Noncoherent Detection in Amplify-and-Forward Relay Systems

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Abstract—In amplify-and-forward single-relay systems that employ an average relay power constraint, one-shot detection at the destination terminal is not optimal when the channel between the source and relay terminals is unknown. In this work, we derive the maximum-likelihood (ML) block noncoherent detector and show that it can be expressed as a reduced-rank quadratic form maximization. We introduce an auxiliary real variable and prove that the maximization form can be closely approximated by a rank-2 positive semidefinite quadratic form through appropriate selection of the auxiliary variable. Motivated by recent developments on the polynomial-time maximization of reduced-rank quadratic forms over finite alphabets, we develop an efficient detector that has polynomial complexity in the block length and demonstrated near-ML performance.

I. INTRODUCTION

Although cooperative diversity as a concept originates in the seventies [1], [2], it is after the work of Sendonaris et al. [3] and Laneman et al. [4] that it has attracted significant research interest which in turn led to the development of a number of (memoryless or not) relaying strategies. Memoryless strategies include amplify-and-forward (AF) [5]-[9] where the relay forwards a scaled version -subject to an average or instantaneous power constraint- of its received signal to the destination, demodulate-and-forward (DF) [6] in which the relay performs symbol-by-symbol demodulation and forwards its ‘hard’ decisions to the source, and estimate-and-forward (EM) [10] where the relay forwarding function is designed to maximize a generalized signal-to-noise ratio (GSNR) criterion at the destination. Strategies with memory involve decode-and-forward [4], [9] in which the relay decodes the entire message, re-encodes it, and transmits it to the destination and compress-and-forward [2] where the relay forwards to the destination a quantized version of its received signal.

Most of the studies of optimum and low-complexity suboptimum detectors for the above schemes as well as optimized power allocation strategies according to symbol-error-rate, outage probability, capacity, diversity order, and/or diversity-multiplexing tradeoff assume that the destination has perfect channel knowledge of all transmission links. Although channel information can be obtained by the use of pilot symbols in slow fading environments, it is questionable how the destination can obtain the source-relay channel coefficient perfectly through pilot signal forwarding without noise amplification [7]. In this context, differential transmission over AF links was studied in [7], differential distributed space-time block codes (STBCs) that achieve full diversity for multiple relay AF links were developed in [11], the performance of noncoherent BFSK with DF relays and a piece-wise linear near-ML decoder was studied in [6], ML and suboptimum receiver structures for noncoherent AF links with multiple relays utilizing OOK and BFSK signalling assuming unfaded relay to destination links were derived in [12], suboptimal one-shot detectors without channel knowledge for multiple relay AF links where developed in [13], while noncoherent receiver structures for distributed STBCs with AF relaying were investigated in [14].

In this present work we consider a single AF relay link and derive the optimum noncoherent detector based on a block of observations of multiple MPSK source transmissions. Since the latter has complexity exponential in the block length, a forbidding factor even for moderate values of the block length, we efficiently reduce the dimensionality of the problem and derive a polynomial-complexity suboptimum detector with demonstrated near-ML symbol error rate.

II. SIGNAL MODEL AND NOTATION

We consider a single1 AF relay model where a source terminal S communicates with a destination terminal D with the assistance of a relay terminal R as shown in Fig. 1. All terminals are equipped with single antenna transceivers. Similarly to [4]-[8], we consider orthogonal [4] (Protocol II [9]) transmission which takes place in a half-duplex mode (the

1The scheme under consideration and the developments in this paper can be extended to a multiple relay system if distributed STBCs are considered.
terminals cannot receive and transmit at the same time) over two separate time slots.

Let \( x, x \in \mathcal{A}_M \triangleq \{ e^{j2\pi m} \mid m = 0, 1, \ldots, M - 1 \} \) be an \( M \)-ary phase-shift keying (MPSK) data symbol emitted at the first time slot by the source terminal. The corresponding down-converted and pulse-matched-filtered received signals at the destination and relay terminals during the first time slot are given by

\[
y_{SD} = \sqrt{P_{SD}} h_{SD} x + n_{SD}
\]

and

\[
y_{SR} = \sqrt{P_{SR}} h_{SR} x + n_{SR},
\]

respectively. In (1) and (2), \( h_{SD} \) and \( h_{SR} \) are zero mean complex Gaussian random variables that represent the Rayleigh fading channel coefficients of the S→D and S→R, respectively, paths. The zero-mean complex Gaussian random variable \( n_{SD} \) and \( n_{SR} \) correspond to additive white Gaussian noise in the respective paths. Without loss of generality (w.l.o.g.), we assume that \( n_{SD} \sim \mathcal{CN}(0, N_o) \), \( n_{SR} \sim \mathcal{CN}(0, N_o) \), \( h_{SD} \sim \mathcal{CN}(0, 1) \), and \( h_{SR} \sim \mathcal{CN}(0, 1) \). The latter implies that the channel power is absorbed by the effectively transmitted power \( P_{SD} \) and \( P_{SR} \).

During the second time slot, the source remains silent and the relay amplifies the received signal \( y_{SR} \) by a factor \( a \) and transmits it to the destination. The received signal at the destination is given by

\[
y_{RD} = a \cdot y_{SR} h_{RD} + n_{RD} = a \sqrt{P_{SR}} h_{SR} h_{RD} x + ah_{SR} h_{RD} + n_{RD}. \]

Once more, w.l.o.g. we assume that \( h_{RD} \sim \mathcal{CN}(0, 1) \) and \( n_{RD} \sim \mathcal{CN}(0, N_o) \). Finally, all channel coefficients and additive noise samples are independent of each other.

In the present work we assume that only the destination terminal knows the channel coefficients \( h_{SD} \) and \( h_{RD} \). The channel coefficient \( h_{SR} \) is unknown to both relay and destination terminals. The scaling factor \( a \) is set to

\[
a = \sqrt{\frac{E_{\{ |y_{SR}|^2 \}}}{h_{SR}^2 + n_{SR}^2}} = \sqrt{\frac{P_{SR}}{P_{SR} + N_o}} \]

so that the transmitted signal by the relay terminal satisfies an average transmitted power constraint \( E\{ |y_{SR}|^2 \} = P_{SR} \).

We note that if \( h_{SR} \) were available at the destination terminal, then maximum-likelihood (ML) detection would simplify to one-shot coherent maximal ratio combining (MRC) of the form

\[
x_{MRC} = \arg \max_{x \in \mathcal{A}_M} \mathbb{E}\left\{ \sqrt{P_{SD}} h_{SD} x + y_{SD} \right\}
\]

Since \( h_{SR} \) is not available at the destination terminal, MRC in (3) cannot be utilized and the optimal receiver takes the form of ML sequence detection due to channel induced memory.

Before we proceed with the derivation of the optimum receiver, we assume that the destination terminal collects \( N \) samples transmitted by the source, say \( y_{SD,1}, y_{SD,2}, \ldots, y_{SD,N} \), and \( N \) samples transmitted by the relay, namely \( y_{RD,1}, y_{RD,2}, \ldots, y_{RD,N} \), where \( y_{SD,i} \) and \( y_{RD,i} \) denote the received samples from the \( i \)th transmission of the source and the relay, respectively, \( i = 1, 2, \ldots, N \), and forms the \( N \times 2 \) observation matrix \( Y = [y_{SD} \ y_{RD}] \) where \( y_{SD} = [y_{SD,1} \ y_{SD,2} \ldots \ y_{SD,N}]^T \) and \( y_{RD} = [y_{RD,1} \ y_{RD,2} \ldots \ y_{RD,N}]^T \). Furthermore, the channel coefficients of all underlying links \( h_{SD}, h_{SR}, \) and \( h_{RD} \) remain constant during the observation period of length \( 2N \) but fade independently over different periods as in [15], [14]. In the sequel, based on \( 2N \) observations at the destination we present ML noncoherent detection developments.

### III. Maximum Likelihood Noncoherent Detection

Given the \( N \times 2 \) observation matrix \( Y \), the ML decision metric for the transmitted sequence \( x = [x_1 \ x_2 \ldots \ x_N]^T \)

maximizes the conditional probability density function of \( Y \) given \( x \). Thus, the optimal decision is given by

\[
\hat{x}_{opt} = \arg \max_{x \in \mathcal{A}_M} f(Y|x) = \arg \max_{x \in \mathcal{A}_M} \left\{ f(y_{RD}|x)f(y_{SD}|x) \right\}.
\]

However, due to the presence of \( h_{SR} \), independence does not hold true for the elements of \( y_{RD} \) given \( x \). The conditional received vector from the relay given the transmitted block \( x \) is

\[y_{RD} = a\sqrt{P_{SR}} h_{SR} x + a h_{SR} n_{RD} + n_{RD}, \]

where \( a\sqrt{P_{SR}} h_{SR} h_{RD} \) is a singular rank-one complex Gaussian vector independent of \( n_{SR} \) and \( n_{RD} \). Therefore, \( y_{RD} \) is a zero mean complex Gaussian vector with covariance matrix

\[R = a^2 P_{SR} |h_{RD}|^2 \mathbf{I} + (a^2 |h_{SR}|^2 N_o + N_o) \mathbf{I}. \]

As a result, the ML receiver in (4) becomes

\[
\hat{x}_{opt} = \arg \max_{x \in \mathcal{A}_M} \left( \ln |R| - \frac{y_{RD}^H R^{-1} y_{RD}}{N_o} - \sum_{i=1}^N |y_{SD,i} - \sqrt{P_{SD}} h_{SD} x_1|^2 \right). \]

Using \(|A + cd^H| = |A| (1 + d^H A^{-1} c) |16\) and the fact that \( \|x\|^2 = N \), we compute \( |R| = (a^2 |h_{RD}|^2 N_o + N_o) \mathbf{I} + (N_o a^2 P_{SR} |h_{RD}|^2 + N_o a^2 P_{SR} |h_{RD}|^2 + 1) \mathbf{I}. \)

Using (6) and after some algebraic manipulations the decision rule in (5) is simplified to

\[
\hat{x}_{opt} = \arg \max_{x \in \mathcal{A}_M} \left\{ \sqrt{P_{SD}} h_{SD} y_{SD}^H x + \sqrt{P_{SD}} h_{SD} y_{SD}^H y_{RD} + \frac{a^2 P_{SR} |h_{RD}|^2 \|y_{RD}^H y_{RD}\|^2}{N_o (a^2 |h_{RD}|^2 + 1)^2} \right\}. \]

Moreover, using Matrix Inversion Lemma, the inverse of \( R \) becomes

\[R^{-1} = \frac{1}{N_o (a^2 |h_{RD}|^2 + 1)^2} \left( \mathbf{I} - \frac{a^2 P_{SR} |h_{RD}|^2 \mathbf{I}^H}{N_o (a^2 |h_{RD}|^2 + 1)^2} \right). \]

Using (6) and after some algebraic manipulations the decision rule in (5) is simplified to

\[
\hat{x}_{opt} = \arg \max_{x \in \mathcal{A}_M} \left\{ \sqrt{P_{SD}} h_{SD} y_{SD}^H x + \sqrt{P_{SD}} h_{SD} y_{SD}^H y_{RD} x + \frac{a^2 P_{SR} |h_{RD}|^2 \|y_{RD}^H y_{RD}\|^2}{N_o (a^2 |h_{RD}|^2 + 1)^2} \right\}.
\]
where \( c_1 = \sqrt{P_{SD} h_{SD}} \) and 
\[ c_2 = \sqrt{N_c^2 (\sigma^2 | h_{RD} |^2 + T_N + 2 \sigma^2 + h_{RD} \sum N_c^2 (\sigma^2 | h_{RD} |^2 + T_N)}. \]

To proceed further, we introduce the augmented vector \( s = [x^H 1]^H \) and the \((N + 1) \times (N + 1)\) matrix 
\[ Q = \begin{bmatrix} c_2 y_{RD} y_{RD}^H \quad c_1 y_{SD}^H \\ c_1 y_{SD}^H \quad y_{SD}^H \end{bmatrix} \] 
and observe that 
\[ s^H Q s = c_1 y_{SD}^H x + c_2 x^H y_{SD} + c_2^2 |y_{SD}^H x|^2 + c. \]
In (8), \( c \) is an arbitrary real constant that does not affect the maximization. Hence, \( \hat{s}_{\text{opt}} = \hat{s}_{\text{opt},1:N} \) where the \((N + 1) \times 1\) vector \( \hat{s}_{\text{opt}} \) is the solution of the equivalent optimization problem
\[ \hat{s}_{\text{opt}} = \arg \max_{s_1, s_2, \ldots, s_N \in A_M, s_{N+1}=1} s^H Q s. \] 

In general, the problem in (9) is NP-hard and can be solved through exhaustive search in \( A_{M+1} \) which requires \( O(MN) \) calculations, an intractable complexity even for moderate values of \( N \). Recently, polynomial complexity solutions for maximization problems of the quadratic form in (9) have been developed ([17]-[20]) for the case where \( Q \) is positive semidefinite and of reduced rank. In the following section, we identify whether the optimization problem in (9) can be solved with polynomial complexity and develop an efficient detection technique that approximates the solution in (9) with polynomial complexity.

**IV. EFFICIENT NONCOHERENT DETECTION**

Let \( q_1, q_2, \ldots, q_{N+1} \) be the eigenvectors of \( Q \) associated with eigenvalues \( \lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_{N+1} \). After straightforward but tedious algebraic manipulations, the characteristic equation \( |Q - \lambda I_{N+1}| = 0 \) becomes
\[ -\lambda^3 + (c + |y_{RD}|^2) \lambda^2 + (|y_{SD}|^2 - c |y_{RD}|^2 \lambda + |y_{SD}^H y_{RD}|^2 |y_{SD}|^2 - 2 |y_{SD}^H y_{RD}|^2) \lambda^{N-2} = 0. \]

The above equation yields \( N - 2 \) zero eigenvalues, hence \( Q \) is a reduced-rank matrix for \( N > 2 \). To identify the remaining three eigenvalues, we define the function \( f(\lambda) \triangleq -\lambda^3 + (c + |y_{RD}|^2) \lambda^2 + (|y_{SD}|^2 - c |y_{RD}|^2 \lambda + |y_{SD}^H y_{RD}|^2 |y_{SD}|^2 - 2 |y_{SD}^H y_{RD}|^2) \lambda \in \mathbb{R} \), and make the following observations:

1. \( f(0) = |y_{SD}^H y_{RD}|^2 - |y_{SD}|^2 |y_{RD}|^2 < 0 \) with probability 1 (w.p.1) by utilizing the Cauchy-Schwarz inequality and \( f(|y_{RD}|^2) = |y_{SD}^H y_{RD}|^2 \) w.p.1.
2. \( f(\lambda) \rightarrow -\infty \) and \( f(\lambda) \rightarrow -\infty \).
3. Combining 1 and 2 implies that \( Q \) has one negative and two positive eigenvalues, i.e. \( \lambda_{N+1} < 0 \) and \( \lambda_1 > \lambda_2 > 0 \).
4. If \( \epsilon \rightarrow \infty \), then \( \lambda_1 \rightarrow c, \lambda_2 \rightarrow |y_{RD}|^2, \) and \( \lambda_{N+1} \rightarrow 0 \).

Since \( \lambda_{N+1} < 0 \), \( Q \) is not positive semidefinite. However, if \( c \) is chosen sufficiently large, then \( \lambda_{N+1} \) can be made arbitrarily close to zero and practically negligible with respect to \( \lambda_1 \) and \( \lambda_2 \). To ensure that \( |\lambda_{N+1}| < \epsilon \), where \( \epsilon \) is an arbitrarily small positive number, we require \( \lambda_{N+1} \in [-\epsilon, 0] \). Such a requirement is met if \( f(0) < 0 \) and \( f(0) = 0 \). After straightforward calculations, we conclude that if \( c = \max\{|y_{SD}|^2, (1+c) |y_{SD}|^2 - |y_{SD}^H y_{RD}|^2\} / |y_{SD}|^2 \), then \( |\lambda_{N+1}| < \epsilon \) which implies that we may ignore \( \lambda_{N+1} \) and approximate \( Q \) with the rank-2 positive semidefinite complex matrix \( Q = \lambda_1 q_1 q_1^H + \lambda_2 q_2 q_2^H \). Accordingly, we form the near-ML maximization problem
\[ \hat{s}_{\text{opt}} = \arg \max_{s_1, s_2, \ldots, s_N \in A_M, s_{N+1}=1} s^H Q s = \arg \max_{s_1, s_2, \ldots, s_N \in A_M, s_{N+1}=1} ||Z^H s||^2 \] 
where \( Z \) is a full rank \((N+1) \times 2\) matrix such that \( ZZ^H = Q \). The computation of \( \hat{s}_{\text{opt}} \) in (10) can be implemented with complexity \( O((MN)^2) \) if we follow the methodology that has been introduced in [17]-[20] and is presented in the sequel tailored to our problem.

We introduce three auxiliary spherical coordinates \( \phi \in (-\pi, \pi), \theta, \omega \in (-\pi/2, \pi/2) \) and define the \( 4 \times 1 \) spherical vector
\[ \tilde{c}(\phi, \theta, \omega) \triangleq \begin{bmatrix} \sin \phi \\ \cos \phi \sin \theta \\ \cos \phi \cos \theta \sin \omega \\ \cos \phi \cos \theta \cos \omega \end{bmatrix} \]

as well as the \( 2 \times 1 \) spherical vector \( c(\phi, \theta, \omega) \triangleq \tilde{c}_{1:2}(\phi, \theta, \omega) + j\tilde{c}_{3:4}(\phi, \theta, \omega) \). Then, the problem in (10) is rewritten as
\[ \hat{s}_{\text{opt}} = \arg \max_{s \in A_M} \max_{s_{N+1}=1} \max_{\phi \in [-\pi, \pi], \theta \in [-\pi/2, \pi/2], \omega \in [-\pi/2, \pi/2]} |s^H Z c(\phi, \theta, \omega)| \]
due to Cauchy-Schwartz Inequality which states that, for any \( v \in \mathbb{C}^2, |v^H c(\phi, \theta, \omega)| \leq |v||c(\phi, \theta, \omega)| \) with equality if and only if \( \phi, \theta, \omega \) are the spherical coordinates of \( v \). Furthermore, \( \forall v \in \mathbb{C}^2, \Re\{v^H c(\phi, \theta, \omega)\} \leq |v^H c(\phi, \theta, \omega)| \) with equality if and only if \( \phi, \theta, \omega \) are the spherical coordinates of \( v \). Hence, the optimization problem in (12) becomes
\[ \hat{s}_{\text{opt}} = \arg \max_{s \in A_M} \max_{s_{N+1}=1} \max_{\phi \in [-\pi, \pi], \theta \in [-\pi/2, \pi/2]} |s^H Z c(\phi, \theta, \omega)| \]

We interchange the maximizations in (13) and obtain the equivalent problem
\[ \max_{\phi \in [-\pi, \pi], \theta \in [-\pi/2, \pi/2]} \max_{s_n \in A_M} \max_{s_{N+1}=1} \sum_{n=1}^{N+1} \Re\{s_n^H Z_{1:2} c(\phi, \theta, \omega)\}. \]

For a given set of angles \( (\phi, \theta, \omega) \in (-\pi, \pi) \times (-\pi/2, \pi/2) \), the maximizing argument of each term of the sum in (14) depends only on the corresponding row of \( Z \). As \( \phi, \theta, \omega \) vary, the decision in favor of \( s_n \) is maintained as long as a decision boundary is not crossed. Due to the structure of \( A_M \), the \( M/2 \) decision boundaries that affect the maximization in (14) are given by \( Z_{n+1:2} c(\phi, \theta, \omega) = Ae^{j\pi/2 \sum_{n=1}^{N+1}} \), \( A \in \mathbb{R} - \{0\}, k = 0, 1, \ldots, M/2 - 1, \) \( n = 1, 2, \ldots, N+1 \), and, hence, can be rewritten as \( \Re\{e^{j\pi/2 \sum_{n=1}^{N+1}} Z_{n+1:2} c(\phi, \theta, \omega)\} = 0, k = 0, 1, \ldots, M/2 - 1, n = 1, 2, \ldots, N+1 \), which is equivalent to
\[ Z_{l:1:4} c(\phi, \theta, \omega) = 0, \quad l = 1, \ldots, M(N+1)/2, \]
where \( \tilde{Z} \triangleq [\mathbb{I}(\tilde{Z}) \cdot \mathbb{R}(\tilde{Z})] \), \( \tilde{Z} \triangleq Z \otimes [e^{-j\pi/4} e^{-j\pi\frac{2}{3}} \ldots e^{-j\frac{M-1}{M-2}\pi}] \), and \( \otimes \) denotes Kronecker product.

The inner maximization rule in (14) motivates us to define a decision function \( s \) that maps a set of angles \((\phi, \theta, \omega)\) to a certain value of set \( A_M \) according to

\[
\begin{align*}
s(z^T; \phi, \theta, \omega) & \triangleq \arg \max_{s \in A_M} \Re \left\{ s^* z^T c(\phi, \theta, \omega) \right\} \tag{16}
\end{align*}
\]

for any \( z \in \mathbb{C}^2 \). Then, for the given \((N+1) \times 2\) matrix \( Z \), each set of angles in \((-\pi, \pi) \times (-\frac{\pi}{2}, \frac{\pi}{2})^2\) is mapped to a candidate \( M\)-ary vector

\[
\begin{align*}
s(Z_{(N+1) \times 2}; \phi, \theta, \omega) & \triangleq \begin{bmatrix}
s(z_{1,1:2}^T; \phi, \theta, \omega) \\
s(z_{2,1:2}^T; \phi, \theta, \omega) \\
\vdots \\
s(z_{N,1:2}^T; \phi, \theta, \omega)
\end{bmatrix} \tag{17}
\end{align*}
\]

and the optimal vector \( \hat{s}_{\text{opt}} \) in (13) belongs to the reduced set \( \bigcup_{\phi \in (-\pi, \pi)} \bigcup_{\theta, \omega \in (-\pi, \pi)} s(Z_{(N+1) \times 2}; \phi, \theta, \omega) \). Furthermore, since opposite \( M\)-ary vectors result in the same metric in (10), we can ignore the values of \( \phi \) in \((-\pi, -\frac{\pi}{2}] \cup [\frac{\pi}{2}, \pi] \)
and consider \( \phi, \theta, \omega \in \Phi \triangleq (-\frac{\pi}{2}, \frac{\pi}{2}] \). Finally, we collect all candidate \( M\)-ary vectors to set

\[
S(Z_{(N+1) \times 2}) \triangleq \bigcup_{\phi, \theta, \omega \in \Phi} \left\{ s(Z_{(N+1) \times 2}; \phi, \theta, \omega) \right\} \subseteq A_M^{N+1}, \tag{18}
\]

hence,

\[
\hat{s}_{\text{opt}} = \arg \max_{s \in S(Z)} \| Z^H s \|. \tag{19}
\]

Therefore, \( \hat{s}_{\text{opt}} \) belongs to a set \( S(Z_{(N+1) \times 2}) \) whose cardinality is later proved to be \( \| S(Z_{(N+1) \times 2}) \| = \mathcal{O}((MN)^3) \) and construction is achieved with complexity \( \mathcal{O}((MN)^4) \).

From (16), we observe that the rows of the \( \frac{M(N+1)}{2} \times 4 \) matrix \( \mathcal{F} \) determine \((N+1)\) hypersurfaces \( \mathcal{F}(Z_{1,1:4}), \mathcal{F}(Z_{2,1:4}), \ldots, \mathcal{F}(Z_{M(N+1),1:4}) \) that partition the hypercube \( \phi^3 \) into \( K \) cells \( C_1, C_2, \ldots, C_K \) such that \( \bigcup_{k=1}^K C_k = \phi^3 \), \( C_k \cap C_j = \emptyset \) if \( k \neq j \), with each cell \( C_k \) corresponding to a unique \( s_k \in A_M^{N+1} \). Let \( \{i_1, i_2, i_3\} \subseteq \{1, 2, \ldots, \frac{M(N+1)}{2}\} \) be a subset of three indices (that correspond to three hypersurfaces) and \( \phi(\tilde{Z}_{M(N+1) \times 2}; i_1, i_2, i_3) \) equal the vector of coordinates of the intersection of hypersurfaces \( \mathcal{F}(Z_{1,1:4}), \mathcal{F}(Z_{2,1:4}), \mathcal{F}(Z_{M(N+1),1:4}) \). It can be shown that a “collection” of \( 3 \) hypersurfaces, say \( \mathcal{F}(Z_{1,1:4}), \mathcal{F}(Z_{2,1:4}), \mathcal{F}(Z_{3,1:4}) \), has a unique intersection (which is a vertex of a cell) if and only if no more than two hypersurfaces originate from the same row of the observation matrix \( Z \). Such a cell, say \( C(\tilde{Z}_{M(N+1) \times 4}; i_1, i_2, i_3) \), is associated with a unique vector \( s(\tilde{Z}_{M(N+1) \times 4}; i_1, i_2, i_3) \). We collect all such vectors to set

\[
J(\tilde{Z}_{M(N+1) \times 4}) \triangleq \bigcup_{i_1, i_2, i_3 \in \{1, \ldots, \frac{M(N+1)}{2}\}} \{ s(\tilde{Z}_{M(N+1) \times 4}; i_1, i_2, i_3) \} \subseteq A_M^{N+1} \tag{20}
\]

with cardinality \( \| J(\tilde{Z}_{M(N+1) \times 4}) \| = \frac{M(N+1)}{2} \times N \) equal to \( \frac{N(N+1)}{2} \times \frac{M(N+1)}{2} \) if \( M > 2 \) and \( \frac{N(N+1)}{6} \times \frac{M(N+1)}{2} \) if \( M = 2 \). Thus, \( J(\tilde{Z}_{M(N+1) \times 4}) \) contains \( \mathcal{O}((MN)^3) \) \( M\)-ary vectors. Then, it can be shown [19] that all candidate vectors form the set

\[
S(Z_{(N+1) \times 2}) = J(\tilde{Z}_{M(N+1) \times 4}) \cup J(\tilde{Z}_{M(N+1) \times 2}) \tag{21}
\]

where \( \| J(\tilde{Z}_{M(N+1) \times 2}) \| = \frac{M(N+1)}{2} \). To summarize, we have utilized three auxiliary spherical coordinates and partitioned the hypercube \( \phi^3 \) into \( \mathcal{O}((MN)^3) \) cells associated with unique \( M\)-ary vectors that constitute the set \( S(Z_{(N+1) \times 2}) \subseteq A_M^{N+1} \) which includes \( \hat{s}_{\text{opt}} \) in (12). Therefore, the initial detection problem in (10) has been converted into a maximization among \( \mathcal{O}((MN)^3) \) candidate vectors.

The construction of \( S(Z_{(N+1) \times 2}) \) is of special interest since it determines the overall complexity of the proposed method. According to (21), it reduces to the parallel construction of \( J(\tilde{Z}_{M(N+1) \times 4}) \) and \( J(\tilde{Z}_{M(N+1) \times 2}) \), which can be also be fully parallelized since cells in the hypersurface arrangement are examined independently from each other. It can be shown that the evaluation of the decision function in (16) at the intersection of the 3 hypersurfaces under consideration determines definitely the corresponding symbol \( s_n \) if and only if no hypersurface originates from \( Z_n \).

For the hypersurfaces that pass through the cell intersection, the rule in (16) becomes ambiguous. In such a case, definite determination of \( s_n \) is attained if \( \omega \) is set to \( \frac{\pi}{2} \) and (16) is examined at the intersection of the same hypersurfaces except from the hypersurface of interest.

The algorithm for the construction of \( S(Z_{(N+1) \times 2}) \) is available at http://www.telecom.tuc.gr/~kary-stinos. The algorithm visits independently the \( |S(Z_{(N+1) \times 2})| = \mathcal{O}((MN)^3) \) cells and computes the candidate vector in \( A_M^{N+1} \) for each cell. The cost of the algorithm for each candidate vector is \( O(MN) \). Therefore, the overall complexity for the construction of \( S(Z_{(N+1) \times 2}) \) becomes \( \mathcal{O}((MN)^3) O(MN) = \mathcal{O}((MN)^4) \).

V. SIMULATION STUDIES AND COMPARISONS

As an illustration, we consider BPSK transmissions and conduct Monte Carlo simulations for 1,000 channels. We assume that \( P_{SD} = P_{BR} \), \( N = 15 \), and \( \epsilon = 0.1 \). In Fig. 2, we set the relay output SNR constraint \( \frac{P_{BR}}{N} \) to 10dB and plot the bit error rate (BER) of the one-shot coherent MRC receiver, the ML block noncoherent receiver of complexity \( \mathcal{O}(M^N) \), and the proposed near-ML receiver of complexity \( \mathcal{O}((MN)^4) \) as a function of the source-destination SNR \( \frac{P_{BR}}{N} \). It is demonstrated that the proposed reduced-complexity detector attains practically ML performance, justifying the validity of our reduced-rank approximation. In Fig. 3 we set \( \frac{P_{BR}}{N} = \frac{P_{BR}}{N} \to 10dB \) and present BER and computational complexity curves versus block length \( N \). In Fig. 3(a), the BER of the MRC receiver is also presented as a performance lower bound.
As the block size \( N \) increases, the performance of the ML noncoherent detector approaches that of the coherent one. Again, the proposed receiver attains practically ML performance. We emphasize that for \( N > 15 \) the exponential-complexity ML receiver cannot be implemented in reasonably small time while the proposed receiver maintains polynomial computational complexity and performs practically at ML levels. Fig. 3(b) demonstrates the significant complexity gain offered by the proposed receiver. For example, if \( N = 40 \), then the ML receiver requires an exhaustive search among \( 2^{40} \approx 10^{12} \) binary vectors of length 40 while the proposed near-ML receiver performs a search among \( \frac{(N+1)!}{N!(N-1)!} + N+1 \approx 10^4 \) binary vectors of length 41.

REFERENCES


