Optimum Bandwidth Partitioning with Analog-to-Digital Converter Constraints

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Abstract

In certain communication channels, such as short copper twisted pairs, it is theoretically possible to perform transmission with very high spectral efficiency using a very wide bandwidth. However, current Analog-to-Digital Converter (ADC) technology limits the allowable sampling rate and resolution, thus severely constraining the transmission speeds. This paper proposes the partitioning of the available bandwidth into multiple bands each employing an independent ADC. The benefit of such a scheme is the reduction of the sampling rates of the ADCs. An increase in the dynamic range is allowed, thus offering the potential to realize very high spectral efficiencies. An analysis of transmission under ADC constraints is performed, where an expression for the achievable data rate is derived based on an empirical rule for the trade-off between the ADC sampling rate and resolution. A bandwidth partitioning problem is formulated, where the objective is the maximization of the data rate and the optimization parameters are the frequency band assignments. Then, a practical example of transmission over CAT-5 cable is considered. The possible impairment factors are outlined, and the essential system elements are described. Using the previously given algorithm, the optimum solution and the corresponding performance are given for two distinct scenarios. These scenarios serve to illustrate the bandwidth partitioning procedure, and provide useful intuition regarding the application of the proposed method. In particular, it is deduced that it is best to have narrower bands in frequencies where the channel is strong, and wider bands in frequencies where the channel is weak.

Keywords

Local area networks, communication systems, analog-digital conversion, frequency division multiplexing.

I. INTRODUCTION

The interest in designing high-speed wireline communication systems remains very strong, despite the huge capacity potential of fiber optics. This interest is exemplified by current efforts to deliver Very-high-speed Digital Subscriber Line (VDSL) systems [1], recent standardization developments in 1000BASE-T and 10GBASE-T Local Area Networks (LANs) [2], [3], as well as the increasing demand for multi-Gbps chip-to-chip communication [4]. Additionally, two trends can be spotted: First, DSL systems seem to converge with LAN systems as demonstrated by the Ethernet in the First Mile (EFM) initiative [5]. Second, chip-to-chip communication systems (e.g. serial links) are starting to employ more advanced techniques such as multi-level transmission and equalization [6], [7], [8], in order to meet the increasing need for higher rates.

There is evidence that a significant number of future DSL systems will be characterized by short loop lengths, possibly approaching the span of today’s LAN systems. Multi-Tenant/Multi-Dwelling Units and Fiber-To-The-Cabinet extensions are already fast growing applications for high-end DSL systems. This development can significantly improve the broadband access speeds, provided that the communication systems take advantage of the increase in the offered capacity. Although a shorter loop implies smaller attenuation, impairment factors such as Inter-Symbol Interference (ISI), Far-End Crosstalk (FEXT), Near-End Crosstalk (NEXT), background and impulse noise remain, and actually FEXT becomes much stronger. It has been shown that with reasonable complexity, VDSL can reach downstream rates in the order of 50-70Mbps at loop lengths around 600m [9]. At a sampling rate of 35Msamples/sec and a required resolution around 12 bits, the specifications of VDSL represent the state of the art in ADC design [10]. If more bandwidth is exploited, then yet higher rates are possible in short loops, provided that the ADC design obstacles are overcome.

Similar limitations exist in the 1000BASE-T system, where 4 pairs of copper cable (CAT-5) are utilized to deliver an aggregate rate of 1Gbps at distances less than 100m. The sampling rate is 125Msamples/sec and the spectral efficiency of each pair is
The new 10GBASE-T system design will surely have to increase both the utilized bandwidth and the spectral efficiency, thus severely straining the ADC design.

A potential remedy to the problem of the ADC constraints in the above mentioned applications is the use of multiple “interleaved” ADCs [11]. This solution has already been adopted in the design of multi-Gbps transceivers for digital interfaces [12], [13]. These transceiver implementations employ multiple Digital-to-Analog Converters (DACs) and multiple ADCs, and perform time-division-multiplexing by adjusting the DAC and ADC phases. The resulting interferences (including ISI) are mitigated through Multiple-Input Multiple-Output (MIMO) equalization and precoding.

The method proposed in this paper is the partitioning of the available bandwidth into a set of contiguous, disjoint bands, each having an independent ADC. The required sampling rate of each ADC is then reduced, so that the resolution can be increased, thus potentially leading to a data rate increase. One of the main conclusions is that, in several cases, optimum partitioning results in significantly superior performance compared to a “naive” equal-band partitioning.

The paper is organized as follows: Section II derives the achievable data rate of the system taking into account the ADC constraints. Specifically, an empirical rule is given that defines the trade-off relationship between the sampling rate and the resolution. The derived data rate expression holds regardless of whether Multi-Carrier Modulation (MCM) or Single-Carrier Modulation (SCM) is employed. Section III formulates the optimization problem with the assumption of multiple bands. The objective is the maximization of the data rate under a Power Spectral Density (PSD) constraint, and the parameters to be optimized are the widths of the sub-bands. Section IV considers the design aspects of a communication system over CAT-5 copper cabling. The channel and noise sources are first outlined, and the essential elements and parameters of the system are described. Section V gives simulation results for the CAT-5 system making use of optimum bandwidth partitioning. The optimization problem is solved using the Nelder-Mead simplex method. Tables of data rates are presented for two distinct loop lengths, and the bandwidth partitioning procedure is illustrated for several scenarios. Finally, Section VI concludes the paper.

II. ADC Analysis for Passband Transmission

The analysis that follows applies equally well to both MCM and SCM systems. Consider a digital transmission system in a passband channel with ISI, as shown in Figure 1. The modulated symbols $x_k$ are pulse shaped by $\phi(t)$, then the signal is injected into the channel $h(t)$ and receiver noise $n_r(t)$, white with variance $\sigma^2_r$ per dimension, is added. At the receiver, the Automatic-Gain-Control (AGC) module scales the channel output by $g_{AGC}$, the resulting signal passes through the anti-aliasing filter $f(t)$, and finally the ADC produces the samples $y_k$. This is assumed to be an equivalent baseband model, so all signals and filters may actually be complex-valued. The above model can be converted to a discrete-time representation as shown in Figure 1. The pulse shaping filter, channel, AGC scaling, and anti-aliasing filter are combined into the $p_k$ filter. The effects of AGC scaling and anti-aliasing filtering on the receiver noise are “absorbed” by $n_{r,k}$, and the finite precision effects of the ADC are modeled by the quantization noise samples $n_{q,k}$.

Note that the actual ADC implementation involves 2 ADC’s, one for the in-phase and one for the quadrature signal component. Obviously, the quantized outputs form a grid on the complex plane. The quantization noise $n_{q,k}$ is approximated as being white and uncorrelated to the ADC input. The smallest difference between quantized outputs is:

$$\Delta = R_m 2^{-b}$$  \hspace{1cm} (1)

where $b$ is the number of bits of the ADC per dimension, and $R_m$ is the signal range. Thus, the variance per dimension of the
uniformly distributed \( n_{q,k} \) is:

\[
\sigma_q^2 = \frac{R_m^2 \sigma^{-2b}}{12}
\]  

(2)

and:

\[
b = \frac{1}{2} \log_2 \left( \frac{\bar{E}_{r,\text{peak}}}{\sigma_q^2} \right) = \frac{1}{2} \log_2 (DR)
\]

(3)

where \( \bar{E}_{r,\text{peak}} = R_m^2/12 \) is a measure of the peak received energy per dimension (obtained by assuming that the received signal is uniformly distributed in the ADC range), and \( DR = \bar{E}_{r,\text{peak}}/\sigma_q^2 \) is the dynamic range of the ADC.

In the rest of the paper, the following definition of the receiver peak-to-average ratio (PAR) is used:

\[
\text{PAR}_r = \frac{\bar{E}_{r,\text{peak}}}{\bar{E}_r}
\]

(4)

where \( \bar{E}_r = \|p\|^2 \bar{E}_x + \sigma^2 \) is the average received energy per dimension. Therefore, letting \( \sigma^2 = \sigma_r^2 + \sigma_q^2 \), and \( SNR_{MFB} = \|p\|^2 \bar{E}_x/\sigma^2 \) (matched-filter-bound SNR), one obtains:

\[
b = \frac{1}{2} \log_2 (\text{PAR}_r) + \frac{1}{2} \log_2 \left( 1 + \frac{\|p\|^2 \bar{E}_x}{\sigma^2} \right) + \frac{1}{2} \log_2 \left( 1 + \frac{\sigma_r^2}{\sigma_q^2} \right)
\]

\[
= \frac{1}{2} \log_2 (\text{PAR}_r) + \frac{1}{2} \log_2 (1 + SNR_{MFB}) + \frac{1}{2} \log_2 \left( 1 + \frac{\sigma_r^2}{\sigma_q^2} \right)
\]

(5)

where \( \bar{E}_x = E (x_k^2)/2 \) is the average transmitted energy per dimension, and \( \|p\|^2 = \sum_k |p_k|^2 \).

Equation (5) is illustrated in Fig. 2, where the levels of the various signals are depicted in logarithmic scale. In the example on the left, the quantization noise is chosen to be much smaller than the receiver noise, so that its effect on the total noise is small, and thus the \( SNR_{MFB} \) is primarily constrained by the receiver noise. However, this implies a large dynamic range, \( DR \), for the ADC, or equivalently a large number of bits \( b \). In the example on the right, the quantization noise dominates the total noise, which consequently restricts the \( SNR_{MFB} \) to a value much smaller than the one that is achievable in the absence of quantization noise. In this case, the required number of bits, \( b \), is also smaller, possibly resulting in a less stringent ADC implementation. These examples are indicative of the possible trade-off between ADC complexity and system performance.

The design of an ADC is fundamentally constrained by a trade-off between sampling rate and resolution. Assuming that some measure of ADC cost is fixed (e.g. die size, or power consumption), one can express this trade-off through the following empirical equation [14]:

\[
b = b_o - b_s \log_2 f_s (\text{MHz})
\]

(6)

where \( b \) is the number of ADC bits, \( f_s \) is the ADC sampling rate in MHz, and \( b_o, b_s \) are constants used for modeling purposes. The constant \( b_o \) is the number of ADC bits attained at a sampling rate of 1MHz, while \( b_s \) is the number of bits by which the resolution is decreased per octave.

Having a fixed sampling rate \( f_s \), and using (3) and (6), one obtains:

\[
\sigma_q^2 = \frac{\bar{E}_{r,\text{peak}}}{2^{2b_o} f_s^{-2b_o}} \approx PAR_r \|p\|^2 \bar{E}_x 2^{-2b_o} f_s^{2b_s}
\]

(7)

(8)
where the approximation is made that the useful signal at the receiver is much larger than the total noise. Therefore, the following result is obtained:

\[
SNR_{MFB} = \frac{\|p\|^2 \hat{\sigma}_x^2}{\sigma_r^2 + PAR_r \|p\|^2 \hat{\sigma}_x^2 2^{-2b_x} f_x^{2b_x}}
\] (9)

Certain interesting observations can now be made: If the sampling rate of the channel is small, then the ADC resolution is allowed to be high, and quantization noise is made negligible compared to the receiver noise, so that \(SNR_{MFB} \approx \|p\|^2 \hat{\sigma}_x^2 / \sigma_r^2\). On the other hand, if the sampling rate is high, then quantization noise dominates, and \(SNR_{MFB} \approx 2^{2b_x} f_x^{-2b_x} / PAR_r\), which in logarithmic scale equals the dynamic range minus the \(PAR_r\).

With certain approximations, the \(SNR_{MFB}\) value can be used to estimate the achievable data rate of the channel. The cases of single carrier transmission with Decision Feedback Equalization (DFE), and multiple carrier transmission with Discrete Multi-Tone modulation (DMT) are both considered here. The basic assumption is that the SNR of the used channel band is high, which is quite realistic in the transmission environments that are later used as examples.

Assuming zero-forcing DFE equalization, a measure of performance is the biased detection SNR, which is expressed as:

\[
SNR_{ZF-DFE} = \frac{SNR_{MFB}}{\gamma_{ZF-DFE}}
\] (10)

where \(\gamma_{ZF-DFE}\) is a constant that depends on the channel. The achievable number of transmitted bits per dimension can then be approximated as [15]:

\[
\bar{c}_{scm} = \frac{1}{2} \log_2 \left( \frac{SNR_{ZF-DFE}}{\Gamma} \right)
\] (11)

where \(\Gamma\) is the SNR-gap and is a function of the modulation scheme, of the coding applied, of the probability of error, and of the required noise margin. Therefore, from (10) and (11), one finds that:

\[
\bar{c}_{scm} = \frac{1}{2} \log_2 \left( \frac{SNR_{MFB}}{\gamma_{ZF-DFE} \Gamma} \right)
\] (12)

Similar analysis can be performed for other equalization methods, such as minimum-mean-square-error DFE, or various forms of linear equalization. With DMT transmission, the achievable number of bits per dimension is approximated as [16]:

\[
\bar{c}_{mcm} = \frac{1}{2} \log_2 \left( \frac{SNR_{geo}}{\Gamma} \right)
\] (13)

with:

\[
SNR_{geo} = \left( \prod_{m=1}^{N_c} SNR_m \right)^{1/N_c}
\] (14)

where \(N_c\) is the number of carriers, \(SNR_m = |P_m|^2 \hat{\sigma}_x^2 / \sigma_r^2\), and \(P_m\) is the sub-channel gain of the \(m\)-th carrier (derived by applying the Discrete Fourier Transform on \(p_k\)). Then,

\[
SNR_{geo} = \frac{SNR_{MFB} \left( \prod_{m=1}^{N_c} |P_m|^2 \right)^{1/N_c}}{\|p\|^2} = \frac{SNR_{MFB}}{\gamma_{DMT}}
\] (15)

Finally:

\[
\bar{c}_{mcm} = \frac{1}{2} \log_2 \left( \frac{SNR_{MFB}}{\gamma_{DMT} \Gamma} \right)
\] (17)
Evidently, when either SCM or MCM is used, the achievable data rate can be expressed as a function of $SNR_{MFB}$ as defined in (9). A conclusion that can be drawn from the above is that a large SNR-gap implies an inefficient utilization of the available ADC dynamic range. A large $\Gamma$ implies that the $SNR_{MFB}$ is “wasted”.

It should be noted that equations (12) and (17) make the tacit assumption that the noise is Gaussian. However, the total receiver noise may not be Gaussian, due to the fact that quantization noise is uniform, and that other noise components (e.g. crosstalk) may deviate from the Gaussian distribution. In those cases where non-Gaussian noise dominates the total noise, these expressions should be interpreted as lower bounds for the achievable data rate.

III. BANDWIDTH PARTITIONING

In certain communication scenarios, the channel has a high SNR, implying that a large number of bits can be transmitted per unit of bandwidth. If additionally the available bandwidth is very large, then it is clear that the ADC is the constraining factor in the system design. In order to circumvent this limitation, the concept of using multiple ADCs has been suggested. In [13], multiple ADCs interleaved in time are employed to achieve transmission at multi-Gbps rates. However, ISI effects lead to the need for MIMO equalization, which is performed through transmitter precoding. Here, a different approach is chosen, where the channel is partitioned into multiple bands, and an ADC is assigned to each band. Passband transmission is taking place in each band, and essentially the corresponding transmitters and receivers operate independently. Having more bands leads to smaller sampling rates, so a larger number of ADC bits can be used, and the dynamic range can thus be made large enough for the system to achieve large transmission. The signals are separated by analog filtering, which implies that the band must be spaced with room for adequate guard bands. The analysis that follows applies equally to either single or multiple carrier modulation.

Assume that the system employs a specified total bandwidth of $B_{total}$, and that cost considerations limit the number of bands to $N$. Also, assume that the energy per dimension is specified to be $\tilde{\mathcal{E}}_x$, which combined with the bandwidth constraint leads to an implicit total power constraint. Notice that transmission optimization through spectrum shaping can be taken into account through the pulse shaping filter $\phi(t)$. The question is how should the available bandwidth be partitioned into bands, so that the transmitted data rate is maximized. Formally, this optimization problem is expressed as follows:

$$\max \sum_{i=1}^{N} 2f_{s,i} \bar{c}_i$$

subject to $$\sum_{i=1}^{N} f_{s,i} (1 + a_i) (1 + g_i) \leq B_{total}$$

where $f_{s,i}$ is the sampling rate of the $i$-th ADC, $\bar{c}_i$ is the achievable data rate per dimension in the $i$-th band, $a_i$ is the percentage of excess bandwidth in the $i$-th band, $g_i$ is the percentage of guard band in the $i$-th band, and $l_i$ is the percentage data rate loss in the $i$-th band due to factors such as the cyclic prefix loss in DMT. Replacing $\bar{c}_i$ in (18) with the expression of either (11) or (17), and utilizing (9) the optimization objective becomes:

$$\sum_{i=1}^{N} 2f_{s,i} \bar{c}_i = \sum_{i=1}^{N} l_i f_{s,i} \log_2 \left( \frac{SNR_{MFB,i}}{\gamma_{t,i}} \right) = \sum_{i=1}^{N} l_i f_{s,i} \log_2 \left[ \frac{\| p^{(i)} \|^2 \tilde{\mathcal{E}}_x}{\gamma_{t,i} \left( \sigma_r^2 + PAR_r \| p^{(i)} \|^2 \tilde{\mathcal{E}}_x 2^{-2b_o} f_{s,i}^{2b_o} \right)} \right]$$

(20)
where $\gamma_{t,i} = \gamma_{ZF-DFE,i}\Gamma$ (SCM), or $\gamma_{t,i} = \gamma_{DMT,i}\Gamma$ (MCM), and $\gamma_{ZF-DFE,i}, \gamma_{DMT,i}, p_{i}^{(i)}$ correspond to the $i$-th band. Finally, the optimization problem becomes:

$$\max \sum_{i=1}^{N} l_{i} f_{s,i} \log_{2} \left[ \frac{p_{i}^{(i)}}{\gamma_{t,i} \left( \sigma_{r}^{2} + PAR_{r} \|p_{i}^{(i)}\|^{2} \mathcal{E}_{x} 2^{-2b_{s}} f_{s,i}^{2} \right)} \right] \tag{21}$$

subject to

$$\sum_{i=1}^{N} f_{s,i} \left( 1 + a_{i} \right) \left( 1 + g_{i} \right) \leq B_{\text{total}} \tag{22}$$

where the quantities that must be optimized are $f_{s,i}, \ i = 1, \ldots, N$.

The constraint of (22) clearly forms a convex set in $f_{s,i}, \ i = 1, \ldots, N$. It is later shown by example that the objective function of (21) is not concave, and it is not even quasi-concave. Still, the function is in most cases well-behaved, and the Nelder-Mead simplex method [17] is in most cases successful in finding the global optimum.

### IV. Multi-Gbps Transmission over Copper Cabling

An application of the above described method is multi-Gbps communication over copper cabling. The IEEE standard 802.3 [18] specifies the Carrier Sense Multiple Access with Collision Detection (CSMA/CD) access method (also known as Ethernet) that is very popular for LANs. The standard includes many supplements, where each addresses the physical layer for different transmission media (coaxial cable, fiber and copper twisted pairs). The latest physical layer specification for copper is 1000BASE-T [2], and refers to operation at 1Gbps over 4-pair CAT-5 balanced copper cabling, where the minimum cable span for which requirements must be met is 100m. Recently, a new study group within IEEE 802.3 has initiated work on the specification of a 10GBASE-T system [3].

Motivated by this, this section describes the fundamental methods and parameters for physical layer communication over CAT-5 cabling at rates of multiple Gbps. The aggregate data stream is divided into 4 sub-streams, which are correspondingly transmitted over the 4 twisted pairs. Taking advantage of the preceding analysis, the available bandwidth is partitioned into multiple bands in order to relax the ADC requirements and better exploit the available capacity. Thus, each of the sub-streams is further subdivided into sub-sub-streams, which are transmitted over a corresponding band.

#### A. Transmission Environment

A CAT-5 cable contains 4 pairs of copper twisted pairs, and all 4 are utilized for transmission. In typical installations, several CAT-5 cables are actually adjacent to each other. Note that crosstalk is induced not only within the pairs of a specific cable, but also between pairs of neighboring cables.

The frequency response of a CAT-5 pair for various lengths is shown in figure 3. These plots are produced using the methodology and parameters of [19, Chapter 3]. It should be noted that these parameters have only been validated for a range of frequencies much smaller than the one that is examined here.

A fundamental consideration in twisted pair communication is crosstalk. A signal transmitted on a specific pair causes FEXT noise at the far-end receivers, as well as NEXT noise at the near-end receivers. Since CAT-5 cable is not shielded, crosstalk is induced on both the pairs within the cable and also on the pairs in adjacent cables. Figure 4 gives the crosstalk coupling functions together with the loop attenuation for a 100m loop. These curves are obtained by the crosstalk models employed in [20] and they refer to 99% worst-case crosstalk within the pairs of a cable. Although measurements regarding crosstalk coupling between pairs
of neighboring cables have not been widely publicized, a common assumption is that it is 10dB weaker than the one between pairs of the same cable.

The typical application of such a system is for a private network in server farms or in office buildings requiring high-bandwidth computing. Thus, it is expected that the crosstalk sources from neighboring cable will primarily be similar CAT-5 systems. Other transmission-impairing factors include the echo signal, impulse noise and radio frequency interference (RFI).

B. System Architecture

The IEEE 802.3 standard allows the use of repeaters, which serve to connect segments of the network. The task of a repeater is to retransmit data to all segments attached to it, thus acting as a “hub”. When the physical medium does not allow multiple access (e.g. twisted pairs), then the function of the repeaters is fundamental for the operation of the network, since otherwise the network is diminished to a single point-to-point connection. In the following, it is assumed that the network contains a single repeater with multiple ports that are connected to corresponding stations (commonly referred as Data Terminal Equipment, or DTE).

Evidently, this topology is identical to the one encountered in DSL systems, where the repeater is replaced by the Central Office (CO), and the DTE is replaced by the Customer Premises Equipment (CPE). This allows the use of a variety of crosstalk suppression techniques previously proposed for DSL.

The solution suggested here for the suppression of NEXT noise is the adoption of Time-Division-Duplexing (TDD). In that case, NEXT originating either from neighboring pairs of the same cable or from adjacent cables is eliminated. However, this task is complicated by the fact that the cables may have different lengths that result in different propagation delays. An elegant solution to this problem has been given in the context of MCM-VDSL and is known as zippering [21]. Since this is not the focus of this paper, no further details will be given about this technique.

Methods for FEXT suppression in MCM-VDSL can also be employed for the CAT-5 system. Specifically, [22] gives a decision-feedback method for the removal of crosstalk in upstream DSL communication, as well as a precoding method capable of attaining crosstalk-free reception in downstream DSL communication. It is conceptually straightforward to adapt these methods for the above topology, so that both FEXT from the same cable, and possibly FEXT from neighboring cables are cancelled. Again, the reader is referred to the paper for further details.

Finally, it is noted that the transmission power should be suppressed in the amateur radio bands, in order to prevent radio frequency ingress and egress.

C. System Parameters

In this subsection, a number of system parameters are defined that are later used in the simulations. The probability of error is set to be $10^{-8}$. Assuming that the coding gain is 6dB, and the noise margin is 3dB, the resulting SNR-gap is $\Gamma = 7.2$dB.

The maximum PSD level is defined to be $-60$dBm/Hz, but no explicit total energy constraint is taken into account. In order to take advantage of the previously mentioned crosstalk cancellation methods, DMT transmission is considered. Assuming that crosstalk suppression is in place, the total receiver noise is set to $\sigma_r^2 = -130$dBm/Hz. Approximating the transmitted samples by a Gaussian random variable, setting the clipping probability to $10^{-8}$, and using no PAR reduction techniques, the transmitter PAR is 15.2dB. In the following, the receiver PAR is assumed to be $PAR_r = 18$dB.

With regard to the ADC technology, the choices $b_o = 15$ and $b_s = 1$ are made. The excess bandwidth parameters are set to
\( a_i = 0.05 \) for all \( i \), and the guard bands are defined by \( g_i = 0.2 \) for all \( i \). The percentage data rate loss is ignored, thus \( l_i = 1 \) for all \( i \). In the absence of relevant data, the AGC scaling is ignored.

V. SIMULATION RESULTS

Figures 5 and 6 show the contour plots of the achievable data rates with 25m and 150m CAT-5 cable correspondingly. The number of bands is taken to be 2, so that visualization is possible. The contour points correspond to different band assignments, where the bandwidths are here defined to include excess bandwidth and guard bands. For 25m, it is clear that the data rate is maximized by utilizing all the available bandwidth, and equally distributing it between the two bands. For 150m, the data rate is maximized at a point where only part of the total bandwidth is used, and the two bands have much different widths. Actually, in this scenario, a “naive” implementation of equal bands occupying the whole available bandwidth achieves only 3.5Gbps compared to the 5.1Gbps of the optimized assignment. These effects are discussed in more detail later. Also, it should be clear that the objective function is not convex and not even quasi-convex (see upper left region of figure 6).

Despite the non-convexity, the objective function is reasonably well-behaved, so that the Nelder-Mead simplex method [17] yields good results. This is a direct search method that does not use gradient information, and is implemented by the fminsearch routine in the MATLAB optimization toolbox [23]. A simplex is defined in the \( N \)-dimensional search space by \( N + 1 \) vertices. At each step of the algorithm, new points are generated, they are compared with the function values at the vertices of the simplex, and they usually replace some of the vertices, thus giving a new simplex. The algorithm is terminated, when the diameter of the simplex is less than some specified tolerance. The method applies for unconstrained problems only. In order to take the constraint of (22) into account, the objective function is modified to equal 0 outside the constraint set. Typically, when the initial point is far from the constraint set boundary, the method converges either to the global optimum or to a point quite close to the global optimum. Despite the simplicity of the method, it has been found to perform better than sequential quadratic programming (as implemented in the fmincon routine of the MATLAB optimization toolbox).

Using the above described optimization algorithm, the achievable data rates for several scenarios have been computed. The different scenarios are characterized by loop lengths of 25m and 150m, between 1 and 4 bands, and maximum total bandwidth constraint of 100, 200, 300, and 400MHz. The results are presented in figures 7 and 8.

Looking at these figure, it is easy to make certain observations: Naturally, increasing the loop length results in lower data rate. However, this effect is influenced by the fact that at short loops the quantization noise is the constraining factor, so the channel improvement does not yield the expected benefit. The communication system is in all cases constrained by the ADC, thus increasing the number of bands always increases the achievable dynamic range that in turn boosts the data rate. Finally, increasing the total bandwidth has a positive effect on the data rate. Still, this effect diminishes in the long loop, where large attenuation at high frequencies means that these frequencies are not beneficial for communication. Under the previously mentioned assumptions, it was found that a 10Gbps CAT-5 system is feasible at 100m, provided that 4 bands are used occupying a total of 400MHz.

The inherent trade-offs between quantization noise, bandwidth and the number of bands can be better illustrated by plotting the bit distributions for various loop lengths. (For the purpose of illustration, an MCM system is here assumed.) Figures 9 and 10 show the bit distributions for 25m and 150m with quantization noise being taken into account, but also in the absence of quantization noise (thus assuming ideal ADCs). The bandwidth partitions for 1-4 bands correspond to the optimum settings obtained by the optimization procedure.
First, the plots corresponding to 25m are examined in Figure 9. If quantization noise is ignored, the achievable number of bits defines an upper bound. If only one band is used, then the ADC-constrained number of bits is significantly lower than the upper bound. Employing more than one bands gradually improves the spectral efficiency, however, even with 4 bands the number of bits is still much lower than the upper bound. Additionally, it should be noted that the bands all have equal widths, implying that the quantization noise is dominant and that trade-offs between quantization noise and sampling rates are not possible.

Then, the case of a 150m loop is investigated in Figure 10, where the channel attenuation is strong at high frequencies. When a single band is used, the optimum choice is not to use all the bandwidth, thus increasing the allowed dynamic range, and consequently the number of bits. If multiple bands are used, the dynamic ranges of the ADCs indeed increase, which enables a larger number of bits per tone, as well as a higher utilization of the available bandwidth. The more the bands the larger the bandwidth used. Clearly, the optimum band assignment corresponds to bands of non-equal width. The justification is the following: In frequencies where the channel is “bad”, some quantization noise can be tolerated, since the total noise is not significantly influenced. Thus, the bands in those frequencies can be made wider. On the other hand, it is important to reduce the quantization noise in frequencies where the channel is “good”, in order to exploit the higher capacity. Therefore, the bands in those frequencies must be made narrower. Finally, the bit distributions approach the upper limits, indicating that the communication is not as constrained by the ADC constraints as it is in the short loops.

It is worth mentioning that these plots validate the high SNR approximation previously made in section II, since the number of bits is in almost all cases quite large.

VI. Conclusion

This paper dealt with the issue of designing a communication system by taking the ADC constraints into account. An approximate analysis of transmission with quantization noise was made, and the solution of using multiple bands and corresponding ADCs was proposed. This gave rise to an optimization problem, where the parameters to be optimized were the band assignments. This treatment is most relevant in channels of large bandwidth and small attenuation, for example short CAT-5 cable. In order to use the CAT-5 system as an example, a possible system architecture was given and its parameters were defined. Simulation results were presented to demonstrate the trade-offs between quantization noise and sampling rate that occur among the multiple bands. It was observed that “good” frequencies need narrower bands, while “bad” frequencies need wider bands.

Acknowledgment

The authors would like to thank the anonymous reviewers for their very helpful comments.

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