k\) fixes its wholesale price \(w_k\) by maximizing the following objective function:

\[
J_{M_k} = (w_k - c_k)\alpha S^\gamma_k (1 + \theta)^\beta w_k^{-\mu_k} w_l^{\epsilon_k}, \quad k, l = 1, 2, \quad k \neq l,
\]

subject to the retailer’s reaction function for the shelf-space allocation given by equation (6) and the equilibrium markup price in equation (7). After replacing these expressions, the manufacturer’s objective function can be rewritten as:

\[
J_{M_k} = (w_k - c_k)\alpha \left(\frac{\beta}{1+\beta}\right)^\beta (S^*(w_1, w_2))^{\gamma} w_k^{-\mu_k} w_l^{\epsilon_k}, \quad k, l = 1, 2, \quad k \neq l.
\]

The next proposition characterizes the manufacturers’ equilibrium wholesale prices.

**Proposition 3.** Assuming interior solutions, the manufacturers’ equilibrium wholesale prices, \(w_k, k = 1, 2\), are the solutions of the following system of equations:

\[
(w_k^\psi + w_l^\psi)[w_k(1 - \mu_k) + \mu_k c_k] - \gamma^\psi w_k^\psi (w_k - c_k) = 0,
\]

where \(\psi > 0, k, l = 1, 2, k \neq l\).
Proof. Assuming interior solutions, first-order optimality conditions for the manufacturers maximization problems read:

\[
\frac{\partial J_{M_1}}{\partial w_1} = \left( \frac{\beta}{1 + \beta} \right)^\beta \alpha w_1^{\beta_1 - 1} \left( S^*(w_1, w_2) \right)^{\gamma - 1} \\
\times \left[ S^*(w_1, w_2) [w_1 - \mu_1(w_1 - c_1)] + \gamma w_1(w_1 - c_1) \frac{\partial S^*}{\partial w_1}(w_1, w_2) \right] = 0,
\]

\[
\frac{\partial J_{M_2}}{\partial w_2} = \left( \frac{\beta}{1 + \beta} \right)^\beta \alpha w_2^{\beta_2 - 1} \left( 1 - S^*(w_1, w_2) \right)^{\gamma - 1} \\
\times \left[ (1 - S^*(w_1, w_2)) [w_2 - \mu_2(w_2 - c_2)] - \gamma w_2(w_2 - c_2) \frac{\partial S^*}{\partial w_2}(w_1, w_2) \right] = 0.
\]

Because interior solutions are assumed, the term in brackets must be null. That is, the following system of equations must be satisfied by the manufacturers’ equilibrium wholesale prices:

\[
S^*(w_1, w_2) [w_1 - \mu_1(w_1 - c_1)] + \gamma w_1(w_1 - c_1) \frac{\partial S^*}{\partial w_1}(w_1, w_2) = 0, \quad (17)
\]

\[
(1 - S^*(w_1, w_2)) [w_2 - \mu_2(w_2 - c_2)] - \gamma w_2(w_2 - c_2) \frac{\partial S^*}{\partial w_2}(w_1, w_2) = 0. \quad (18)
\]

System (16) is obtained after substitution of \( \frac{\partial S^*}{\partial w_k}(w_1, w_2), k = 1, 2, \) by their expressions in equations (17) and (18).

The solution of system (16) gives the expressions of \( w_k, k = 1, 2, \) which have to be replaced in the expression of the retailer’s reaction function given by equation (6) to obtain the equilibrium strategy for the shelf-space allocation. Unfortunately, system (16) is heavily nonlinear and we could not solve it analytically. We can, however, derive lower and upper bounds for wholesale prices and thus provide some insight.

Assuming that \( w_k > c_k, \) otherwise manufacturer \( k \) would be better off not selling, and taking into account the relationships in equations (10) and (11), system (16) has a solution, if

\[
w_k(1 - \mu_k) + \mu_k c_k > 0, \quad k = 1, 2, \quad (19)
\]

which is equivalent to

\[
w_k < \frac{\mu_k}{\mu_k - 1} c_k, \quad k = 1, 2.
\]
Now from system (16), we can express one wholesale price as a function of the other one, that is,

$$w_k = w_l \left[ \frac{(\gamma \psi + \mu_l)(w_l - c_l) - w_l}{w_l(1 - \mu_l) + \mu_l c_l} \right]^{1/\psi}, \quad k, l = 1, 2, \quad l \neq k,$$

(20)

if, and only if, \(\frac{(\gamma \psi + \mu_l)(w_l - c_l) - w_l}{w_l(1 - \mu_l) + \mu_l c_l} > 0\) (otherwise there is no real solution to system (16)). Under condition (19) this is equivalent to having a positive numerator. This leads to the lower bound,

$$\frac{\gamma \psi + \mu_k}{\gamma \psi + \mu_k - 1} c_k < w_k, \quad k = 1, 2.$$

Putting the two bounds together, we have

$$c_k < \frac{\gamma \psi + \mu_k}{\gamma \psi + \mu_k - 1} c_k < w_k < \frac{\mu_k}{\mu_k - 1} c_k, \quad k = 1, 2.$$

Replacing the expression of \(\psi\) in the above conditions in terms of the model’s parameters, we obtain

$$\frac{\mu_k + \gamma (\epsilon_l - 1)}{\mu_k - 1 + \gamma \epsilon_l} c_k < w_k < \frac{\mu_k}{\mu_k - 1} c_k, \quad k, l = 1, 2, \quad k \neq l.$$

(21)

A sensitivity analysis on the above bounds leads to the following intuitive results:

(i) Lower and upper bounds are increasing functions of the unit cost \(c_k\).

(ii) The lower the direct-price elasticity \(\mu_k\), the higher the lower and upper bounds.

(iii) The lower bound increases when the cross-price elasticity \(\epsilon_l\) and/or shelf-space elasticity \(\gamma\) decrease. In particular, if the shelf-space elasticity tends to zero, which is equivalent to assuming a demand model where the shelf space does not have any influence on demand, then the wholesale price would be given by \(w_k = \frac{\mu_k}{\mu_k - 1} c_k\). Thus, each manufacturer’s wholesale price would be independent of cross-price elasticity.

Further, it is straightforward to check from equation (20) that \(w_k\) is an increasing function of \(w_l\). Managerially speaking, these bounds provide to each manufacturer an admissible interval for setting their wholesale price and a clear link between the latter and all the parameters of the model. For instance, it is admitted that the price of a product should be somewhere between its cost and what the consumer is maximally willing to pay. These bounds provide a quantitative and more precise content to this last statement. Further, the bounds (and the sensitivity analysis) show how the admissible zone for the wholesale price would be affected if there is a slight change in the values of the parameters.

Remark 2. It can easily be shown that manufacturer \(k\)’s objective function \(J_{M_k}\) is concave in \(w_k\), for all the wholesale prices satisfying the bounds in equation (21). Therefore, the necessary conditions for optimality are also sufficient.
Remark 3. If the two manufacturers have the same cost, that is, $c_1 = c_2$ and if $\mu_1 = \mu_2$ and $\varepsilon_1 = \varepsilon_2$, then $w_1 = w_2$ is a solution to (16).

NUMERICAL RESULTS

As mentioned earlier, we could not find an analytical equilibrium solution in the general case. In this section, we provide numerical simulations to gain some insight into the behavior of the strategies and the outcomes in the asymmetric scenario.

To solve the nonlinear system (16), we substitute $w_k$ by its value from equation (20). The resulting one variable nonlinear equation for $w_l$ is numerically solved using the MATLAB function ‘fzero.’ The algorithm uses a combination of bisection, secant, and inverse quadratic interpolation methods (see the MATLAB documentation, The MathWorks, Inc.). The MATLAB code for generating numerical results is available from us upon request.

Our model contains eight parameters, namely,

Cost parameters: $c_k, \ k = 1, 2,$

Price elasticities: $\varepsilon_k, \mu_k, \ k = 1, 2,$

Baseline demand parameter: $\alpha,$

Shelf-space elasticity: $\gamma.$

Because the baseline parameter is the same for both demand functions (and it is actually a scaling parameter with no particular important role in our context), we shall fix it once for all ($\alpha = 1,000$). We shall vary the other parameters’ values in an ordered manner around the following fully symmetric ones,

$c_k = 1, \gamma = .5, \varepsilon_k = 1.5, \mu_k = 5, \ k = 1, 2.$

We consider two asymmetric scenarios. In the first scenario, manufacturer 1 has a cost advantage with respect to the manufacturer’s competitor ($c_1 = 1, c_2 = 1.1$). In the second scenario, we look at a case where price elasticities are not the same for the two brands ($\varepsilon_1 = 1, \varepsilon_2 = 1.5, \mu_1 = 4.5, \mu_2 = 5$).

The results for the first scenario are reported in Table 1. The cost-advantaged manufacturer sells at a lower wholesale price than her competitor and this translates

<table>
<thead>
<tr>
<th>$c_1 = 1, c_2 = 1.1$</th>
<th>$\gamma = .1$</th>
<th>$\gamma = .4$</th>
<th>$\gamma = .5$</th>
<th>$\gamma = .6$</th>
<th>$\gamma = .8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_1^*$</td>
<td>1.2367</td>
<td>1.1893</td>
<td>1.1706</td>
<td>1.1503</td>
<td>1.1068</td>
</tr>
<tr>
<td>$w_2^*$</td>
<td>1.3508</td>
<td>1.2723</td>
<td>1.2440</td>
<td>1.2146</td>
<td>1.1533</td>
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<tr>
<td>$S_1^*$</td>
<td>.6317</td>
<td>.6499</td>
<td>.6614</td>
<td>.6786</td>
<td>.7562</td>
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<td>.4000</td>
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<td>$q_{1l}^*$</td>
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<td>156.3883</td>
<td>158.1316</td>
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<td>183.6699</td>
</tr>
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<td>$q_{2l}^*$</td>
<td>85.2339</td>
<td>78.7273</td>
<td>76.1846</td>
<td>72.7519</td>
<td>56.8283</td>
</tr>
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<td>$J_{R1}^*$</td>
<td>125.0330</td>
<td>114.4619</td>
<td>111.9523</td>
<td>109.9778</td>
<td>107.5302</td>
</tr>
<tr>
<td>$J_{R2}^*$</td>
<td>37.7886</td>
<td>29.5994</td>
<td>26.9736</td>
<td>24.3849</td>
<td>19.6152</td>
</tr>
<tr>
<td>$J_{M1}^*$</td>
<td>21.3764</td>
<td>13.5669</td>
<td>1.9725</td>
<td>8.3374</td>
<td>3.0292</td>
</tr>
<tr>
<td>$J_{M2}^*$</td>
<td>21.3764</td>
<td>13.5669</td>
<td>1.9725</td>
<td>8.3374</td>
<td>3.0292</td>
</tr>
</tbody>
</table>
into a lower retail price for her brand. (Recall that retail prices are obtained by applying the same markup to the wholesale prices.) Consequently, demand and profit are higher. The retailer allocates a bigger shelf space to the most efficient manufacturer because demand for its brand is higher. Now increasing shelf-space elasticity leads both manufacturers to lower their wholesale prices and the most efficient one sees its share of the shelf increase. For example, for $\gamma = .1$, the shelf space for the efficient brand is .63 and it is, everything else being equal, .76 for $\gamma = .8$. In terms of profit, increasing shelf-space elasticity is detrimental to all channel members, especially to the manufacturers. Table 2 reports the same type of results with, however, a more pronounced cost advantage ($c_1 = 1, c_2 = 1.3$). It is apparent that the qualitative effects are the same; however, there are deeper quantitative effects. For instance, for $\gamma = .8$, the less efficient brand is almost thrown off the shelf.

Table 3 reports the results for the second scenario. In this series of experiments, manufacturer 2 faces a higher price elasticity, to which it adapts by selling at a lower wholesale price (and hence at a lower retail price). This brings us to the chain rule of the first scenario, that is, the lower the wholesale price, the higher the demand, profits, and shelf space. The simulations again show that the higher the shelf-space elasticity, the lower the wholesale prices. Note that the effect of

Table 2: Base case, except $c$’s: Sensitivity results to $\gamma$.

<table>
<thead>
<tr>
<th>$c_1 = 1, c_2 = 1.3$</th>
<th>$\gamma = .1$</th>
<th>$\gamma = .4$</th>
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<th>$\gamma = .6$</th>
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<tr>
<td>$w^*_1$</td>
<td>1.2432</td>
<td>1.2208</td>
<td>1.2132</td>
<td>1.2067</td>
<td>1.2046</td>
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<td>$w^*_2$</td>
<td>1.5889</td>
<td>1.4822</td>
<td>1.4476</td>
<td>1.4139</td>
<td>1.3518</td>
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<tr>
<td>$S^*$</td>
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<td>.8555</td>
<td>.8746</td>
<td>.8984</td>
<td>.9597</td>
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<tr>
<td>$\theta^*$</td>
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<td>.4000</td>
<td>.4000</td>
<td>.4000</td>
<td>.4000</td>
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<tr>
<td>$q^*_1$</td>
<td>203.5706</td>
<td>192.5876</td>
<td>190.8462</td>
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<tr>
<td>$q^*_2$</td>
<td>35.5630</td>
<td>28.7894</td>
<td>22.9308</td>
<td>18.3240</td>
<td>6.9155</td>
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<tr>
<td>$J^*_R$</td>
<td>123.8357</td>
<td>109.9238</td>
<td>105.8941</td>
<td>101.9762</td>
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</tr>
<tr>
<td>$J^*_M1$</td>
<td>49.5120</td>
<td>42.5159</td>
<td>40.6955</td>
<td>39.2284</td>
<td>37.7831</td>
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<tr>
<td>$J^*_M2$</td>
<td>10.2747</td>
<td>4.8798</td>
<td>3.3835</td>
<td>2.0875</td>
<td>.3580</td>
</tr>
</tbody>
</table>

Table 3: Base case, except $\varepsilon$’s and $\mu$’s: Sensitivity results to $\gamma$.

<table>
<thead>
<tr>
<th>$\varepsilon_1 = 1, \varepsilon_2 = 1.5$</th>
<th>$\mu_1 = 4.5, \mu_2 = 5$</th>
<th>$\gamma = .1$</th>
<th>$\gamma = .4$</th>
<th>$\gamma = .5$</th>
<th>$\gamma = .6$</th>
<th>$\gamma = .8$</th>
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<tr>
<td>$w^*_1$</td>
<td>1.2635</td>
<td>1.1909</td>
<td>1.1642</td>
<td>1.1359</td>
<td>1.0733</td>
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</tr>
<tr>
<td>$w^*_2$</td>
<td>1.2347</td>
<td>1.1787</td>
<td>1.1560</td>
<td>1.1308</td>
<td>1.0721</td>
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</tr>
<tr>
<td>$S^*$</td>
<td>.4681</td>
<td>.4786</td>
<td>.4822</td>
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<tr>
<td>$\theta^*$</td>
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<td>.4000</td>
<td>.4000</td>
<td>.4000</td>
<td>.4000</td>
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<tr>
<td>$q^*_1$</td>
<td>123.0605</td>
<td>123.5136</td>
<td>124.7224</td>
<td>127.2779</td>
<td>136.4146</td>
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<tr>
<td>$q^*_2$</td>
<td>143.0892</td>
<td>135.5774</td>
<td>138.4077</td>
<td>135.2884</td>
<td>140.3880</td>
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<tr>
<td>$J^*_R$</td>
<td>132.8644</td>
<td>122.5950</td>
<td>120.4478</td>
<td>119.0270</td>
<td>118.7731</td>
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<tr>
<td>$J^*_M1$</td>
<td>32.4227</td>
<td>23.5136</td>
<td>20.4836</td>
<td>17.3027</td>
<td>10.0025</td>
<td></td>
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<tr>
<td>$J^*_M2$</td>
<td>143.0892</td>
<td>24.2307</td>
<td>21.0378</td>
<td>17.6985</td>
<td>10.1276</td>
<td></td>
</tr>
</tbody>
</table>
shelf-space elasticity on the allocation of the latter between the two brands is much less than in the previous scenario.

Other simulations were conducted where price elasticities were varied, subject to satisfying the condition in equation (3), and providing the same qualitative results.

To conclude on the subject of these simulations, one important lesson seems to be that there is a strong link between wholesale price of a brand and its share of the shelf. This shows that a manufacturer must pay considerable attention to its costs, in order to be able to offer a competitive price to retailers.

CONCLUDING REMARKS

This article provides answers to the three questions raised in the introduction. More specifically, it provides a rule for allocating shelf space, which resembles qualitatively what is done in practice. However, the important difference is that manufacturers can act on this rule. Further, our numerical results show that the manufacturers’ costs, and thus the wholesale prices, which are precisely the instruments by which manufacturers can intervene, have a considerable impact on the shelf-space-allocation process.

Based on our theoretical results, in this section, we first provide a series of testable conjectures and second, make some suggestions for further investigation.

The following conjectures can be tested by suitable experimental methods or by a survey of a sample of category managers.

Conjecture 1. Everything else being equal, the most efficient manufacturer secures the highest share of the shelf space.

All of our numerical results, printed or not, support this relationship.

Conjecture 2. Everything else being equal, the higher the shelf-space elasticity, the lower the wholesale prices.

All of our numerical results show this effect. The higher the consumer’s sensitivity to the shelf space (with everything that shelf space means for consumers, as discussed in the introduction), the tougher the battle is for shelf space between manufacturers.

Conjecture 3. Everything else being equal, the lower the price elasticity, the less shelf space is allocated to that brand.

The brand with the lower price elasticity sells at a higher price and attracts lower demand and thus deserves a smaller share of the shelf space.

Conjecture 4. Everything else being equal, the profits of all channel members decrease with shelf-space elasticity.

All of our numerical results suggest this. Further, the retailer’s profit seems to be less sensitive to an increase in shelf-space elasticity than the manufacturers’
The Impact of Manufacturers’ Wholesale Prices

profits. How to act on the importance of shelf space for consumers is more than a serious matter for manufacturers.

On top of testing the above conjectures, future research should be devoted to eliminating the restrictive assumptions we made. More specifically, the constraint on the choice of direct- and cross-price elasticities in condition (3) should be relaxed if we wish to be able to fully assess the impact of consumer sensitivity to shelf-space allocation as well as to wholesale and retail prices. This is, however, a challenging task, because it would require numerically solving a nonlinear system to obtain the retailer’s reaction functions and then solving, also numerically, another highly nonlinear system to obtain an equilibrium in wholesale prices.

We also made the assumption that the retailer adopts a cost-based pricing strategy and applies the same markup to both brands. Although this assumption does not seem to be too restrictive, because it does not yield the same retail prices (see simulation results), it would still be interesting to attempt to remove it and assess its impact on the results.

Finally, we assumed that the shelf-space cost is zero. This could be a strong assumption in a context where the retailers are gaining much power and are in a position of asking the manufacturers to pay for the shelf space allocated to their brands. It will certainly be interesting to include such considerations in the model.

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REFERENCES


**APPENDIX: THE CASE OF TWO DIFFERENT MARKUPS**

We made in this article the simplifying assumption that the two markups ($\theta_1$ and $\theta_2$) applied by the retailer to the wholesale prices of the two brands are the same. Unfortunately, the case where $\theta_1 \neq \theta_2$ does not allow one to determine analytically the reaction functions of the retailer to the wholesale prices. However, some insight can be obtained regarding the relationship between the markups and the shelf space allocated by the retailer to each brand.

Assume that the price of brand $k$ is now given by

$$p_k = (1 + \theta_k)w_k, \quad k = 1, 2.$$

Inserting in the retailer’s profit and optimizing leads to

$$\left( \frac{1 - S}{S} \right)^{\gamma - 1} = K \frac{w_1^{1-\mu_1-\varepsilon_2}}{w_2^{1-\mu_2-\varepsilon_1}},$$

where

$$K = \frac{\theta_1 (1 + \theta_1)^{-\mu_1 - \varepsilon_2}}{\theta_2 (1 + \theta_2)^{-\mu_2 - \varepsilon_1}}.$$  

Recalling that

$$\psi = \frac{1 - \mu_2 - \varepsilon_1}{\gamma - 1} = \frac{1 - \mu_1 - \varepsilon_2}{\gamma - 1} > 0,$$

solving for $S$ yields

$$S = \frac{w_2^\psi}{K w_1^{\psi \psi} + w_2^\psi}.$$
Differentiating the retailer’s profit with respect to $\theta_1$ and $\theta_2$ and equating to zero gives

$$S = \frac{-\varepsilon_2}{\theta_1 + 1 - \mu_1 - \varepsilon_2}, \quad (A1)$$

$$S = \frac{1}{\theta_2 + 1 - \mu_2 - \varepsilon_1}. \quad (A2)$$

Note that if we impose $\theta_1 = \theta_2$, we recover the result of the optimal markup in (7).

Now (A1) and (A2) are equivalent to

$$\theta_1 = \frac{-S}{\varepsilon_2 + S(1 - \mu_1 - \varepsilon_2)},$$

$$\theta_2 = \frac{1 - S}{S(1 - \mu_2 - \varepsilon_1) - (1 - \mu_2)}.$$  

It is easy to verify that the derivatives of $\theta_1$ ($\theta_2$) with respect to $S$ is negative (positive). This shows that the markup and the shelf space are substitutes; that is, increasing the value of one of them leads to a decrease in the value of the other one. Note that the shelf space allocated to brand 2 is $(1 - S)$ and hence the same relationship stands for both brands. This result can be summarized by saying that in the context of our demand function and fixed total shelf space, more shelf space to a brand means more capacity/supply, and thus a reduction of price is in order.

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