Analytical Evaluation of Splice and Bending Losses of Photonic Crystal Fibers based on Empirical Relations

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Abstract—Photonic crystal fibers (PCFs) have been under intensive study for the past several years as they offer a number of unique and useful properties not achievable in conventional silica glass fibers. Important issues regarding the practical development of these PCFs are their splice and bending loss properties. In this work, we present an analytical evaluation of the splice losses as well as bending losses for PCFs using simple empirical relations in order to determine the Petermann-I and Petermann-II spot sizes as a function of the fundamental geometrical parameters - the air-hole diameter and the hole-pitch of the PCF. We consider the splice loss due to lateral, longitudinal as well as angular offsets, when a PCF is spliced to an identical PCF or to a step index single mode fiber (SMF). To deduce macro-bending loss in PCFs, we apply the standard radiation model for bend conventional fibers, making a full transformation of conventional SIF parameters to PCF effective parameters. The spectral response of bending loss is also investigated. The results can be utilized in the design and development of telecommunication and sensor systems based on PCFs.

I. INTRODUCTION

Photonic crystal fiber (PCF) is a new type of optical fiber [1]–[3] incorporating an array of air holes that run along its length, reminiscent of a crystal lattice, which gives to this type of fiber, its name. There are two main types of PCF: air-guiding which guides light via a photonic band-gap effect and index-guiding which guides light via a modified total internal reflection mechanism. In air-guided PCF, the core is hollow, and light is guided by the photonic band gap (PBG) effect, a mechanism that does not require a higher refractive index in the core in order to confine and guide light. The PBG guidance effect relies on coherent backscattering of light into the core. In index-guided PCFs the core area is solid and the light is confined to a central core as in conventional fibers. An index guiding PCF is usually formed by a central solid defect region surrounded by multiple air holes in a regular triangular lattice as it is shown in (Fig.1). The lower effective refractive index of the surrounding holes forms the cladding resulting in an index guidance mechanism analogous to total internal reflection in standard fibers.

PCFs have been shown to possess many significant properties like single mode operation over wide range of wave-lengths, highly tunable dispersion, propagation of high power densities without exciting unwanted non-linear effects and high birefringence. Index guiding PCFs present great controllability in chromatic dispersion by varying the hole diameter (d) and pitch (Λ). So far, various PCFs with remarkable dispersion properties such as, zero dispersion wavelengths shifted to visible and near-infrared, ultra-flattened chromatic dispersion and a large positive dispersion with negative slope in the 1.55 μm range have been reported [4], [5].

To realize the full potential of PCFs, it is necessary to efficiently splice PCFs or couple light from conventional single-mode fibers (SMF) to PCFs. However, because PCFs have microhole structures that are totally different from conventional fibers, splicing different PCFs to conventional fibers is a significant challenge [6]–[9]. Recently, attention has been paid to methodologies of splicing large air-filling factor PCFs [10]. Splice losses mainly come from fundamental mode mismatch, a difference in the mode field diameters of connected fibers as well as lateral, longitudinal and angular offsets [11]. Micro-hole collapse effect in PCFs is another source of splice loss, but it also can be used to reduce splice loss when the two fibers have mode mismatch [12]. Furthermore, in many applications, PCF is required to be cabled and placed in the form of a coil, which leads to a macro-bending-loss in the PCF and, therefore,
bending loss in PCF have to be evaluated. It is well known [13] that the transmission bandwidth in PCFs is limited by macro-bending loss due to the existence of short and long wavelength bending loss edges.

A scalar effective index model can be a valuable tool for aiding fabrication efforts because it is easy to use and provides good qualitative information [14]. The accuracy of this model is limited by the difficulty to assign an appropriate equivalent step-index profile of the PCF. A few numerical approaches regarding splice and macro-bending losses of PCFs were reported, using finite-difference time-domain (FDTD) method [15], [16] or Finite Element method (FEM) with perfectly matched layers (PML) boundary conditions [17]–[19]. However, these numerical calculations are tedious and intense computational effort is needed. Nevertheless, the influence of varying PCF parameters on the splice loss is only partially addressed ignoring any transverse, longitudinal or angular misalignment at the splice point.

In this paper, we present an analytical evaluation of the splice and bending losses for PCFs using simple empirical relations introduced firstly by Saitoh and Koshiba [20]. We consider the splice loss due to lateral, longitudinal as well as angular offsets, when a PCF is spliced to an identical PCF or to a step index single mode fiber (SMF). To deduce macro-bending loss in PCFs, we apply the standard radiation model for bend conventional fibers, making a full transformation of conventional SIF parameters to PCF effective parameters. The spectral response of the bending loss is also investigated.

II. ANALYTICAL MODEL

We consider a PCF with a triangular lattice of holes running along the length of the fiber, as shown in Fig.1. The PCF can be characterized by the pitch Λ and the diameter d of the air holes. The fiber core is formed by a missing air-hole. Saitoh and Koshiba [20] claimed that the triangular PCF can be well parameterized in terms of an effective $V_{eff}$-parameter given by

$$V_{eff} = \frac{2\pi}{\lambda} \alpha_{eff} \sqrt{n_{co}^2 - n_{FSM}^2} = U_{eff}^2 + W_{eff}^2$$  \hspace{1cm} (1)$$

where $\alpha_{eff}$ is the effective core radius assumed to be $\Lambda/\sqrt{3}$, $n_{co}$ is the refractive index of the fiber obtained by the Sellmeier equation, $\lambda$ is the operating wavelength and $n_{FSM}$ is the refractive index of the so-called fundamental space-filling mode in the air-hole lattice, respectively. The parameters $U_{eff}$ and $W_{eff}$ are called, respectively, the effective normalized transverse phase and attenuation constants and are given by

$$U_{eff} = \frac{2\pi}{\lambda} \alpha_{eff} \sqrt{n_{co}^2 - n_{eff}^2}$$  \hspace{1cm} (2)$$

$$W_{eff} = \frac{2\pi}{\lambda} \alpha_{eff} \sqrt{n_{eff}^2 - n_{FSM}^2}$$  \hspace{1cm} (3)$$

The effective index of the guided mode can be accurately determined as follows: Firstly, by using Table 1 from Ref. [20], we calculate $V_{eff}$ based on the empirical relation

$$V_{eff} = \frac{\lambda}{\Lambda} \frac{d}{\Lambda} = A_1 + \frac{A_2}{1 + A_3 \exp \left( A_4 \frac{\lambda}{\Lambda} \right)}$$  \hspace{1cm} (4)$$

where

$$A_1 = a_{i0} + a_{i1}(d/\Lambda)^{b_{1i}} + a_{i2}(d/\Lambda)^{b_{2i}} + a_{i3}(d/\Lambda)^{b_{3i}}$$  \hspace{1cm} (5)$$

and the coefficients $a_{ij}$ and $b_{ij}$ are given in Table 1 of Ref. [20]. Afterward, the refractive index of the fundamental space-filling mode $n_{FSM}$ is obtained by Eq. (1). Then, the attenuation constant $W_{eff}$ is calculated using the empirical relation

$$W_{eff} = \frac{\lambda}{\Lambda} \frac{d}{\Lambda} = B_1 + \frac{B_2}{1 + B_3 \exp \left( B_4 \frac{\lambda}{\Lambda} \right)}$$  \hspace{1cm} (6)$$

where

$$B_i = c_{i0} + c_{i1}(d/\Lambda)^{d_{i1}} + c_{i2}(d/\Lambda)^{d_{i2}} + c_{i3}(d/\Lambda)^{d_{i3}}$$  \hspace{1cm} (7)$$

and the coefficients $c_{ij}$ and $d_{ij}$ are also given in Table 2 of Ref. [20]. Finally, for given $W_{eff}$ and $n_{FSM}$, the effective index $n_{eff}$ can be obtained by using Eq. (3). The empirical relations approach described above has been compared to full vectorial FEM-method and excellent agreement has been observed [20]. The limitations of the above approach appears for small air filling fractions, when it does not answer for all range of wavelengths [21].

Spot size is a very important parameter characteristic that can be used to determine both the splice and bending losses. The Petermann-I spot size of the near field $w_{PI}$ is related to the splice loss due to angular offset at the joint between the two fibers and is defined as

$$w_{PI}^2 = \frac{2}{\int_0^\infty E^2(r) r^3 dr} \int_0^\infty E^2(r) r dr$$  \hspace{1cm} (8)$$

In general, one has to evaluate the above integrals numerically, but for the ‘equivalent step index fiber’, an analytical expression is obtained as

$$w_{PI}^2 = \frac{2\alpha_{eff}}{\sqrt{3}} \left[ \frac{J_0(U_{eff})}{U_{eff} J_1(U_{eff})} + \frac{1}{W_{eff}^2} - \frac{1}{U_{eff}^2} + \frac{1}{2} \right]$$  \hspace{1cm} (9)$$

The Petermann-II spot size of the near field $w_{PII}$ is related to the splice loss due to lateral or longitudinal offsets at the joint between the two fibers as well as to the bending loss and is defined as

$$w_{PII}^2 = \frac{2}{\int_0^\infty (dE(r)/dr)^2 r dr} \int_0^\infty (dE(r)/dr)^2 r dr$$  \hspace{1cm} (10)$$

Evaluating the integrals for the ‘equivalent step index fiber’, an analytical expression is also obtained

$$w_{PII} = \alpha_{eff} \sqrt{\frac{2}{3}} \frac{J_0(U_{eff})}{U_{eff} J_1(U_{eff})}$$  \hspace{1cm} (11)$$
The spot size of SI-SMFs is calculated using the Petermann’s formula [22]:

\[ w_p = w_M - (0.016 + 1.561 V^{-7})a \]  \hspace{1cm} (12)

where \( w_M \) is the Marcuse spot size given by

\[ w_M = (0.650 + 1.619 V^{-3/2} + 2.879 V^{-6})a \]  \hspace{1cm} (13)

and \( \alpha \) denotes the core radius of the SI-SMF. Equation (12) is accurate to 1% in the range of our interest \( 1.5 < V < 2.5 \).

Given the modal spot sizes \( w_1 \) and \( w_2 \) of the two spliced fibers, Gaussian beam propagation theory states that the splice loss due to lateral offset \( r_d \) can be obtained as [23]

\[ \eta_d = \frac{4w_1^2w_2^2}{(w_1^2 + w_2^2)^2} \exp\left(-\frac{2r_d^2}{w_1^2 + w_2^2}\right) \]  \hspace{1cm} (14)

When there is a longitudinal offset \( z \) between the two fibers, the splice loss can be expressed as:

\[ \eta_z = \frac{4w_1^2w_2^2}{(w_1^2 + w_2^2)^2} \left(\frac{z}{K_0}\right)^2 \]  \hspace{1cm} (15)

Splice losses due to angular offset is also important for designing high performance optoelectronic components and to fiber-optic angular alignment automation. When there is an angular offset \( \theta \) between two spliced fibers, loss can be expressed as follows:

\[ \eta_\theta = \frac{4w_1^2w_2^2}{(w_1^2 + w_2^2)^2} \exp\left(-\frac{k^2\theta^2w_1^2w_2^2}{2(w_1^2 + w_2^2)^2}\right) \]  \hspace{1cm} (16)

Though the above formulas Eqs. (14)-(16) are extremely easy to calculate, there are some limitations because Gaussian mode profiles for the spliced fibers are assumed. In case of highly non-Gaussian modes, the rigorous overlap integral of both fiber mode distributions must be calculated.

One of the important issues related to practical development of PCFs is their bending loss properties. Also, bending loss estimation plays a central role when defining the spectral window in which the fiber may be operated with low loss. When a fiber is bent, the modal field distorts outwards in the bend region leading to a radiation loss. Predictions of macrobending loss in PCFs have been made using the antenna theory of Sakai and Kimura [24] or a phenomenological model within the tilted-index representation [25]. The tilted index model is an exact mapping of the scalar description which works well for large mode area PCFs with \( \lambda << \Lambda \). Here, we apply the standard radiation model [14] for bend standard fibers using the empirical relations of Saitoh and Koshiba [20]. Thus, using the effective parameters for PCFs defined by Eqs. (1)-(3), the power attenuation coefficient of standard fiber due to macrobending is transformed to the following expression:

\[ a = \left(\frac{dB}{m}\right) \approx 4.343 \left(\frac{\pi}{4a_{eff}R_c}\right)^{1/2} \left(\frac{1}{W_{eff}}\right)^{3/2} \exp\left(-\frac{4R_c W_{eff}^3}{3a_{eff}V_{eff}^2}\right) \left(\frac{U_{eff}}{V_{eff}K_1(W_{eff})}\right)^2 \]  \hspace{1cm} (17)

where \( R_c \) denotes the curvature of the bend, \( \Delta_{eff} = (n_{co} - n_{FSM})/n_{co} \) is the relative effective refractive index difference and \( K_1(W_{eff}) \) is a modified Bessel function of second kind.

III. NUMERICAL RESULTS AND DISCUSSION

In this section, two representative fibers, a large mode area PCF LMA-5 and a single mode fiber SMF-28, are considered to calculate the splice loss between two identical PCFs or between a PCF and a SMF. The LMA-5 PCF has a core diameter 4.5 \( \mu m \). Figs. 2-4 show the splice loss between two identical LMA-5 PCFs in (dB) versus lateral, longitudinal and angular offset, respectively, for different values of the ratio \( d/\Lambda \) where the operating length is \( \lambda = 1.55 \mu m \). As the ratio \( d/\Lambda \) increases, the splice loss increases drastically. Thus, the tolerance of lateral offset is less than 1 \( \mu m \) if the allowed loss is 0.5 dB. It is clear from Fig. 2 that the loss dominates as the air-hole size in cladding is increased. From Fig. 3 we can see that as the ratio \( d/\Lambda \) increases, the splice loss due to longitudinal offset increases drastically and as long as the offset is less than 10 \( \mu m \), the loss is less than 1 dB for all ratios \( d/\Lambda \). This result is important for free-space interconnections in optical switching systems. The splice loss due to angular offset, for increasing ratios \( d/\Lambda \), is shown in Fig. 4. As it is seen, PCFs become very sensitive as the hole diameter
decreases or the pitch $\Lambda$ increases. At 1.55 $\mu m$, an angular offset of 1°, results in 2.5 dB penalty for PCFs with $d/\Lambda$=0.49 but 1 dB penalty when the ratio $d/\Lambda$ is increased to 0.56. This sensitivity of PCFs to angular offset can be explained by the reduction of numerical aperture or equivalently the increase of directivity of the outgoing beam from the end-face of the PCF, as the ratio $d/\Lambda$ increases. Therefore, during the design and manufacturing process of optoelectronic components utilizing PCFs, the angular offset issue must be handled properly.

Figures 5-7 illustrate the dependence of splice loss between a LMA-5 PCFs and a SMF-28 on lateral, longitudinal and angular offset, respectively, for different values of $d/\Lambda$, where the operating wavelength is $\lambda = 1.55\mu m$. As the ratio $d/\Lambda$ increases, the splice loss due to lateral or longitudinal offset, increases drastically but the loss due to angular offset decreases. It is observed that the sensitivity to these offsets in the case of PCF-SMF splice is lower than in the case of PCF-PCF splice. The minimum loss, when the lateral or longitudinal offset is zero, varies from less than 1 dB to 3 dB when $d/\Lambda$ is changed from 0.37 to 0.62 which agree approximately with the measured butt-coupling loss in Refs. [8], [12]. The large splice loss even at zero offsets is attributed to the mode field mismatch between the PCF and SMF. Moreover, the experimental values are expected to be larger due to micro-hole collapse effect in the vicinity of the splice joint during the splicing process [12].

In the following we present our numerical results concerning the macro-bending loss of a LMA-5 PCF. Firstly, bending loss has been calculated as a function of the operating wavelength at different ratios $d/\Lambda$ for two different values of bending radius $R_c = 5 \text{ mm}$ and $R_c = 10 \text{ mm}$. As it is seen from Figs. 8 and 9, a short wavelength loss edge appears for PCFs, in contrast to conventional SMF, where only a long wavelength loss edge is found. These two loss edges determine the range of wavelengths in which PCF can operate for low bend loss. At a particular value of bending radius, a larger hole-diameter results in broader window of operating wavelengths, whereas the minimum loss decreases as the ratio $d/\Lambda$ increases. More specifically, the figures reveal that for the realization of signif-
Fig. 8. Variation of bending loss with wavelength for different configurations of PCFs at a bend radius $R_c = 5 \text{ mm}$.

Fig. 9. Variation of bending loss with wavelength for different configurations of PCFs at a bend radius $R_c = 10 \text{ mm}$.

Fig. 10. Bending loss as a function of bending radius for a LMA-5 PCF for increasing air-hole diameters at $\lambda = 1.55 \mu m$.

Fig. 10. Bending loss as a function of bending radius for a LMA-5 PCF for increasing air-hole diameters at $\lambda = 1.55 \mu m$.

by lateral, longitudinal and angular offsets. The effective Petermann-I and Petermann-II spot sizes of the near field for different configurations of PCFs are obtained, which is used for computing the splice losses. Petermann-I spot size is related to the splice loss due to angular offset whereas Petermann-II spot size is related to the splice loss due to lateral or longitudinal offsets at the joint between the two fibers. The wavelength dependence of the cladding index of the PCF has been taken properly into account.

The splice losses due to lateral, longitudinal as well as angular offsets are strongly dependent on the geometric characteristics of the PCF. We found a high sensitivity of PCF-splicing to angular offset and thus angular offsets must be handled properly in designing PCF-components. Loss due to splicing LMA-5 PCF to standard SMF-28 has also been investigated because of its importance to broad band applications. The minimum loss, when the lateral or longitudinal offset is zero, varies from less than 1 dB to 3 dB when $d/\Lambda$ is changed from 0.37 to 0.62 which agree approximately with the measured butt-coupling loss. The large splice loss even at zero offsets is attributed to the mode field mismatch between the PCF and SMF. It is worth noting that although air-hole collapse is often inevitable during the splicing between PCFs, is not considered in our study [12].

To deduce macrobending loss in PCFs, we apply the standard radiation model for bend conventional fibers, making a full transformation of conventional SIF parameters to effective index-guiding PCF parameters which can be easily calculated using the empirical relations. It is shown that macrobending loss can be controlled by varying the fiber parameters. The spectral window, in which a PCF can operate with low loss is observed for different values of air-hole sizes. The effect of tailoring the size of air-holes on the spectral window is also investigated and it is found that the range of operating wavelengths for minimum bending loss, increases as the hole size increases. PCFs seems to be more bend resistant than the conventional fibers, for a particular value of bend radius and

IV. CONCLUSION

We have presented a simple method to calculate the splice and bending losses for index-guiding PCFs. The calculations were based on empirical relations, proposed firstly by Saitoh and Koshiha, in order to determine the Petermann-I and Petermann-II spot sizes as a function of the fundamental geometrical parameters - the air-hole diameter and the hole-pitch of the PCF. The splice losses due to lateral, longitudinal as well as angular offsets, when a PCF is spliced to an identical PCF or to a step index single mode fiber (SMF), are investigated. The Gaussian beam propagation theory leads to simple closed-form expressions for the splice losses produced
operating wavelength range. We expect that the above study on macrobending and splice losses maybe helpful in the design and development of telecommunication and sensor systems utilizing PCFs.

REFERENCES


