Contouring Control of Machine Tool Feed Drive Systems: A Task Coordinate Frame Approach

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Abstract—In this paper, we will show that contouring performance can be viewed as a regulation problem in a moving task coordinate frame that is attached to the desired contour. By transforming the machine tool feed drive dynamics to this task coordinate frame, a control law is designed to assign different dynamics to the tangential and normal directions. The transformation also illustrated the effect of contour curvature and feed rate in the control action as well as the system dynamics in the task coordinate frame. The resulting control law consists of a linear time-varying proportional-plus-derivative (PD) position error feedback control law and a linear time-invariant trajectory feedforward control law. Experimental results of the proposed control law on the motion axes of a machining center, Matsuura MC510V, showed significant improvement of the contouring accuracy compared to the existing servo controller as well as the successful decoupling between the tangential dynamics and the normal (contouring) dynamics for feed rates up to 6 m/min (4 in/s).

Index Terms—Industrial control, machine tools, manufacturing, motion control, servo systems.

I. INTRODUCTION

In machining applications, one of the most important issues is the elimination of machining errors to ensure the quality of the final product. One important measure of product quality is the dimensional accuracy of the final workpiece. Machine tool literature contains many references on the subject of reducing machining errors. A summary of the various strategies to compensate for machine tool errors is given by Blaedel [1]. Donmez et al. [2] have developed a real-time compensation scheme to compensate for geometric and thermally induced errors in a computer numerical controlled (CNC) turning process. Many researchers have studied process models to estimate cutting forces and surface errors in milling processes [3]–[5]. Ulsoy and Koren [6] have provided an overview concerning the control of machining processes.

Among the causes of machining errors, it has been shown that dynamic positioning errors can contribute as much as 90% of the machining error [7]. Two major causes to the degradation of positioning accuracy are: “radial reduction” which stems from the lack of bandwidth in the feed drive servos of the machine tool and nonlinearities such as low velocity friction. Although with the addition of feedforward compensation, e.g., zero phase error tracking controller (ZPETC) [8], the system bandwidth can be increased, the contouring performance of the feed drive axes under disturbances, e.g., friction, is not directly addressed. Since the coordinated responses of all the feed drive axes are necessary to maintain good dimensional accuracy when the system is subjected to disturbances, coordination control of the machine tool feed drive system can provide a mean to improve the overall dimensional accuracy of the workpiece. In this paper, coordinated control algorithms are implemented on an industrial CNC machine tool feed drive system to demonstrate the possible improvement in contouring performance over the existing servo control.

In most machine tool feed drive systems, the servo controllers are usually individually designed for each of the motion axes. This results in a collection of decoupled single input and single output (SISO) systems. Decoupled design may be preferable if the disturbance in one axis will not affect the performance of other axes. For contouring applications, however, decoupling is sometimes damaging to the overall performance objective. For example, when one axis of the feed drive system under decoupling control is subject to a disturbance, other axes will perform as if the disturbed axis is functioning normally. The result is a loss of coordination and degradation of the overall contouring accuracy. Fig. 1 shows the axial tracking errors and contouring errors of two contouring experiments, both tracking a circular contour of radius 10 mm. As can be seen in Fig. 1, case I has the smaller axial tracking errors but case II has the smaller contouring error. A better way for controlling machine tool feed drive systems is to introduce “intelligent” coupling actions in the controller so that the coordination objective of the desired contour is maintained during the transient and disturbance recovery process as well as at steady state. Since in actual operation, motion axes are constantly under both internal and external disturbances, the ability to maintain coordination under these circumstances is imperative to the overall system performance.

The characterizations above suggest that the coordinated motion controller will be multivariable and trajectory dependent. The design process should be closely related to the desired contour, since the coordination objective of the desired motion needs to be addressed explicitly. Motivated to improve the contouring performance of machine tool feed drive systems, many researchers have studied the motion coordination problem as early as 1950s.

Poo et al. [9] studied the dynamic errors in contouring systems and concluded that contouring performance can be improved by applying control to each axis so that the dynamic characteristics of all of the axes are similar to each other. Bin et al. [10] arrived at the same conclusion when investigating...
microprocessor-based control schemes to improve contouring accuracy. Other researchers have approached the problem from a trajectory planning point of view. Butler and Tomizuka [11], [12] have shown that contouring performance of CNC feed drive systems can be improved by taking into account the dynamics of the axes when planning the command trajectories for each axis. Imamura and Kaufman [13] formulated an feedrate optimization problem with explicit contouring error tolerance in the cost function. Chou and Yang [14], [15] proposed a systematic approach to generating desired reference trajectories for multi-axis CNC machines or coordinate measuring machines (CMMs) for coordinated motions. The approaches discussed above assumed the feed drive axes are decoupled, and they focused in generating the desired reference trajectories to each axis to improve the overall contouring performance. These are essentially open-loop approaches where the reference generation are done off-line. The contouring objective is explicitly considered in the problem formulation but lacks the necessary coupling effect to ensuring contouring performance under the ever present system uncertainties and disturbances.

By introducing coupling effects among multiple axes, coordinated motion can be achieved by either the “equal-status” approach or the “master–slave” approach [16]. In the equal-status approach, the coordination controller treats multiple axes in a similar manner without favoring one axis over the others. When the dynamics are significantly different among multiple axes, the equal-status approach may not be the best because the controller may saturate the slow axis actuator in attempting to improve its response and under-utilize the fast axis actuator. In a two-axes problem with significantly different dynamics, it will make more sense to consider the master–slave approach. In this case, the slow axis acts as the master for the fast axis. The fast axis is the slave and follows the slow axis. In 1957, Sarachik and Ragazzini [17] first treated the problem using the “master–slave” approach. In 1980, Koren [18] introduced the cross-coupled controller which took the “equal-status” viewpoint to address contouring performance in two-axis feed drive systems. Kulkarni and Srinivasan [19], [20] have also investigated the cross-coupled compensator scheme in detail. They introduced an optimal version in 1989 [21]. Recently, Tomizuka et al. [22] added an adaptive feedforward scheme to the cross-coupled compensator to improve transient response and disturbance rejection.

A limitation of the approaches described above is that the coordination objective is linear; that is, the desired goal is a linear relationship among the multiple axes. For example, in an $X$-$Y$ table positioning system, a linear coordination objective is to maintain $x = k_y$ where $k$ is a constant. Srinivasan and Fosdick [23] have proposed a multivariable analysis approach to motion coordination, but their extension makes the assumption that the desired contour is a combination of piecewise linear segments and the resulting controller is similar to gain scheduling. Recently, Koren and Lo [24]–[26] introduced a variable-gain cross-coupling controller for a general class of contours. However, the effect of a time-varying gain in the cross-coupling controller to system stability and the effect of introducing time-varying cross-coupling controller on overall system dynamics are yet to be examined. One main difficulty in analyzing the cross-coupling control system for general (nonlinear) contour is the fact that the cross-coupling gains are time-varying (changing with the desired contour). It is not trivial to incorporate time-varying parameters in a transfer function formulation of an coupled multivariable system. Recently, there are efforts to examine the stability and robustness of the cross-coupled control system [27].

Using implicit representation of curves to model desired contours, Chiu and Tomizuka [28] extended contour tracking to general $C^2$ curves. The limitation of this approach lies in the fact that the desired contour needs to be represented in an analytical form. McNab and Tsao [29] approached the problem using a receding horizon linear quadratic formulation and the resulting controller required extensive on-line calculation.

In this paper, we will formulate the contour tracking problem in a “task” coordinated frame that is attached to the desired contour. Without the need for an analytical contour representation, the coordinate transformation requires the knowledge of the feedrate, velocity direction, and instantaneous curvature for the desired contour. This information is calculated in the existing upstream processes, as the CAD and tool path generation. By transforming the machine tool feed drive dynamics to this task coordinate frame, a control law can be designed to assign different dynamics to the normal and tangential directions relative to the desired contour. The approach is similar to those used in constraint robot motion control [30], [31]. It is also shown that for a general contour, the transformed dynamic is trajectory-dependent. The time varying geometric property of the desired trajectory directly affects the error dynamics in the task coordinate frame. It easily explains the degradation of performance under high speed (feedrate) and/or during sharp corner (large curvature) applications when a linear time-invariant approach is used to design the contouring controller. The effectiveness of the resulting control law is demonstrated on the feed drive system of a Matsuura MC510V high-speed machining center. The rest of the paper is organized as follows: problem formulation and the design and analysis of the proposed control law will be given in...
II. CONTOURING IN THE TASK COORDINATE FRAME

A. Problem Formulation

In this section we will treat the position control of a three axis machine tool feed drive system. First, we will introduce the “task” coordinate frame and the corresponding transformation properties. Then, the error dynamics of the positioning system in the physical (fixed) coordinate frame will be derived and transformed into the task coordinate frame.

A desired contour for the feed drive system is given by

\[ x_{pd}(t) = \begin{bmatrix} x_{pd1}(t) \\ x_{pd2}(t) \\ x_{pd3}(t) \end{bmatrix} \]

which represents a regular curve in the \( \mathbb{R}^3 \) space [32]. The time derivative of \( x_{pd} \), denoted by \( \dot{x}_{pd} \), is referred to as the desired velocity vector. Combining \( x_{pd} \) and \( \dot{x}_{pd} \), we can define

\[ x_{d}(t) = \begin{bmatrix} x_{pd}(t) \\ \dot{x}_{pd}(t) \\ \ddot{x}_{pd}(t) \end{bmatrix} \]

Given \( x_{pd}(t) \) and \( x_{d}(t) \), the unit tangent vector, \( t(t) \), is defined as

\[ t(t) = \frac{x_{d}(t)}{||x_{d}(t)||} \]

The curvature vector at \( x_{pd}(t) \), \( k(t) \), can be represented by

\[ k(t) = \kappa(t)n(t) \]

where \( \kappa(t) \) is the curvature of the contour at \( x_{pd}(t) \) and \( n(t) \) is the unit normal vector. It is easy to show that \( t(t) \) and \( n(t) \) are mutually orthogonal. By defining the binormal vector, \( b(t) = t(t) \times n(t) \), we can construct a moving coordinate frame \( \{t(n), n(t), b(t)\} \) in \( \mathbb{R}^3 \) that is attached to the desired contour \( x_{pd}(t) \) and changes its orientation at different points on the curve, see Fig. 2. Let \( E \) denote the physical coordinate frame composed of \( \{e_1, e_2, e_3\} \). In this frame, \( x \) is expressed in terms of the vectors \( \{t, n, b\} \). Any vector in \( \mathbb{R}^3 \) with representation \( x \) with regard to \( E \) will have a different representation \( x_F \) with regard to \( F \). Let

\[ F = [t \times n \times b] \]

be a \( 3 \times 3 \) matrix, where \( t \), \( n \) and \( b \) are represented with regard to the \( E \) frame. Then the coordinate transformation:

\[ x_F = F^T x \]

\[ x = Fx_F \]

holds. Note that \( F \) is unitary, i.e., \( F^{-1} = F^T \). Since \( F \) is a moving coordinate frame that moves along the desired contour \( x_{pd}(t) \), the corresponding transformation matrix \( F(t) \) is a function of time. It can be shown that the time derivative of \( F(t) \) is

\[ \frac{d}{dt} F^T(t) = v(t)R(t)F^T(t) \]

where \( v(t) = ||x_{pd}(t)|| \) is the desired feedrate and

\[ R(t) = \begin{bmatrix} 0 & \kappa(t) & 0 \\ -\kappa(t) & 0 & \tau(t) \\ 0 & -\tau(t) & 0 \end{bmatrix} \]

In the above equation, \( \kappa \) and \( \tau \) are the curvature and torsion that are unique to the desired contour, respectively. Detailed discussion of the above material can be found in [32] and [33].

Taking time derivative of (2) and using (4) we can obtain

\[ \frac{d}{dt} x_F = \frac{d}{dt} F^T x + F^T \frac{d}{dt} x = vRF^T x + F^T \frac{d}{dt} x \]

Combining this with (2) we have the following:

\[ \begin{bmatrix} x_F \\ \frac{d}{dt} x_F \end{bmatrix} = \begin{bmatrix} F^T & 0 \\ vR & F^T \end{bmatrix} \begin{bmatrix} x \\ \frac{d}{dt} x \end{bmatrix} \]

Consider the position coordination of a 3 axes feed drive system with the following equation of motion:

\[ \dot{x} = Ax + Bu \]

where

\[ x = \begin{bmatrix} x_p \\ x_v \end{bmatrix}, \quad A = \begin{bmatrix} 0 & I & 0 \\ A_p & A_v \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ B_v \end{bmatrix} \]

In (7), \( x_p, x_v \) are the \( 3 \times 1 \) position and velocity vectors, respectively. \( A_p, A_v, \) and \( B_v \) are \( 3 \times 3 \) matrices. Without loss of generality, we will assume that matrix \( B \) has full rank, i.e., \( B_v \) has rank 3. The error vector \( e(t) \) is defined as

\[ e(t) = \begin{bmatrix} e_p(t) \\ e_v(t) \end{bmatrix} = x_d(t) - x(t) = \begin{bmatrix} x_{pd}(t) - x_{pd}(t) \\ x_{d}(t) - x_{d}(t) \end{bmatrix} \]

where \( e_p \) and \( e_v \) are referred to as the position error and velocity error, respectively.

The error dynamics of the system can be obtained by differentiating \( e \) with regard to time \( t \). Using (6), it can be shown that

\[ \dot{e} = Ae - Bu + w \]
where \( \mathbf{w}_d = (\dot{\mathbf{x}}_d - A\dot{\mathbf{x}}_d) \). Equation (9) is the error dynamics represented in the fixed (physical) coordinate frame.

The position error vector represented in the task coordinate frame, denoted by \( \mathbf{e}_p(t) \), is related to \( \mathbf{e}_p(t) \) through the transformation matrix \( \mathbf{F}(t) \)

\[
\mathbf{e}_p(t) = \mathbf{F}(t)\mathbf{e}_p(t).
\]

By construction, the first component of \( \mathbf{e}_p(t) \) is the tangential component of the error vector and the remaining components constitute the normal components of the error vector. From (5), we see that

\[
\begin{bmatrix}
\mathbf{e}_p(t) \\
\dot{\mathbf{e}}_p(t)
\end{bmatrix} =
\begin{bmatrix}
\mathbf{F}(t) & 0 \\
\dot{\mathbf{F}}(t) & \mathbf{F}(t)
\end{bmatrix}
\begin{bmatrix}
\mathbf{e}_p(t) \\
\dot{\mathbf{e}}_p(t)
\end{bmatrix}.
\]

By defining \( \mathbf{e}_p(t) \equiv \dot{\mathbf{e}}_p(t) \) and letting

\[
\mathbf{e}_p(t) =
\begin{bmatrix}
\mathbf{e}_p(t) \\
\dot{\mathbf{e}}_p(t)
\end{bmatrix}
\]
and

\[
\mathbf{T}(t) =
\begin{bmatrix}
\mathbf{F}(t) & 0 \\
\dot{\mathbf{F}}(t) & \mathbf{F}(t)
\end{bmatrix}
\]

the error vector represented in the task coordinate frame \( \mathbf{e}_T(t) \) is related to \( \mathbf{e}(t) \) by the trajectory dependent transformation matrix \( \mathbf{T}(t) \)

\[
\mathbf{e}_T(t) = \mathbf{T}(t)\mathbf{e}(t).
\]

For simplicity, we will not explicitly write out the time \( t \) in the following derivation. Since \( \mathbf{F} \) is unitary, \( \mathbf{T} \) is invertible and

\[
\mathbf{T}^{-1} = 
\begin{bmatrix}
\mathbf{F} & 0 \\
\mathbf{F} & \mathbf{F}
\end{bmatrix}.
\]

The time derivative of \( \mathbf{T} \) can be written as

\[
\frac{d}{dt}\mathbf{T} = (\alpha\mathbf{R}_{2n} + \Delta)\mathbf{T} \tag{11}
\]

where \( \mathbf{R}_{2n} \) and \( \Delta \) are defined as

\[
\mathbf{R}_{2n} = 
\begin{bmatrix}
\mathbf{R} & 0 \\
0 & \mathbf{R}
\end{bmatrix}
\]
and

\[
\Delta = 
\begin{bmatrix}
0 & 0 \\
\frac{d}{dt}(\alpha\mathbf{R}) & 0
\end{bmatrix}.
\]

For a regular contour \( \mathbf{x}_d(t) \), both \( \mathbf{T}(t) \) and \( \dot{\mathbf{T}}(t) \) are continuous and bounded. Also note that \( \det\mathbf{T}(t) = \det\mathbf{F}(t)\det\mathbf{F}(t) = 1 \) for all \( t \). Therefore, \( \mathbf{T}(t) \) is a Lyapunov transformation that preserves the stability properties of a linear system [34]. The stability analysis can be discussed either in the fixed coordinate frame or in the task coordinated frame.

To obtain the error dynamics in the task coordinate frame, differentiating \( \mathbf{e}_T \), we have

\[
\dot{\mathbf{e}}_T = \dot{\mathbf{T}}\mathbf{e} + \dot{\mathbf{T}}\mathbf{e} = (\mathbf{R}_{2n} + \Delta)\mathbf{T} + \mathbf{A}(\mathbf{e} - \mathbf{B}u + \mathbf{w}_d) \tag{12}
\]

Note that in (12), matrices \( \mathbf{T}, \mathbf{vR}_{2n} \), and \( \Delta \) are trajectory dependent. The error dynamics of the system represented in the task coordinate frame is linear but time-varying. From (12) we notice that in addition to the similarity transformation of the system matrix \( \mathbf{A} \) in (9), the derivatives of the trajectory dependent coordinate transformation, namely \( \mathbf{R}_{2n} \) and \( \Delta \), will also affect the error dynamics in the task coordinate frame.

In contour tracking, we would like to specify different dynamic characteristics for the tangential component and the normal components of the error vector. This can be achieved if a desired system matrix \( \mathbf{A}_{T_d} \) is assigned in place of \( (\mathbf{TAT}^{-1} + \mathbf{vR}_{2n} + \Delta) \) in (12) and the term \( \mathbf{Tw}_d \) is compensated. The contour tracking problem in the task coordinate frame can then be stated as: given a three axes machine tool feed drive system with the equation of motion, (12), and a desired trajectory \( \mathbf{x}_d(t) \), find a stabilizing control law that achieves a desired error dynamics \( \mathbf{A}_{T_d} \) in the task coordinate frame. In the next section we will construct a control law that solves this contour tracking problem.

B. Controller Design and Analysis

In this subsection we will first construct a control law that solves the contour tracking problem in the task coordinate frame. The control law will be transformed back to the fixed coordinate frame for implementation.

The desired error dynamics represented in the task coordinate frame is

\[
\dot{\mathbf{e}}_T = \mathbf{A}_{T_d}\mathbf{e}_T. \tag{13}
\]

For contour tracking, we would like to decouple the tangential and normal error dynamics and focus more on the normal error. Equating (13) and (12), we have

\[
\mathbf{A}_{T_d}\mathbf{e}_T = (\mathbf{TAT}^{-1} + \mathbf{vR}_{2n} + \Delta)\mathbf{e}_T - \mathbf{TB}u + \mathbf{Tw}_d. \tag{14}
\]

Since \( \mathbf{B} \) has rank 3, \( \mathbf{B}^+ = (\mathbf{B}^T\mathbf{B})^{-1}\mathbf{B}^T \) exists. Multiplying \( \mathbf{T}^{-1} \) from the left to both sides of the above equation and solve for \( \mathbf{u} \), we have

\[
\mathbf{u} = \mathbf{B}^T\mathbf{T}^{-1}\mathbf{K}\mathbf{e}_T + \mathbf{B}^T\mathbf{w}_d \tag{14}
\]

where \( \mathbf{K} = (\mathbf{TAT}^{-1} + \mathbf{vR}_{2n} + \Delta - \mathbf{A}_{T_d}) \) is the feedback gain matrix in the task coordinate frame. Recall that \( \mathbf{e}_T \) consists of \( \mathbf{e}_p \) and \( \dot{\mathbf{e}}_p \), hence the resulting control law is a time-varying proportional-plus-integral (PD) controller in the task coordinate frame with desired feedforward compensation \( \mathbf{w}_d \). The time-varying characteristic comes from the effect of the trajectory dependent coordinate transformation, \( \mathbf{T}, \mathbf{R}_{2n}, \) and \( \Delta \).

To transform the control law, (14), to the physical coordinate frame, substituting \( \mathbf{e}_T = \mathbf{T}\mathbf{e} \) into (14)

\[
\mathbf{u} = \mathbf{B}^T(\mathbf{K}\mathbf{e} + \mathbf{w}_d)
\]

where the feedback gain \( \mathbf{K} \) is defined by

\[
\mathbf{K} = (\mathbf{T}^{-1}\mathbf{R}_{2n} + \mathbf{T}^{-1}\Delta - \mathbf{T}^{-1}\mathbf{A}_{T_d}). \tag{15}
\]

From (14) and the above equation, we see that

\[
\mathbf{K} = \mathbf{T}^{-1}\mathbf{K}_T\mathbf{T}.
\]
Hence, a control law that solves the position coordination problem stated in the previous section is

$$u(t) = B^+(K(t)e + w_d(t)) \tag{16}$$

Since the control law, (16), will achieve the desired error dynamics in the task coordinate frame, namely $\dot{e}_T = \mathbf{A}_{T,d} e_T$, the stability of the closed-loop system is insured if $\mathbf{A}_{T,d}$ is chosen to be stable ($\mathbf{T}$ is a Lyapunov transformation, [34]). Fig. 3 shows the block diagram of the proposed control law.

The contour tracking control law in (16) consists of a time-varying PD controller $K$ and a feedforward compensation term $w_d$. The first and last terms of $K$ in the right-hand side (RHS) of (15) cancel the original system dynamics and insert the desired dynamics, respectively. The middle two terms compensate for the effect of trajectory dependent coordinate transformation that shows up in the task coordinate frame. These two terms are not incorporated in the cross-coupling control design [24], [26] where the trajectory dependent effects of the system dynamics in the task coordinate frame are not taken into consideration. Without compensating for these trajectory dependent terms, the tangential and normal dynamics will still be coupled. The effect of the coupling will be amplified during high-speed (high-feedrate) and/or sharp corner operations. This has been demonstrated in recent work of Lacerda and Belo [35], where they demonstrated that under high feedrate, without taking into account of the curvature and feedrate, the estimated contouring error used in the cross-coupled controller can not accurately approximate the actual contouring error. This will explain the degradation in performance for cross-coupling controller at high feedrate and large curvature applications. The second term in the RHS of (15), $\mathbf{A}_{T,d} e_T$, contains the information of the geometric properties of the desired trajectory. As discussed before, $v(t) = [x_{\text{goal}}(t)]$ is the feedrate along the desired contour at time $t$, and $R_{2m}$ represents the rate of change of the basis of the task coordinate frame. In $\mathbb{R}^3$, $R_{2m}$ includes the curvature $\kappa(t)$ and the torsion $\tau(t)$ of the contour at time $t$. In order to specify the systems error dynamics in the task coordinate frame, the control has to account for the desired feedrate and the changes in the geometric profile of the curve. For example, trajectories with sharp corners (large curvature) and/or tracking at high speeds requires larger magnitude of control action. Since it is the product of $v$ and $R_{2m}$ that appears in the control gain $K$, for large $R_{2m}$, e.g., sharp corners, the desired speed $v$ should be kept small. This is consistent with the common practice that with finite control action, to achieve good contouring performance near sharp corners, the corresponding speed should be kept small. It also explained that if the controller design does not consider the trajectory dependednt effect in the contouring direction, the result will not be desirable under high feedrate (large $v$) and/or sharp corners (large $R_{2m}$).

In the proposed control law (16), the design parameter is the matrix $\mathbf{A}_{T,d}$, which specifies the desired error dynamics in the task coordinate frame. Since the error vector $e_T$ is defined to be $[e_{p,T}^T, e_{n,T}^T]^T$, the matrix $\mathbf{A}_{T,d}$ can take the form

$$\mathbf{A}_{T,d} = \begin{bmatrix} 0_{3\times 3} & \mathbf{I} \\ -\mathbf{A}_{p,F} & -\mathbf{A}_{n,F} \end{bmatrix} \tag{17}$$

where $\mathbf{A}_{p,F}$ and $\mathbf{A}_{n,F}$ are $3 \times 3$ matrices. The matrix $\mathbf{A}_{T,d}$ with the structure defined above specifies a second-order error dynamics

$$\dot{e}_{p,F} + \mathbf{A}_{p,F} \dot{e}_{p,F} + \mathbf{A}_{p,F} e_{p,F} = 0.$$ 

Recall that the first component of $e_{p,F}$ is the tangential error component and the remaining components constitute the normal error. To specify different dynamics for the tangential and normal errors, $\mathbf{A}_{p,F}$ and $\mathbf{A}_{n,F}$ can have the following structure:

$$\mathbf{A}_{p,F} = \begin{bmatrix} a_{p,t} & 0_{1 \times 2} \\ 0_{2 \times 1} & \mathbf{A}_{p,n} \end{bmatrix} \quad \text{and} \quad \mathbf{A}_{n,F} = \begin{bmatrix} a_{n,t} & 0_{1 \times 2} \\ 0_{2 \times 1} & \mathbf{A}_{n,n} \end{bmatrix}$$

where $\mathbf{A}_{p,n}$ and $\mathbf{A}_{n,n}$ are $2 \times 2$ matrices. The pairs $(A_{p,F}, A_{n,F})$ specify the dynamics in the tangential and normal directions, respectively.

III. EXPERIMENTAL RESULTS

In this section, we will present experimental results applying the control law designed in the previous section to an experimental feed drive system on a Matsuura MC510V machining center.

Fig. 4 shows the experimental setup. The feed drive system for each of the $X$ and $Y$ axes consists of a three-phase ac servo...
motor, a 12 mm/pitch leadscrew, and the table. The existing feed drive servo for each axis uses an Yaskawa SR15-AF servo pack that provides digital commutation for the ac motor, current amplification, analog proportional plus integral (PI) velocity feedback control and digital proportional (P) position control with 2 ms sampling period. Position sensing is provided by a 6000 count/revolution optical shaft encoder which translates to a linear resolution of 1 μm for the X-Y table. A frequency-to-voltage converter provides the analog velocity measurement. In the experiment, the digital position loop and the analog I action is replaced by a 486 computer. Position measurement is obtained through a quadrature decoder board and the control signal is send through a 16-bit D/A interface board to the servo amplifier. The block diagram of one of the feed drive axes is shown in Fig. 5. To make meaningful comparisons, the experimental controller is also running at 2-ms sampling period.

Frequency response of the X and Y axes of the experimental feed drive system from the control input to the velocity are shown in Figs. 6 and 7, respectively. First-order models were used to fit the experimental data. The resultant transfer functions from the input voltage to the output velocity (m/s) for the X and the Y axes are $1.9779/(s+62.8319)$ and $1.3196/(s+45.5531)$, respectively. These transfer functions, augmented with an integrator, 1/s, are the plant models used in the controller design.

Following Poo et al. [9], circular trajectories are used in this study to examine the performance of the proposed control algorithm. One advantage of using circular trajectories is that the effectiveness of the control algorithm to disturbances can be examined by its ability to reduce quadrant glitches (caused by stiction and friction when one of the feed drive axes’ velocity changes sign). Another advantage is that the contouring error can be exactly computer by $\sigma_C = R - \sqrt{(x-x_c)^2 + (y-y_c)^2}$ where $x_c$ and $y_c$ defines the center of the circle and $R$ is the radius. The test contour is a 20-mm diameter circular contour with desired feedrate of 3 m/min and 6 m/min.

Since the Matsuura X-Y feed drive system is a two-dimensional (2-D) contouring system, the parameterization based on the feed direction, see Fig. 8, can be used to represent the coordinate transformation matrix $\mathbf{F}(t)$. Let $\theta(t)$ be the incline angle between the desired velocity vector $\mathbf{x}_{vel}(t)$ and the X axis, then $\mathbf{F}(t)$ can be represented by

$$
\mathbf{F}(t) = \begin{bmatrix}
\cos \theta(t) & -\sin \theta(t) \\
\sin \theta(t) & \cos \theta(t)
\end{bmatrix}.
$$

For a straightline trajectory the matrix $\mathbf{R}$ is the zero matrix. For a circular contour

$$
\mathbf{R} = \begin{bmatrix}
0 & \frac{1}{R} \\
-\frac{1}{R} & 0
\end{bmatrix}
$$

where $R$ is the radius. For a general 2-D trajectory, $\mathbf{R}$ is a matrix function of the incline angle $\theta$. In addition, for a circular trajectory with constant feedrate, the matrix $\Delta$ in the proposed control law (16) is zero.

The desired system matrix $\mathbf{A}_{tel}$ is designed as

$$
\mathbf{A}_{tel} = \begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
-a_{pt} & 0 & -a_{et} & 0 \\
0 & -a_{pt} & 0 & -a_{et}
\end{bmatrix}
$$

which specifies the following second-order tangential and normal dynamics:

**Tangential:**

$$
(s^2 + 2(2\pi f_l)\alpha_{et}s + (2\pi f_l)^2)\epsilon_t = 0
$$

**Normal:**

$$
(s^2 + 2(2\pi f_l)\alpha_{pt}s + (2\pi f_l)^2)\epsilon_t = 0
$$
In the above equations, $f_t$ and $f_n$ are the undamped natural frequencies, i.e., bandwidths, for the tangential and normal dynamics, respectively. The parameters $\zeta_t$ and $\zeta_n$ are the corresponding damping coefficients. In the following experiments, we will use $f_t$ and $f_n$ as the major design parameters and set both $\zeta_t$ and $\zeta_n$ to be one, i.e., critically damped dynamics.

Although there are many criterions for evaluating control system performance, for machine tool contouring applications, the maximum and the average contouring errors are the main contributors to the quality of the final part. Hence, in the subsequent discussion, the maximum absolute contouring error $\|\mathbf{e}_c\|_{\text{MAX}}$ and the average contouring error $\bar{\mathbf{e}}_c$ will be used to compare the performance of the proposed control algorithm. The root mean square (rms) value of the $X$ and $Y$ axes control inputs, $U_{\text{X RMS}}$ and $U_{\text{Y RMS}}$, will be used to document the control efforts.

Fig. 9 shows the contouring errors for the circular part of the 3 m/min feedrate trajectory when $f_t$ was fixed at 5 Hz and $f_n$ increased from 5 to 45 Hz. As expected, when the normal dynamics became more stiff, the average contouring error reduced from 62.19 $\mu$m to 7.00 $\mu$m while the maximum contouring error reduced from 176.43 $\mu$m to 12.06 $\mu$m, respectively. As can be seen in Fig. 10, with the tangential bandwidth fixed, the tangential errors remain at about the same level of magnitude. On the other hand, the normal errors decrease as the normal bandwidth increases. Although large $f_t$ and $f_n$ can achieve good tracking and contouring performance, due to the effect of sampling, quantization and unmodeled dynamics, increasing both $f_n$ and $f_t$ to higher than 20 Hz causes chattering and instability. However, maintaining $f_t$ at 20 Hz, we can further increase $f_n$ and improve the contouring performance without inducing instability. As can be seen in Figs. 11 and 12, by increasing $f_n$ from 20 to 45 Hz, the maximum absolute and the average contouring error are reduced to 3.43 $\mu$m and 0.006 $\mu$m, respectively. The tangential errors remain at the same level of magnitude.

Fig. 13 and Table I summarize the performance of the proposed approach for the feedrate of 3 m/min. Similar improvement in contouring performance can be observed for tracking a circular trajectory at 6 m/min, see Figs. 14 and 15. When $f_t$ was set to 20 Hz and $f_n$ was increased from 20 to 45 Hz, the corresponding maximum absolute and average contouring error reduced to 7.61 and 1.29 $\mu$m, respectively.
IV. CONCLUSION

In this paper we formulated the contouring control problem in the task coordinated frame. By transforming the error dynamics of a machine tool feed drive system to the task coordinated frame, a controller was designed to assign different dynamics to the tangential and normal components of the error vector. The proposed control law consists of a linear time-varying PD error feedback term and a linear time invariant trajectory feedforward compensation. The controller can be designed to focus more on the normal components of the error vector by making

Table II summarizes the results for the 6 m/min feedrate experiments.

Fig. 12. Tangential and normal error at 3 m/min feedrate with $f_t = 20$ Hz.

Fig. 13. Contouring error at 3 m/min feedrate—comparison with existing servo.

Fig. 14. Contouring error at 6 m/min feedrate with $f_t = 20$ Hz.

Fig. 15. Tangential and normal error at 6 m/min feedrate with $f_t = 20$ Hz.
the normal dynamics of the error more stiff than that of the tangential dynamics. Implementation of the proposed control law on the Matsuura feed drive system confirmed that the tangential and normal error dynamics can be selectively manipulated. Experimental results showed improvements in contouring performance over the existing servo controller.

Methods to incorporate the robustness of the proposed control algorithm to model uncertainties and external disturbances in under investigation. Additional experiments are planned including three-dimensional contouring and actual machining operation.

REFERENCES


TABLE II

| $f_t$ | $e_C$ | $||e_C||_{MAX}$ | $e_C$ | $||e_C||_{MAX}$ | $e_n$ | $||e_n||_{MAX}$ | $f_t$ | $U_{RMS}$ | $U_{RMS}$ |
|-------|-------|-----------------|-------|-----------------|-------|-----------------|-------|-----------|-----------|
| 20 Hz | 28.3  | 9.31            | 28.27 | 9.27            | 43.82 | 25.04           | 8.25  | 8.25      |           |
| 45 Hz | 7.81  | 1.29            | 7.63  | 1.25            | 45.81 | 27.05           | 8.22  | 8.22      |           |


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